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Math Matters

A Brief Look at How Mathematics Has Influenced Modern Life

Mathematics Matters in the Modern World

In 2003, the National Academy of Engineering (USA) published *A Century of Innovation: Twenty Engineering Achievements that Transformed Our Lives*. This book celebrates the top twenty technological advances of the twentieth century that fundamentally changed our lives. These advances have influenced where and how we live, what we eat, what we do for work or leisure, and even how we think about our world and the universe.

Mathematics has played a major role in bringing these innovations to reality. Many mathematical theories and models of real world problems have helped scientists and engineers grapple with seemingly impossible tasks. Today, mathematical techniques reach even further into our society. In addition to making technology more efficient and effective, mathematical techniques help organizations deal with financial, manufacturing, and even marketing issues.

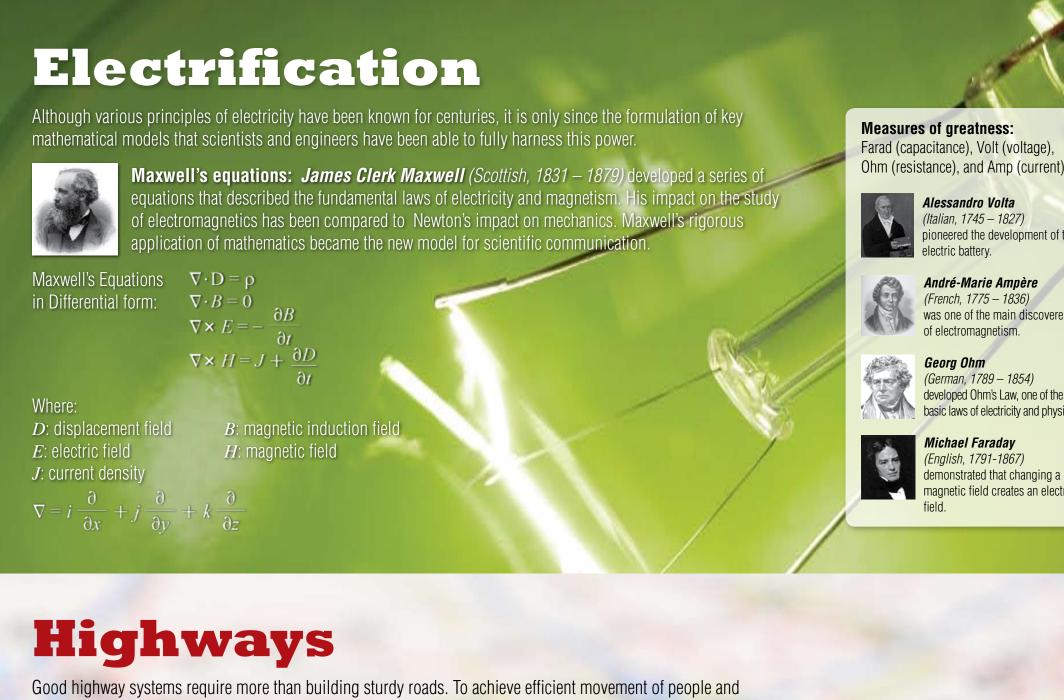This poster is a tribute to the National Academy of Engineering as well as the men and women who have focused their brilliance to transform the modern world. The poster is a mosaic of the ways mathematics helps us utilize and benefit from these great technological achievements. Some achievements will be familiar. Some will be a surprise. All, hopefully, will encourage you to investigate these topics further.

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Electrification

Although various principles of electricity have been known for centuries, it is only since the formulation of key mathematical models that scientists and engineers have been able to fully harness this power.

Maxwell's equations: James Clerk Maxwell (Scottish, 1831 – 1879) developed a series of equations that described the fundamental laws of electricity and magnetism. The impact on the theory of electromagnetics has been compared to Newton's impact on mechanics. His mathematical analysis of mathematics became the new model for scientific communication.

