Maple User Manual

Maple User Manual

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Contents

Preface ... xiii
1 Getting Started ... 1
  1.1 In This Chapter ... 1
  1.2 Introduction to Maple ... 1
    Working in Maple ... 1
    Starting the Standard Document Interface ... 2
    Entering 2-D Math ... 4
    Toolbar Options ... 7
    The Context Panel and Copy & Drag ... 10
    Saving a Maple Document ... 13
    Saving a Maple Document as a Workbook ... 13
  1.3 Entering Expressions ... 14
    Execution Groups ... 14
    Math Mode vs. Text Mode ... 14
    Palettes ... 15
    Symbol Names ... 21
    Toolbar Icons ... 22
  1.4 Point-and-Click Interaction ... 24
    Assistants ... 24
    Tutors ... 25
    Math Apps ... 26
    The Context Panel ... 27
    Task Templates ... 27
    Exploration Assistant ... 29
  1.5 Commands ... 31
    The Maple Library ... 31
    Entering Commands ... 32
    Document Blocks ... 35
  1.6 The Maple Help System ... 37
    Accessing the Help System ... 37
    Using the Help Navigator ... 38
    Viewing Help Pages as Documents ... 39
    Viewing Examples in 2-D Math ... 39
    Copying Examples ... 39
  1.7 Available Resources ... 40
    Resources Available through the Maple Help System ... 40
    Quick Reference Card ... 41
    Website Resources ... 41
2 Document Mode ... 43
  2.1 In This Chapter ... 43
  2.2 Introduction ... 43
  2.3 Entering Expressions ... 44
    Example 1 - Enter a Partial Derivative ... 45
    Example 2 - Define a Mathematical Function ... 46
  2.4 Evaluating Expressions ... 46
  2.5 Editing Expressions and Updating Output ... 47
  2.6 Performing Computations ... 48
    Computing with Palettes ... 48
    Computing with the Context Panel ... 48
    Assistants and Tutors ... 53
3 Worksheet Mode ... 55

3.1 In This Chapter 55
3.2 Input Prompt 56
Suppressing Output 56
1-D Math Input 56
Input Separators 57
3.3 Commands 57
The Maple Library 58
Top-Level Commands 58
Package Commands 59
3.4 Palettes 61
3.5 The Context Panel 63
Example - Using the Context Panel 63
3.6 Assistants and Tutors 64
Launching an Assistant or Tutor 64
3.7 Task Templates 64
3.8 Text Regions 65
3.9 Names 66
Assigning to Names 66
Unassigning Names right single 67
Valid Names 68
3.10 Equation Labels 68
Displaying Equation Labels 69
Referring to a Previous Result 69
Execution Groups with Multiple Outputs 70
Label Numbering Schemes 70
Features of Equation Labels 70
4 Basic Computations 73
4.1 In This Chapter 73
4.2 Symbolic and Numeric Computation 74
Exact Computations 74
Floating-Point Computations 75
Converting Exact Quantities to Floating-Point Values 75
Sources of Error 76
4.3 Integer Operations 76
Non-Base 10 Numbers and Other Number Systems 78
4.4 Solving Equations 80
Solving Equations and Inequations 80
Other Specialized Solvers 87
4.5 Units, Scientific Constants, and Uncertainty 93
Units 93
Scientific Constants and Element Properties 97
Uncertainty Propagation 101
4.6 Restricting the Domain 103
Real Number Domain 103
Assumptions on Variables 104
5 Mathematical Problem Solving 109
5.1 In This Chapter 109
5.2 Algebra 109
Polynomial Algebra 110
5.3 Linear Algebra 116
Creating Matrices and Vectors 116
Accessing Entries in Matrices and Vectors 122
Linear Algebra Computations 124
Student LinearAlgebra Package 128

5.4 Calculus ... 128
Limits ... 128
Differentiation ... 130
Series ... 133
Integration ... 134
Calculus Packages ... 136
Differential Equations ... 137
5.5 Optimization ... 138
Point-and-Click Interface ... 138
Large Optimization Problems ... 140
MPS(X) File Support ... 141
Optimization Package Commands ... 141
5.6 Statistics ... 141
Probability Distributions and Random Variables ... 141
Statistical Computations ... 142
Plotting ... 143
Student Statistics Package ... 145
Additional Information ... 145
5.7 Teaching and Learning with Maple ... 145
Student Packages and Tutors ... 147
Calculus Problem Solving Examples ... 151
5.8 Clickable Math™ ... 157
Smart Popups ... 157
Drag-to-Solve ... 157
Examples ... 158
6 Plots and Animations ... 183
6.1 In This Chapter ... 183
6.2 Creating Plots ... 183
Interactive Plot Builder ... 184
Context Panel ... 189
The plot and plot3d Commands ... 191
The plots Package ... 198
Multiple Plots in the Same Plot Region ... 202
6.3 Customizing Plots ... 206
Interactive Plot Builder Options ... 206
Context Panel Options ... 207
The plot and plot3d Options ... 211
6.4 Analyzing Plots ... 213
Point Probe, Rotate, Pan, and Zoom Tools ... 213
6.5 Representing Data ... 214
6.6 Creating Animations ... 215
Interactive Plot Builder ... 215
The plots[animate] Command ... 215
The plot3d(...,viewpoint) Option ... 217
6.7 Playing Animations ... 219
Animation Context Bar ... 219
6.8 Customizing Animations ... 221
Interactive Plot Builder Animation Options ... 221
Context Panel Options ... 221
The animate Command Options ... 222
6.9 Exporting ... 226
6.10 Code for Color Plates ... 226
7 Creating Mathematical Documents ... 227
7.1 In This Chapter ... 227