Measures of greatness:
Farad (capacitance), Volt (voltage), Ohm (resistance), and Amp (current)

Alessandro Volta (Italian, 1745 – 1827) pioneered the development of the electric battery.

André-Marie Ampère (French, 1775 – 1836) was one of the main discoverers of electromagnetism.

Georg Ohm (German, 1789 – 1854) developed Ohm's Law, one of the basic laws of electricity and physics.

Michael Faraday (English, 1791 – 1867) demonstrated that changing a magnetic field creates an electric field.

Highways

Good highway systems require more than building sturdy roads. To achieve efficient movement of people and goods, scientists and engineers develop and apply a wide range of mathematical techniques.

Operations Research (OR) was developed by the military. Its goal was to develop a mathematical framework for solving complex logistics problems. Through techniques called mathematical programming, optimization, and others, engineers can design highway networks that balance transportation efficiency with resource constraints.

One of the first traffic problems:

Leonard Euler (Swiss, 1707 – 1783) introduced a famous math problem called the Seven Bridges of Königsberg – how do you cross these seven bridges without crossing over any bridge more than once?

To analyze this problem, Euler introduced graph theory, which represents connected elements (like places connected by bridges), as points and connects them with lines. Graph theory is an important mathematical technique for designing networks of fun named and routing networks of highways.

Antoine Lavoisier (French, 1743 – 1794) who is considered the "father of modern chemistry," discovered the element silicon. Silicon is the main component in semiconductor devices. Lavoisier also introduced the Metric System of measurement.

Electronics

The invention of the transistor in 1947 at Bell Labs launched the electronics age. This small, three-terminal device allows current to flow between two of its terminals depending on the current or voltage applied on the third. It can be used for amplification, switching, modulation, and many other tasks that had only been possible by large, less robust vacuum tubes or mechanical relay switches. Over time, transistors became further miniaturized within integrated circuits (chips).
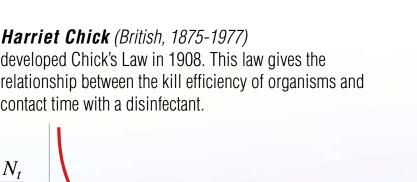
Sample mathematics of a Bipolar Junction Transistor (Ebers-Moll model):

Emitter current
$$I_E = I_{ES}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

Collector current
$$I_C = \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

Base-internal current
$$I_B(base) = \frac{qD_p}{L_p} \cdot \frac{p_n}{N_D} \cdot \frac{1}{e}$$

Where:
α_F is the common base forward short circuit current gain (0.98 to 0.998)
I_{ES} is the reverse saturation current of the base-emitter diode (on the order of 10^{-15} to 10^{-12} amperes)
V_T is the thermal voltage (approximately 26 mV at room temperature = 300 K)
V_{BE} is the base-emitter voltage

Water Supply & Distribution

The protection of public health from the transmission of waterborne diseases by disinfecting water has been recognized since the early 1900's. Of the existing processes used to produce safe drinking water, chlorination of public water supplies ranks as one of the most important.

Harriet Chick (British, 1875 – 1977) developed Chick's Law in 1908. This law gives the relationship between the kill efficiency of organisms and contact time with a disinfectant.

Chick's Law was later modified by H.E. Watson (1908) to include the coefficient of specific lethality (λ). The Chick-Watson Equation is shown below.

$$\ln\left(\frac{N_t}{N_0}\right) = -\lambda C_n t$$

Where:
N_0 = initial number of organisms
N_t = number of organisms at time t
C = concentration of disinfectant (mg/L)
t = contact time (min.)
λ = coefficient of specific lethality
n = coefficient depending on disinfectant type and pH

Household Appliances

Remarkable technology enters our home via household appliances. Microwave ovens, vacuum cleaners, and dishwashers are all direct descendents of the scientific and mathematical research done decades, and even centuries, earlier.

Statistical Process Control (SPC): William Edward Deming (American, 1900 – 1993) was a leading force in the 19th century market called *Positivism*, which saw mathematics and statistics as valuable tools in making decisions about people and society. Today we experience his legacy in business practices that use mathematics to model consumer behavior and determine the right marketing tactics for persuading us to buy household appliances and other items.

SPC uses statistical techniques to measure and manage the quality of manufactured consumer items such as household appliances. Innovative use of classical mathematics has dramatically increased factory productivity and lowered prices.

Measuring the market: Auguste Comte (French, 1798 – 1857) was a leading force in the 19th century movement called *Positivism*.

Petroleum & Petrochemicals

Robert Brown (Scottish, 1773 – 1858) is the botanist in honor of whom Brownian motion was named.

Louis Jean-Baptiste Alphonse Bachelier (French, 1870 – 1946) is credited with being the first person to model Brownian motion.

The importance of petroleum to the global economy cannot be overstated. The price of petroleum has a large impact on global growth, and in the long term, the greatest single factor influencing petroleum prices is the cost of crude oil. As such, its trading activity is closely watched and routinely scrutinized.

The first successful attempt to model speculative prices was made by Louis Bachelier in his 1900 Ph.D. thesis "The Theory of Speculation". Bachelier used stochastic processes such as Brownian motion to model the market noise of the Paris Bourse.

Geometric Brownian Motion:

With time measured in trading days from initial purchase, $\mu S_t dt$ is the rate of the predicted change, $\sigma S_t dz$ is the standard deviation and dz is the increment of Brownian motion.

Almost 60 years later, Bachelier's ideas were taken up with significant effect by the Nobel prize-winning economist **Paul Anthony Samuelson** (American, 1915), who argued that discounted future prices must fluctuate randomly. His work provided the basis of what is now known as the "efficient market hypothesis", which caused a revolution in empirical finance.

Today, stochastic processes are used to model a wide range of financial activity from stock prices, interest rates and commodity prices.

Airplanes

Navier-Stokes equations are partial differential equations that describe motions of fluids (liquids and gases). In addition to airflow analysis for airplanes, they are useful for modeling weather, ocean currents and many other systems. A simplified form of these equations within a Cartesian frame of reference is as follows:

These equations establish the relations among the fluxes of the variables of interest. By solving these equations, you can describe the motion of fluid substances such as liquids and gases.

Claude-Louis Navier (French, 1785 – 1836) was an engineer and physicist. The Navier-Stokes equations are named after him and George Gabriel Stokes.
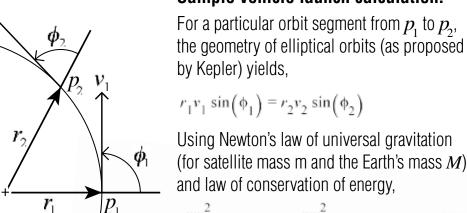
George Gabriel Stokes (Irish, 1819 – 1903) was a mathematician and physicist.

FEM: Because of the complexity of the Navier-Stokes equations, it is impossible to solve them algebraically. The techniques of Finite Element Methods (FEM) have been developed to compute approximations of solutions. FEM divides the area of interest into small related regions, makes approximations for each region, then combines the results to produce a single overall approximation to the fluid behavior. FEM techniques are also used to analyze deformation of metals and electro-magnetic fields.

Spacecraft

Although the science and mathematics behind space exploration is mind-boggling, many key concepts relate back to the fundamentals that many of us are familiar with. For example, modern orbital mechanics calculations still rely directly on the work of Kepler and Newton, among others.

Sample vehicle launch calculation:

For a particular orbit segment from p_1 to p_2, the geometry of elliptical orbits (as proposed by Kepler) yields.

Using Newton's law of universal gravitation (for satellite mass m and the Earth's mass M) and conservation of energy.