7.2 Document Formatting 228
Copy and Paste 228
Quick Character Formatting 229
Quick Paragraph Formatting 231
Character and Paragraph Styles 232
Sections 237
Headers and Footers 239
Show or Hide Worksheet Content 239
Indentation and the Tab Key 240
7.3 Commands in Documents 241
Document Blocks 241
Typesetting 243
Auto-Execute 243
7.4 Tables 245
Creating a Table 245
Cell Contents 245
Navigating Table Cells 245
Modifying the Structural Layout of a Table 246
Modifying the Physical Dimensions of a Table 248
Modifying the Appearance of a Table 248
Printing Options 251
Execution Order Dependency 251
Editable Tables 252
Additional Examples 252
7.5 Canvas 254
Insert a Canvas 255
Drawing 255
Canvas Style 256
Inserting Images 257
7.6 Hyperlinks 258
Inserting a Hyperlink in a Document 258
Bookmarks 261
7.7 Embedded Components 263
Adding Graphical Interface Components 263
Task Template with Embedded Components 264
7.8 Spell Checking 265
How to Use the Spellcheck Utility 266
Selecting a Suggestion 266
User Dictionary 266
8 Maple Expressions 269
8.1 In This Chapter 269
8.2 Creating and Using Data Structures 269
Expression Sequences 269
Sets 270
Lists 270
Arrays 271
Tables 273
Matrices and Vectors 273
Functional Operators 274
Strings 277
8.3 Working with Maple Expressions 278
Low-Level Operations 278
Manipulating Expressions 282
Evaluating Expressions 285

9 Basic Programming 295
9.1 In This Chapter 295
9.2 Flow Control 295
Conditional Execution (if Statement) 295
Repetition (for Statement) 298
9.3 Iterative Commands 303
Creating a Sequence 303
Adding and Multiplying Expressions 303
Selecting Expression Operands 304
Mapping a Command over a Set or List 305
Mapping a Binary Command over Two Lists or Vectors 305
Additional Information 305
9.4 Procedures 305
Defining and Running Simple Procedures 306
Procedures with Inputs 306
Procedure Return Values 306
Displaying Procedure Definitions 307
Displaying Maple Library Procedure Definitions 307
Modules 308
Objects 308
9.5 Programming in Documents 308
Code Edit Region 308
Startup Code 309
9.6 Additional Information 310
10 Embedded Components and Maplets 311
10.1 In This Chapter 311
10.2 Using Embedded Components 311
Interacting 311
Printing and Exporting a Document with Embedded Components 314
10.3 Creating Embedded Components 314
Inserting Components 314
Editing Component Properties: General Process 315
Removing Graphical Interface Components 315
Integrating Components into a Document 315
Example 2 - Creating Embedded Components 317
10.4 Using Maplets 320
Maplet File 320
Maple Document 321
10.5 Authoring Maplets 321
Simple Maplet 321
Maplet Builder 322
Maplets Package 325
Saving 326
11 Input, Output, and Interacting with Other Products 327
11.1 In This Chapter 327
11.2 Writing to Files 327
Saving Data to a File 327
Saving Expressions to a File 328
Saving Data as Part of a Workbook 329
11.3 Reading from Files 329
Reading Data from a File 329
Reading Expressions from a File 330
Reading Data From Workbook Attachments 331
11.4 Exporting to Other Formats 331

Exporting Documents 331
MapleNet 333
11.5 Connectivity 333
Translating Maple Code To Other Programming Languages 333
Accessing External Products from Maple 333
Accessing Maple from External Products 334
Sharing and Storing Maple Content 335
Index 337

List of Figures

Figure 1.1: The Maple Environment 2
Figure 1.2: Text and Math Buttons on the Toolbar 14
Figure 1.3: Accessing the Assistants from the Tools Menu 24
Figure 1.4: Accessing Tutors from the Tools Menu 25
Figure 1.5: Calculus - Single Variable → Differentiation Methods Tutor 26
Figure 1.6: Click the expression to see applicable operations in the context panel 27
Figure 1.7: Click the plot to see plot options in the context panel 27
Figure 1.8: Browse Tasks Dialog 28
Figure 1.9: Equation Label 34
Figure 1.10: Inserting an Equation Label 35
Figure 1.11: Controlling Equation Label Format 35
Figure 1.12: Label Reference 35
Figure 1.13: Document Block Markers 36
Figure 1.14: Expanded Document Block 36
Figure 1.15: Sample Help Page 38
Figure 2.1: Context Panel 49
Figure 2.2: Approximating the Value of a Fraction 51
Figure 2.3: Finding the Approximate Solution to an Equation 52
Figure 2.4: Units Palette 52
Figure 3.1: Calculus Palette 62
Figure 3.2: Integer Context Panel 63
Figure 3.3: ODE Analyzer Assistant 64
Figure 3.4: Task Browser 65
Figure 3.5: Insert Label Dialog 69
Figure 3.6: Format Labels Dialog: Adding a Prefix 70
Figure 4.1: Context Panel for an Integer 77
Figure 4.2: Context Panel for an Equation 81
Figure 4.3: ODE Analyzer Assistant 88
Figure 4.4: ODE Analyzer Assistant: Solve Numerically Dialog 89
Figure 4.5: ODE Analyzer Assistant: Solve Symbolically Dialog 90
Figure 4.6: Units Converter Assistant 94
Figure 4.7: Units Palette 95
Figure 5.1: Sorting a Polynomial Using the Context Panel 113
Figure 5.2: Matrix Palette 117
Figure 5.3: Matrix Palette: Choosing the Size 117
Figure 5.4: Insert Matrix or Insert Vector 118
Figure 5.5: Matrix Browser 120
Figure 5.6: Computing the Infinity Norm of a Matrix 126
Figure 5.7: Directional Derivative Tutor 132
Figure 5.8: Optimization Assistant 138
Figure 5.9: Optimization Assistant Plotter Window 139
Figure 5.10: Calculus 1 Derivatives Tutor 147
Figure 5.11: Calculus 1 Differentiation Methods Tutor 148
Figure 5.12: Multivariate Calculus Gradient Tutor 149
Figure 5.13: Multivariate Calculus Gradient Tutor Showing x-y Plane 149
Figure 5.14: Flowchart of solving a problem 152
Figure 5.15: Volume of Revolution Tutor 154
Figure 5.16: Inserted Task Template 155
Figure 5.17: Example Worksheet 156
Figure 6.1: Plot an Expression Using the Context Panel 191
Figure 7.1: Select Color Dialog 229