Some key calculations are performed when the distance between the two masses is at a minimum (periapsis) or a maximum (apoapsis). For periapsis, $v_1 = V_p$ and $r_1 = R_p$. Rearranging the above equations gives,

Solving for gives altitude predictions.

High Performance Materials

A major category of high performance materials is that of synthetic polymers, commonly known as plastics. A new invention of the 20th century, they are now found everywhere in a wide variety of forms.

One application of high performance plastics is the artificial lung. The main component of the artificial lung is a microporous membrane that allows the diffusion of oxygen into the blood, while carbon dioxide diffuses out of the blood into the gas stream and is thereby removed.

Fick's First Law of Diffusion: *Steady-state diffusion*

Fick's Second Law of Diffusion: *Non-steady state diffusion*

where J is the diffusion flux, D is the diffusion coefficient or diffusivity, c is the concentration, and x is the position.

Adolf Eugen Fick (German, 1829 – 1901) introduced Fick's Laws of Diffusion in 1855, which describe the diffusion of a gas across a fluid membrane. In 1887 he constructed and fitted what was the first successful model of a contact lens.

Air Conditioning & Refrigeration

The refrigerator cycle is a thermodynamic process of transporting heat from a low temperature area to a high temperature area. The liquid, or refrigerant, evaporates at an extremely low temperature so it can create freezing temperatures.

The most widely used refrigeration principle is vapor compression, also referred to as the reverse Rankine cycle. It operates around the refrigerant's saturation curve (shown in green).

This cycle takes advantage of the universal gas law:
$$PV = nRT$$
where P is pressure, V is volume, R is the universal gas constant, T is temperature, and n is the number of moles of gas (1 mole = 6.022×10^{23} molecules).

The refrigerant starts as a gas and is compressed and cooled to a liquid. The liquid is throttled to a lower pressure and evaporated to absorb heat, and recycled to continuously remove heat.

Johannes Kepler (German, 1640 – 1630) derived mathematical models (ellipses) for orbital motions.

Isaac Newton (English, 1643 – 1727) developed the laws of motion and gravity. He also developed integral calculus and differential calculus, which are probably the most important fields of mathematics.

Gottfried Leibniz (German, 1646 – 1716) independently developed integral and differential calculus. His notation and terminology is the standard today.

Sadi Nicolas Léonard Carnot (French, 1796 – 1832) hypothesized that the efficiency of a heat engine depended only on its input and output temperatures. This thought is now the Carnot cycle efficiency calculation.

William John Macquorn Rankine (Scottish, 1820 – 1872) was an engineer and physicist. He was a founding contributor to the science of thermodynamics.

Nuclear Technology

$E=mc^2$ is the world's most famous equation. Introduced by **Albert Einstein** (Swiss, 1879-1955) in 1905, it establishes the relationship of energy and matter. Einstein, along with major contributions from his contemporaries including Bohr, Heisenberg, Rutherford, and Fermi, dramatically increased science's understanding of the nature of atoms and atomic energy. Einstein, himself, had no direct involvement with the development of the atomic bomb.

Einstein was also notable for his early adoption of tensor notation, which offered more efficient manipulation of multidimensional equations. The following is a simple example of compact tensor notation for a vector dot product:

The following shows equivalence:

Gregorio-Ricci Curbastro (Italian, 1853-1925) invented tensor calculus. Tensor calculus and tensor algebra are essential tools in many fields of engineering, including fields and solid mechanics, and in modern physics. All these subjects are critical to the safe and effective management of nuclear energy.

Maria Skłodowska Curie (Polish, 1867-1934) discovered the new elements polonium and radium. Her work deepened our understanding of radioactive materials, which ultimately lead to the development of nuclear energy technology. Curie also pioneered the use of radiation for cancer treatment.

Radio & TV

Signal processing is one of the most important mathematical fields that supports the design and effective operation of radio and TV.

Amplitude Modulation (AM) is the earliest form of radio transmission. It applies a simple combination of a carrier wave and a modulating wave (the message). For example, a carrier wave is,
$$c(t) = C\sin(\omega_c t)$$

The signal that we wish to broadcast could be
$$m(t) = M\sin(\omega_m t + \varphi)$$
The modulated signal is then
$$y(t) = \left(C + M\sin(\omega_m t + \varphi)\right)\sin(\omega_c t)$$

Frequency Modulation (FM) uses the modulating signal to vary the frequency of the carrier signal. FM signals are generally more robust and are capable of carrying higher fidelity signals.

Claude Shannon (American, 1916 – 2001) introduced Information Theory in 1948. Information Theory mathematically predicts the amount of information contained in a signal and the ability of a channel to transmit the information. One of his equations states that the capacity of a channel (how much information can be transmitted) is given by
$$C = B\log_2\left(1 + S/N\right)$$
where B is the channel bandwidth and S/N is the signal-to-noise ratio of the channel. This type of analysis was instrumental in improving the quality of communication systems and even other branches of computer science and engineering, such as image processing, data mining, and pattern recognition. Today we owe Shannon's legacy in virtually all aspects of the Information Revolution.

Guglielmo Marconi (Italian, 1874-1937) was the first to successfully transmit radio signals across the Atlantic Ocean using a telegraph, in 1901.

Computers

George Boole (English, 1815-1864) invented Boolean algebra, the mathematical foundation of the logic used by the digital computer. In particular, computers use the special case of Boolean logic where entities can have only two values (e.g. 1 or 0, the binary system) and a set of clear rules defines their operations.
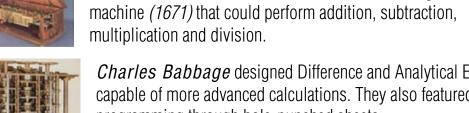
"If X is true and Y is false, or X is true and W is true, then Z is true" is a typical decision that computer programs make. In Boolean mathematical form, this decision would look like:
$$Z = (X \cdot \overline{Y}) + (X \cdot W)$$

Ada Byron, Countess of Lovelace (English, 1815-1852) was the world's first computer programmer. She wrote a brilliantly detailed account of the functioning and potential of Charles Babbage's Analytical Engine. Her work demonstrated the modern concepts of computer programming. The programming language Ada honors her work.

Alan Turing (British, 1912-1954) is often presented as the father of modern computing. He developed seminal concepts for the design and function of modern computers. The Turing Test was his proposal for assessing the true intelligence of computers.

Computers Doing Math

The idea of a machine to do mathematical calculations is not modern at all. Pascal, Leibniz, and Babbage are well-known for their attempts at automating calculations long before Microsoft®, Apple®, or Intel®.

Blaise Pascal's machine (1652) could add or subtract numbers. His most advanced models could work with numbers up to 9,999,999.

Co-father of calculus **Gottfried Leibniz** also designed a machine (1671) that could perform addition, subtraction, multiplication and division.

Charles Babbage designed Difference and Analytical Engines capable of more advanced calculations. They also featured programming through hole-punched sheets.

Today, a modest machine with the right software can crunch through math that would have taken a hundred *Gausses* a hundred years to do. Spreadsheets help business people use math to make business decisions. Advanced software like **Maple** replicates the mathematical thinking that is needed to build a spacecraft or cure a disease.

Imaging

Modern imaging relies heavily on digital image manipulation. The algorithms to enhance pictures or to automatically detect items in pictures are all relatively modern.

Jean Baptiste Fourier (French, 1768-1830) lends his name to one of the most important techniques in image processing, Fourier Transforms, which converts images (or signals in general) into frequency representations. This alternate view of images makes it easier to analyze and manipulate the data mathematically.

The general one-dimensional Fourier transform of a function $x(t)$ is:
$$X(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x(t)\,e^{-i\omega t}\,dt$$

For image processing, we require a two-dimensional variation:
$$F(u,v) = \int\int f(x,y)\,e^{-2\pi i(ux+vy)}\,dx\,dy$$

Euclid of Alexandria (Greek, 325 – 265 BC) wrote the earliest book on Geometric Optics (300 BC). His principles still survive today.