Figure 7.2: Character Style Dialog 230
Figure 7.3: Paragraph Style Dialog 231
Figure 7.4: Style Management Dialog 232
Figure 7.5: Defining a Character Style 234
Figure 7.6: Defining a Paragraph Style 237
Figure 7.7: Style Set Management Dialog 237
Figure 7.8: Header and Footer Dialog - Custom Header 239
Figure 7.9: Show Contents Dialog 240
Figure 7.10: Working with Document Blocks 241
Figure 7.11: Delete Table Contents Verification Dialog 247
Figure 7.12: Table Paste Mode Selection Dialog 247
Figure 7.13: Two Cells 248
Figure 7.14: Merged Cells 248
Figure 7.15: Drawing Tools and Canvas 255
Figure 7.16: Drawing Outline Color Icon 256
Figure 7.17: Drawing Properties Canvas Icon - Change the Gridline Color 257
Figure 7.18: Hyperlink Properties Dialog 258
Figure 7.19: Bookmark Indicator 261
Figure 7.20: Create Bookmark Dialog 262
Figure 7.21: Components Palette 264
Figure 7.22: Interactive Application Task Template 265
Figure 7.23: Spellcheck Dialog 266
Figure 8.1: Function Definition Palette Items 274
Figure 8.2: Evaluate at a Point 286
Figure 9.1: Code Edit Region 308
Figure 9.2: Collapsed Code Edit Region 309
Figure 9.3: Startup Code Editor 309
Figure 10.1: Components Palette 315
Figure 10.2: Label Properties Dialog 316
Figure 10.3: Slider Properties Dialog 316
Figure 10.4: The Inserted Components 318
Figure 10.5: DialComponent Action Code Editor 319
Figure 10.6: A Simple Maplet 321
Figure 10.7: Maplet Builder Interface 322
Figure 10.8: Image of the Maplet 323
Figure 10.9: Body Elements Used to Define This Maplet 323
Figure 11.1: Import Data Assistant 329

List of Tables

Table 1.1: Common Keystrokes for Entering Symbols and Formats 5
Table 1.2: Maple Toolbar Options 7
Table 1.3: Context Bar Tools 9
Table 1.4: Context Selector Options 9
Table 1.5: Text Mode vs. Math Mode 14
Table 1.6: Palette Categories 16
Table 1.7: Managing Palettes 18
Table 1.8: Complex Numbers using I 22
Table 1.9: Complex Numbers using i 22
Table 1.10: Help Page Icons 39
Table 3.1: Top Commands 59
Table 3.2: Top Packages 60
Table 4.1: Select Integer Commands 77
Table 4.2: Modular Arithmetic Operators 79
Table 4.3: Overview of Solution Methods for Important Equation Types 80
Table 4.4: Sample Dimensions 93
Table 4.5: Scientific Constants 98
Table 4.6: Restricting to Real Numbers 104
Table 5.1: Polynomial Arithmetic Operators 110
Table 5.2: Polynomial Coefficient and Degree Commands 114
Table 5.3: Select Other Polynomial Commands 115
Table 5.4: Additional Polynomial Help 115
Table 5.5: Matrix and Vector Arithmetic Operators 124
Table 5.6: Select Matrix and Vector Operators 125
Table 5.7: Select LinearAlgebra Package Commands 127
Table 5.8: Limits 129
Table 5.9: Optimization Package Commands 141
Table 5.10: Student and Instructor Resources 146
Table 6.1: The plot and plot3d Commands 191
Table 6.2: Common Plot Options 211
Table 6.3: Plot Context Bar Analysis Options 213
Table 6.4: The animate Command 215
Table 6.5: Animation Options 220
Table 9.1: Default Clause Values 298
Table 9.2: Iterative Commands 303
Table 9.3: The seq Command 303
Table 9.4: The add and mul Commands 304
Table 9.5: The select, remove, and selectremove Commands 304
Table 9.6: The map Command 305
Table 9.7: The zip Command 305
Table 10.1: Embedded Component Descriptions 311
Table 11.1: Summary of Content Translation When Exporting to Different Formats 332

Preface

Maple Software

Maple^{TM} software is a powerful system that you can use to solve mathematical problems from simple to complex. You can also create professional quality documents, presentations, and custom interactive computational tools in the Maple environment.

You can access the power of the Maple computational engine through a variety of interfaces.

Interface	Description
Standard (default)	A full-featured graphical user interface that helps you create electronic documents to show all your calculations, assumptions, and any margin of error in your results. You can also hide the computations to allow your reader to focus on the problem setup and final results. The advanced formatting features lets you create the customized document you need. Because the documents are *live*, you can edit the parameters and, with the click of a button, compute the new results. The Standard interface has two modes: *Document* mode and *Worksheet* mode.
Command-line version	A command-line interface for solving very large complex problems or batch processing with scripts. No graphical user interface features are available.
Maplet^{TM} Applications	Graphical user interfaces containing windows, textbox regions, and other visual interfaces, which gives you point-and-click access to the power of Maple. You can perform calculations and plot functions without using the worksheet.

This manual describes how to use the Standard interface. As mentioned, the Standard interface offers two modes: *Document* mode and *Worksheet* mode. Using either mode, you can create high quality interactive mathematical documents. Each mode offers the same features and functionality, the only difference is the default input region of each mode.

Shortcut Keys by Platform

This manual will frequently refer to shortcut keys and command completion when entering expressions. The keyboard keys used to invoke these features differ based on your operating system.

The keystrokes given in this document are for Windows. There will be differences for other platforms. If you are using a different platform, see **Shortcut Keys**.

Command Completion

- **Esc**, Macintosh, Windows, and Linux
- **Ctrl + Space**, Windows
- **Ctrl** + **Shift** + **Space**, Linux

Begin entering a command in a Maple document. Press the **Esc** key. Alternatively, use the platform-specific keys. For Windows, press and hold the **Ctrl** key and then press the **Space** bar.

For more information on Command Completion, see *Command Completion (page 33)*.

In This Manual

This manual provides an introduction to the following Maple features:

- Ease-of-use when entering and solving problems
- Point-and-click interaction with various interfaces to help you solve problems quickly
- Maple commands and standard math notation

- Clickable Math^{TM}
- The help system
- Online resources
- Performing computations
- Creating plots and animations
- The Maple programming language
- Using and creating custom Maplet applications
- File input and output, and using Maple with third party products
- Data structures

For a complete list of manuals, study guides, toolboxes, and other resources, visit the Maplesoft website at **http://www.maplesoft.com**

Audience

The information in this manual is intended for first-time Maple users and users looking for a little more information.

Conventions

This manual uses the following typographical conventions.

- **bold** font - Maple command, package name, option name, dialog, menu, or text field
- *italics* - new or important concept
- **Note** - additional information relevant to the section
- **Important** - information that must be read and followed

Customer Feedback

Maplesoft welcomes your feedback. For suggestions and comments related to this and other manuals, contact **support@maplesoft.com**.

1 Getting Started

Don't worry about your difficulties in Mathematics. I can assure you mine are still greater.

~Albert Einstein

Mathematics touches us every day—from the simple chore of calculating the total cost of our purchases to the complex calculations used to construct the bridges we travel.

To harness the power of mathematics, Maplesoft provides a tool in an accessible and complete form. That tool is Maple.

1.1 In This Chapter

Section	Topics
Introduction to Maple (page 1) - The main features of Maple's Standard Interface	• Starting the Standard Document Interface • Entering commands and mathematical expressions • Toolbars • Context Panel • Copy and drag keys • Saving Maple documents
Entering Expressions (page 14) - Methods of entering expressions in 1-D and 2-D Math	• Execution groups • Math Mode and Text Mode • Palettes • Symbol names • Toolbar icons
Point-and-Click Interaction (page 24) - An introduction to the point-and-click features in Maple	• Assistants • Tutors • Context Panel • Task Templates • Exploration Assistant
Commands (page 31) - An introduction to the commands of the Maple language	• Using commands from the Maple library • Entering commands • Document blocks
The Maple Help System (page 37) - Accessing help on commands, packages, point-and-click features, and more	• How to access help for Maple features • Interacting with help pages • Viewing and interacting with examples
Available Resources (page 40) - Both online and from within Maple	• New user resources, including tutorials and the Maple Portal • Examples • Maple website resources

1.2 Introduction to Maple

Working in Maple

With Maple, you can create powerful interactive documents. The Maple environment lets you start solving problems right away by entering expressions in 2-D Math and solving these expressions using point-and-click interfaces. You can combine text and math in the same line, add tables to organize the content of your work, or insert images and sketch regions. You can visualize and animate problems in two and three dimensions, format text for academic papers or books, and insert hyperlinks to other Maple files, websites, or email addresses. You can embed and program graphical user interface components, as well as devise custom solutions using the Maple programming language.

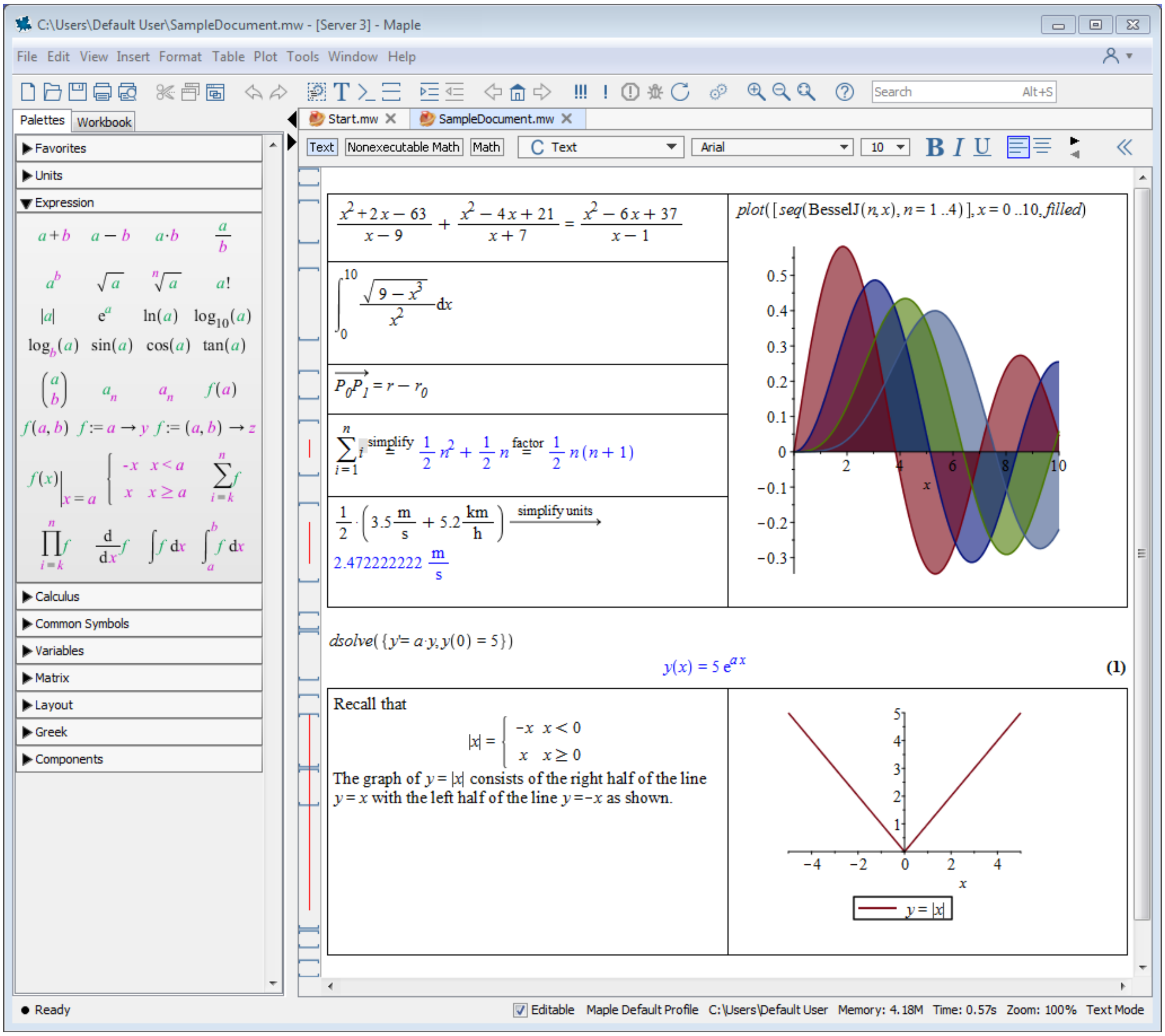


Figure 1.1: The Maple Environment

Starting the Standard Document Interface

To start Maple on:

Windows	From the **Start** menu, select **All Programs** → **Maple 2022** → **Maple 2022**. **Alternatively:** Double-click the **Maple 2022** desktop icon.