Fast Fourier Transforms (FFT) are versions of the general Fourier Transforms where the mathematics is broken into portions (discretized) to allow efficient computation on computers.

Telephone

We take it for granted that someone in America can make a phone call to Japan. Almost every home has its own telephone line, which is connected to a service of exchanges that span the globe. The reliable and efficient operation of telephone networks is of vital importance to both personal and commercial users.

Mathematics have developed ingenious ways of routing calls which can cope with unpredictable events like everyone on a block trying to phone at the same time.

Agner Krarup Erlang (Danish, 1878-1929) is considered the father of telephone networks in 1917. The dimensionless unit named the Erlang is a statistical measure of telecommunications traffic.

Erlang's Formula:

Where C is the number of lines, with an average of a calls starting per minute, and that the average length of a call is one minute. $E_1(C)$ represents the proportion of callers that find all the lines already full, and so would not be able to make their call until later.

Alexander Graham Bell (Scottish, 1847-1922) was credited with the first patent of the telephone.

The Internet

Part of the growth of the internet has been due to mathematical techniques designed to maintain privacy and authenticity of information. Cryptography is the mathematical science behind the encryption schemes that do this.

Modern cryptography is highly mathematical, utilizing advanced number theory and complexity theory, among other fields.

Julius Caesar (Roman, 100 BC – 44 BC) used encryption to communicate with his generals. Named after his most recorded use, the *Caesar cipher* shifts each letter of a message by a constant number of letters. The constant is the "key".

The RSA (Rivest-Shamir-Adleman) algorithm is an example of a public key encryption algorithm that offers an elegantly simple way of storing and sharing the keys needed to unlock sensitive information. RSA is the main algorithm behind the security infrastructure that we use on the Web. The math in RSA centers around the difficulty in factoring large integers, that is, to defeat an RSA scheme you must be able to solve a virtually impossible factoring problem.

Choose a number e that is relatively prime to m, that is, no number other than 1 divides into e and m.

Find d such that $de \mod m = 1$. If p and q are large, solving for d by breaking the key is virtually impossible.

This number d is part of the secret (or private) key that unlocks hidden information. The number e is part of the public key that allows others to lock up information destined for specific people.

Another important public key algorithm uses elliptic curves over a finite field to supply the impossible problem. The advantage over RSA is that smaller numbers can be used with equal security. This is important for mobile devices where speed and space are issues.

Health Technologies

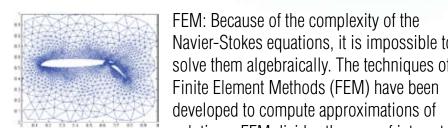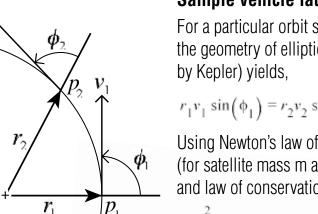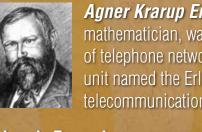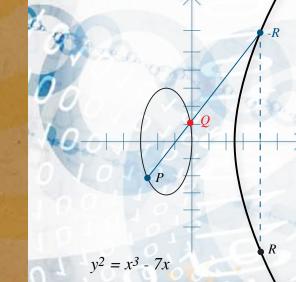Using x-rays to peer into the human body was a major milestone in modern medicine. **Wilhelm Röntgen** (German, 1845-1923) discovered the portion of the electromagnetic spectrum that he called "x-rays". X-ray wavelengths range from 0.1 to 10 Angstroms compared to 4000 to 7000 Angstroms for visible light.

In its simplest form, the wave equation refers to a scalar quantity u that satisfies

where c is a fixed constant equal to the propagation speed of the wave.