Mac	1. From the Finder, select **Applications** and **Maple 2022**. 2. Double-click **Maple 2022**.
Linux	Enter the full path, for example, **/usr/local/maple/bin/xmaple** **Alternatively:** 1. Add the Maple directory (for example, **/usr/local/maple/bin)** to your command search path. 2. Enter **xmaple**.

When the first Maple session opens, a **Start Page** displays shortcuts to useful tasks and topics.

To start a Maple session:

- In the **Start Page**, select **New Document** or **New Worksheet**. A blank document displays.

or

- From the **File** menu, select **New**, and then either **Document Mode** or **Worksheet Mode**. A blank document displays.

You can opt to start Maple with a blank document instead of the start page. You can also replace the default start page with a custom start page. For instructions, refer to the **startpage** help page.

To invoke the Start Page at any time, click the home button ( ) on the worksheet toolbar.

Document and Worksheet Modes

Maple offers two modes, *Document Mode* and *Worksheet Mode*. Using either mode, you can create high quality interactive mathematical documents. Each mode offers the same features and functionality; the only difference is the default input region of each mode.

Document Mode

Document mode uses *Document Blocks* as the default input region to hide Maple syntax. A Document Block region is indicated by two triangles located in the vertical Markers column along the left pane of the Maple Document, . If the Markers column is not visible, open the **View** menu and select **Markers**. This allows you to focus on the problem instead of the commands used to solve the problem. For example, when using the context-sensitive operations from the Context Panel on Maple input in Document mode (invoked by moving your mouse cursor over your input expression, then selecting the appropriate operation from the displayed context panel), input and output are connected using an arrow or equal sign with self-documenting text indicating the calculation that had taken place. The command used to solve this expression is hidden.

$$x^2 + 7x + 10 \xrightarrow{\text{solve}} \{x = -2\}, \{x = -5\}$$

To create a new document, select **File** → **New** → **Document Mode**.

Worksheet Mode

Worksheet mode uses a Maple prompt as the default input region. The Maple input prompt is a red angle bracket, [>. When using context-sensitive operations on input in Worksheet mode, all commands are displayed.

> $solve(x^2 + 7 \cdot x + 10 = 0)$

$$-2, -5$$

```
> solve(x^2 + 7*x + 10 = 0);
                              -2, -5
```

To create a new worksheet, select **File** → **New** → **Worksheet Mode**.

Full Flexibility in Either Mode

Regardless of which mode you begin working in, you have the opportunity to use both document blocks and command prompts.

For example, you can hide commands in Worksheet Mode by adding a document block from the **Edit** menu, **Edit** → **Document Blocks** → **Create Document Block** (see *Document Blocks (page 35)*), or you can show commands in Document mode by adding a Maple prompt from the **Insert** menu, **Insert** → **Execution Group** → **Before** / **After Cursor** (see *Input Prompt (page 56)*).

This chapter discusses features common to both modes. Specific aspects of Document mode are explained in *Document Mode (page 43)*, and aspects of Worksheet mode are explained in *Worksheet Mode (page 55)*.

The Maple Workbook

The Maple Workbook acts as a container that lets you collect Maple worksheets, library archives and language files, data (such as images or spreadsheets), and other items into a single file, stored in the **.maple** file format. This lets you better organize your Maple-based projects. For more information, see **Workbook Overview**.

Navigation of Workbook files is accomplished through the workbook Navigator palette, available in the Workbook tab. For more information, see **The Workbook Navigator**.

Entering 2-D Math

In documents, the default format for entering mathematical expressions is 2-D Math. This results in mathematical expressions that are equivalent to the quality of math found in textbooks. Entering 2-D Math in Maple is done using common key strokes or palette items. For more information on palettes, see *Palettes (page 15)*. An example of entering an expression using common key strokes is presented in the following section. An example of entering an expression using palette items is presented in *Example 3 - Enter an Expression Using Palettes (page 20)*.

Common Operations

Entering mathematical expressions, such as $\frac{35}{99} + \frac{1}{9}$, $x^2 + x$, and $x \cdot y$ is natural in 2-D Math.

To enter a fraction starting with the numerator:

1. Enter the numerator.
2. Press the forward slash (/) key.
3. Enter the denominator.
4. To leave the denominator, press the right arrow key.

To enter a fraction starting with the denominator:

1. Enter the denominator.
2. Press // (two forward slashes).
3. Enter the numerator.
4. To leave the numerator, press the right arrow key.

To enter a power:

1. Enter the base.
2. Press the caret (^) key.
3. Enter the exponent, which displays in math as a superscript.
4. To leave the exponent, press the right arrow key.

To enter a product:

1. Enter the first factor.
2. Press the asterisk (*) key, which displays in 2-D Math as a dot, $\cdot$.
3. Enter the second factor.

Implied Multiplication:

In general, the best practice is to enter a multiplication symbol (*) for multiplication in any calculation. In some cases, you can instead insert a space character between two quantities to multiply them. This is called *implicit multiplication*.

For example, in the expression $\frac{-b+\sqrt{b^2-4\ a\ c}}{2\ a}$ a space is used for the multiplication 4*a*c and 2*a.

In the case of a number followed by a variable, Maple interprets the expression as meaning multiplication even without the presence of * or a space character.

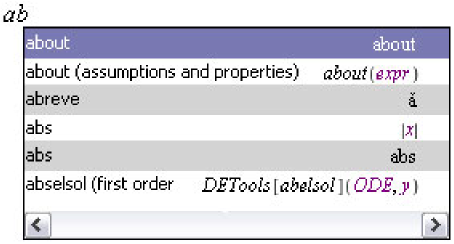
However, it's easier to identify and correct mistakes in your formulas if you use the multiplication symbol (*) regularly.

Important: Maple interprets a sequence of letters, for example, xy, as a single variable. To specify the product of two variables, you must insert a space character (or multiplication operator), for example, $x\ y$ or $x \cdot y$. For more information, refer to the **2DMathDetails** help page.

Shortcuts for Entering Mathematical Expressions

Table 1.1: Common Keystrokes for Entering Symbols and Formats

Symbol/Formats	Key	Example
implicit multiplication	**Space** key	$(x^2 - 7\ x\ y + 3\ y^2)\ x\ y$
explicit multiplication [1]	**Shift** + *	$2 \cdot 3$
fraction [2]	/ (forward slash) // (two forward slashes)	$\frac{1}{4}$
exponent (superscript) [2]	**Shift** + ^	x^2
indexed subscript [2]	**Ctrl** + **Shift** + _ (**Command** + **Shift** + _ in Mac)	x_a
literal subscript (subscripted variable name)	__ (two underscores)	x_{max}
navigating expressions	Arrow keys	

Symbol/Formats	Key	Example
command / symbol completion[3]	• **Esc** in Mac, Windows, and Linux • **Ctrl** + **Space** in Windows • **Ctrl** + **Shift** + **Space** in Linux	*ab* 
square root	*sqrt* and then command completion	$\sqrt{25}$
exponential function[2]	*exp* and then command completion	e^x
enter / exit 2-D Math	• **F5** key • **Math** and **Text** icons in the toolbar	$\frac{1}{4}$ versus 1/4
[1] required for products of numbers [2] use the right arrow key to leave a denominator, numerator, superscript, or subscript region [3] for more information, see *Command Completion (page 33)*.		

For a complete list of shortcut keys, refer to the **2-D Math Shortcut Keys and Hints** help page. To access this help page in the Maple software, in Math mode enter ?MathShortcuts and then press Enter. For information on the Maple Help System, see *The Maple Help System (page 37)*.

Example 1 - Enter and Evaluate an Expression Using Keystrokes

Review the following example:

$$\frac{x^2+y^2}{2}$$

In this example, you will enter $\frac{x^2+y^2}{2}$ and evaluate the expression.

Action	Result in Document
To enter the expression: 1. Enter **x**.	x
2. Press **Shift** + ^ . The cursor moves to the superscript position.	$x^{\square}$
3. Enter **2**.	x^2
4. Press the right arrow key. The cursor moves right and out of the superscript position.	x^2
5. Enter the + symbol.	x^2+
6. Enter **y**.	x^2+y
7. Press **Shift** + ^ to move to the superscript position.	$x^2+y^{\square}$
8. Enter **2** and press the right arrow key.	x^2+y^2

Action	**Result in Document**
9. With the mouse, select the expression that will be the numerator of the fraction.	$x^2 + y^2$
10. Enter the / symbol. The cursor moves to the denominator, with the entire expression in the numerator.	$\frac{x^2 + y^2}{\mid}$
11. Enter **2**.	$\frac{x^2 + y^2}{2}$
12. Press the right arrow key to move right and out of the denominator position.	$\frac{x^2 + y^2}{2}$
To evaluate the expression and display the result inline: 13. Press **Ctrl** + **=** (**Command** + **=** in Mac).	$\frac{x^2 + y^2}{2} = \frac{x^2}{2} + \frac{y^2}{2}$

To execute 2-D Math, you can use any of the following methods.

- Pressing **Ctrl** + **=** (**Command** + **=** in Mac). That is, *press and hold* the **Ctrl** (or **Command**) key, and then press the equal sign (=) key. This evaluates and displays results inline.
- Pressing the **Enter** key. This evaluates and displays results on the next line and centered.
- Select the input and from the context panel, select **Evaluate and Display Inline**. See *The Context Panel (page 27)* for more details.
- Using the **Evaluate** menu items **Evaluate** and **Evaluate and Display Inline**.

Toolbar Options

The Maple toolbar offers easy access to a number of actions. See **Table 1.2**.

Table 1.2: Maple Toolbar Options

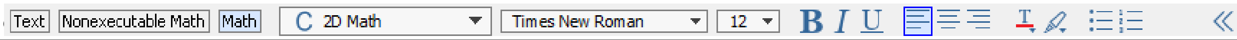
Basic Usage	**Icon**	**Equivalent Menu Option or Command**
Create a new Maple document		From the **File** menu, select **New** and then **Document Mode**
Open an existing document or worksheet		From the **File** menu, select **Open...**
Save the active document or worksheet		From the **File** menu, select **Save...**
Print the active document or worksheet		From the **File** menu, select **Print...**
Print preview the active document or worksheet		From the **File** menu, select **Print Preview...**
Toggle the Print Layout Mode		From the **File** menu, select **Print Layout Mode**.
Cut the selection to the clipboard		From the **Edit** menu, select **Cut**
Copy the selection to the clipboard		From the **Edit** menu, select **Copy**
Paste the clipboard contents into the current document or worksheet		From the **Edit** menu, select **Paste**
Undo the last operation		From the **Edit** menu, select **Undo**

Basic Usage	**Icon**	**Equivalent Menu Option or Command**
Redo the last operation		From the **Edit** menu, select **Redo**
Insert the Code Edit Region		From the **Insert** menu, select **Code Edit Region**
Inserts plain text after the current execution group.		From the **Insert** menu, select **Text**.
Inserts Maple Input after the current execution group. For details, refer to *Execution Groups (page 14)*.		From the **Insert** menu, select **Execution Group** and then **After Cursor**
Encloses the selection in a document block. If nothing is selected, it creates a new document block.		From the **Edit** menu, select **Document Blocks** and then **Create Document Block**
Encloses the selection in a subsection. For details, refer to *Sections (page 237)*.		From the **Format** menu, select **Insert Section**
Remove one level of indentation enclosing the selection.		From the **Format** menu, select **Remove Section**
Move backward to previous document in the hyperlink history		
Open the start page		
Move forward to next document in the hyperlink history.		
Executes all commands in the worksheet or document		From the **Execute** menu, select **Execute Worksheet**.
Executes a selected area.		From the **Evaluate** menu, select **Execute Selection**.
Interrupt the current operation.		
Debug the current operation		
Clears Maple's internal memory. For details, refer to the **restart** help page.		Enter *restart.*
Add and edit Maple code that is executed each time the worksheet is opened. For details, refer to the **startupcode** help page.		From the **Edit** menu, select **Startup Code**.
Adjusts the display size of document content. **Note:** Plots, images, and sketches remain unchanged.		From the **View** menu, select **Zoom Factor** and then a zoom size.
Reset zoom level.		From the **View** menu, select **Zoom Factor → Default**.
Opens the Maple help system in a new window. For details, refer to *The Maple Help System (page 37)*.		From the **Help** menu, select **Maple Help**.
Search box provides quick access to the help system.	Search for help, tasks, apps...	
Access to the MapleCloud on the web.	Sign in	

The Context Bar shows icons that are relevant to the location of the cursor in the document. For example, place the cursor at an input region and the **Text** and **Math** tools are accessible. See **Table 1.3** for a list of the tools available in each icon.