Christiaan Huygens (Dutch, 1629-1695) first proposed the wave nature of visible light.

Paul Adrien Maurice Dirac (British, 1902-1984) was a theoretical physicist and founder of the field of quantum physics. He is also known for the Dirac Delta Function.

Allan Cormack (South African, 1924-1998) laid the mathematical foundation for what we know as the CAT Scan (Computed Axial Tomography). This is often based on the mathematical function, "can we reconstruct a body if we can compute all of the line integrals that pass through it?"

Agricultural Mechanization

Increased efficiency in food production is vital to the development of a society. From the days of the ancients, people understood that better knowledge of nature and mathematics helps ensure secure food supplies.

Archimedes (Greek, 287 BC – 212 BC) was considered the first "mathematical physicist." He was able to relate known mathematical concepts to physical objects. His treatment of the lever, pulley systems, and screws shares the same core concepts as modern techniques used in the design of agricultural machinery. Later scholars such as Newton were able to elaborate on Archimedean concepts to develop modern science.

Today, agricultural efficiency is improved not just by better machines but also by working with biological processes, including genetics.

Andrey Andreivich Markov (Russian, 1856-1922) developed mathematical methods to model the interaction among elements of a complex system. Genetic scientists have used a variation of his method called the Hidden Markov Model (HMM) to decipher the mappings and interactions within the genomes of living organisms.

Archimedean treatment of the lever

$$\frac{L}{l} = \frac{R}{r} = \frac{E}{F}$$

L = length of effort arm
l = length of resistance arm
R = resistance weight or force
E = effort force.

Laser & Fiber Optics

Laser is an acronym for **light amplification by stimulated emission of radiation**. It is a device that controls the way energy is emitted and released. The principles that govern lasers are rooted in quantum mechanics.

The Dirac Equation is a relativistic quantum mechanical wave equation:

where m is the rest mass of the electron, c is the speed of light, p is the momentum operator, $\hbar$ is the reduced Planck's constant, x and t are the space and time coordinates respectively, and $\psi(x, t)$ is a four-component wavefunction.

Automobiles

The modern automobile is a complex collection of mechanical, chemical, electrical, hydraulic, and other types of physical systems, all of which have strong connections to mathematics.
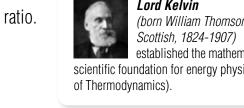
During the 19th century, scientists developed ways to mathematically model energy, and through these models they were able to build the practical engines that led to the modern car.

Otto (4 stroke) Cycle:
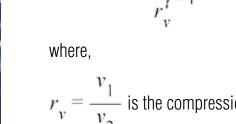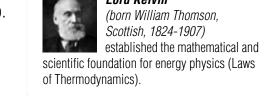
1 → 2 Intake stroke (isentropic compression)
2 → 3 Compression stroke (constant volume heating)
3 → 4 Power stroke (isentropic expansion)
4 → 1 Exhaust stroke (constant volume cooling)

The maximum efficiency for engines that use this cycle is given:

where,

r_v is the compression ratio

γ is the heat capacity ratio.

Nikolaus Otto (German, 1832 – 1891), a scientist, developed a version of the internal combustion engine that was efficient enough for practical automobiles. Today, his Otto thermodynamic cycle is the scientific foundation for most gas-powered cars.

Gottlieb Daimler (German, 1834 – 1900) is considered the inventor of the modern gasoline engine, the world's first motorcycle and the first important physical the twentieth century.

Max Karl Ernst Ludwig Planck (German, 1858-1947) is the founder of quantum theory, and is regarded as one of the most important physicists of the twentieth century.

André Villiot (German, 1866 – 1922) established Volhard's theorem, a useful result in theoretical physics that expresses the one-to-one correspondence between symmetries and conservation laws. It is deeply tied to quantum mechanics.

Lord Kelvin (born William Thomson, Irish, 1824-1907) successfully founded the energy physics (thermodynamics).