Table 1.3: Context Bar Tools

Context Bar Tools
Text

Math

Drawing tools

2-D Plot tools

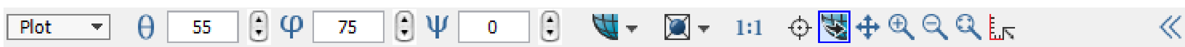
3-D Plot tools

Animation tools


Table 1.4: Context Selector Options

Region	**Available Tools**
Input region	Text and Math icons
Plot region	Drawing and Plot icons
Animation region	Drawing, Plot, and Animation icons
Drawing Canvas and Image regions	Drawing icon

The **Text** and **Math** icons allow you to enter text and math in the same line by choosing the appropriate input style at each stage when entering the sentence.

$\sin(x)$ is $\cos(x)$

For an example, see *Example 6 - Enter Text and 2-D Math in the Same Line Using Toolbar Icons (page 22)*.

The meaning of the **Text** and **Math** icons differs while at a Maple input prompt. The Math icon displays input as 2-D Math, whereas the Text icon displays Maple input. For details, refer to *Math Mode vs. Text Mode (page 14)*.

> $\frac{x^2}{2}$

```
> x^2/2;
```

To access the tools available in the **Plot** and **Drawing** icons, click a plot region. These tools allow you to manipulate the plot or draw shapes and enter text on the plot region. By clicking an animation region, you have the same features available for a plot region, in addition to tools for playing the animation in the **Animation** icon. For details on plots and animations, refer to *Plots and Animations (page 183)*.

For the description of any icon, hover the mouse over the icon to display at tooltip.

The Context Panel and Copy & Drag

Context Panel

Maple dynamically generates a collection of applicable options when you select or hover your mouse over an object, expression, or region. The options are organized and collectively displayed in the context panel to the right-hand side of the Maple user interface. The options available in the context panel depend on the selected input region. For example, you can manipulate and graph expressions, enhance plots, format text, manage palettes, structure tables, and more. When using menu items from the context panel to perform an action on an expression, the input and output are connected with a self-documenting arrow or equal sign indicating the action that had taken place. For more information, see *The Context Panel (page 27)*.

Copy & Drag

With Maple, you can drag input, output, or curves in a plot region into a new input region. This is done by highlighting the input or selecting the curve and dragging it with your mouse into a new input region. Dragging the highlighted region will cut or delete the original input. To prevent this, use the copy and drag feature.

- **Ctrl** + drag, Windows and Linux
- **Command** + drag, Mac

That is, highlight the region you want to copy. Press and hold the **Ctrl** key while you drag the input to the new region using the mouse. The analogous operation on Mac uses the **Command** key.

Example 2 - Solve and Plot an Equation Using Context Panel Options and Copy & Drag

Review the following example:

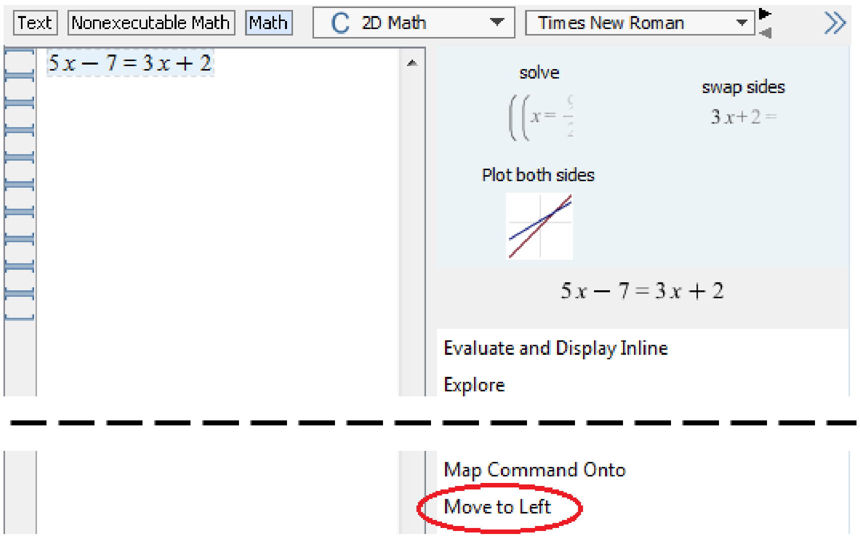
$$5\,x - 7 = 3\,x + 2$$

In this example, you will enter the equation and then solve and plot the equation using the context panel and the copy & drag feature. This example will only refer to the keystrokes needed on a Windows operating system to invoke the context panel and the copy & drag feature. For your operating system, refer to section *Shortcut Keys by Platform (page xiii)* for the equivalent keystrokes.

To solve the equation:

1. Enter the equation.
2. Click the equation and select **Move to Left**.

Input:



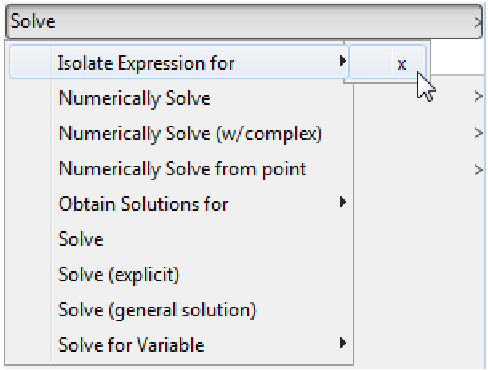
Result:

$$5\,x - 7 = 3\,x + 2 \xrightarrow{\text{move to left}} 2\,x - 9 = 0$$

A brief description, "move to left" is displayed above the arrow that connects the input and output.

3. Right-click the output from the previous action, $2\,x - 9 = 0$, and select **Solve** → **Isolate Expression for** → **x**.

Input:



Result:

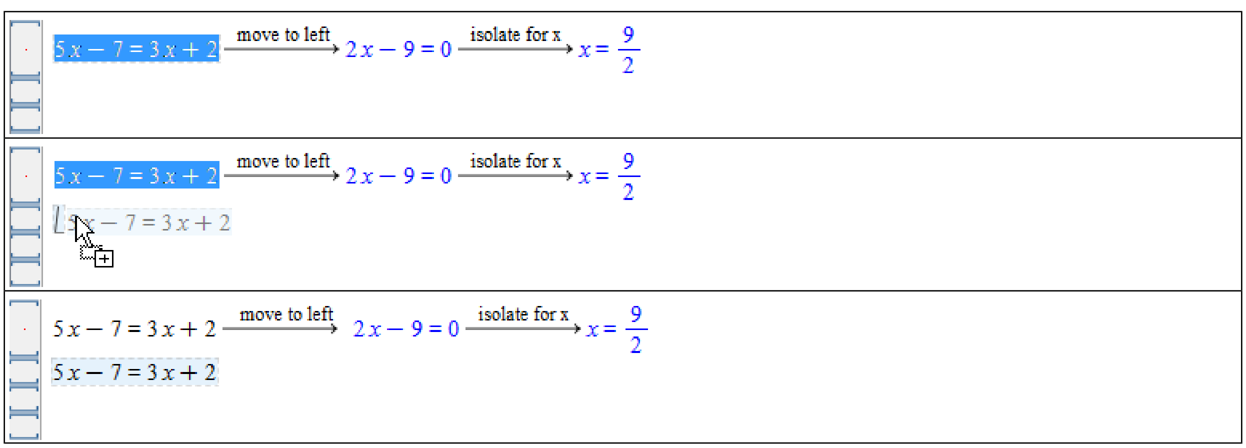
$$5\,x-7=3\,x+2 \xrightarrow{\text{move to left}} 2\,x-9=0 \xrightarrow{\text{isolate for x}} x=\frac{9}{2}$$

Now that we have solved the equation, we can plot it. To do this, we will copy the equation $2\,x-9=0$ to a new document block and use the context panel again.

4. From the **Edit** menu, select **Document Blocks→ Create Document Block**.

5. To copy the expression $5\,x-7=3\,x+2$, highlight this expression from the previous line. Press and hold the **Ctrl** key and drag the expression to the new document block region.

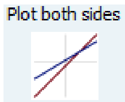
Result:



To plot the expression:

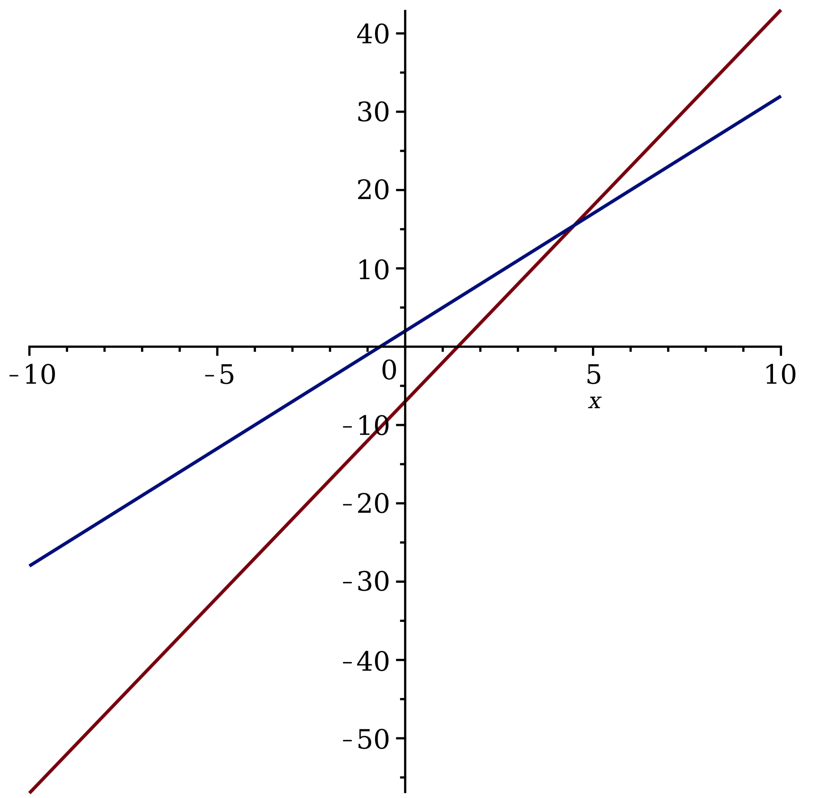
6. From the context panel, select **Plot both sides**.

Input:



Result:

$5\,x - 7 = 3\,x + 2 \xrightarrow{\text{Plot both sides [5*x-7, 3*x+2]}}$



Saving a Maple Document

To save these examples you created, from the **File** menu, select **Save**. Maple documents are saved as **.mw** files.

Saving a Maple Document as a Workbook

To save these examples you created as part of a new Maple workbook, from the **Workbook** tab, click **Save As Workbook**. For more information about saving Maple content see **worksheet,managing,saving**.

1.3 Entering Expressions

Execution Groups

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket, called a *group boundary*, at the left. An execution group may also contain any or all of the following: a plot, text, embedded components, and a drawing canvas.

Execution groups are the fundamental computation and documentation elements in the document. If you place the cursor in an input command and press the **Enter** or **Return** key, Maple executes all of the input commands in the current execution group.

Math Mode vs. Text Mode

The default mode of entry in Document or Worksheet mode is Math Mode, which displays input in 2-D Math. In Worksheet Mode or at a prompt, you can opt to use Maple Input (1-D Math). This is 1-D math input:

```
> simplify(cos(alpha)^2+sin(alpha)^2);
```

$$1$$

```
> a*int(exp(sqrt(2)*x),x);
```

$$\frac{a\sqrt{2}\,\mathrm{e}^{\sqrt{2}\,x}}{2}$$

```
> limit(f(x),x=infinity);
```

$$\lim_{x\to\infty} f(x)$$

```
> sum(a[k]*x^k, k=0..m)=product(b[j]*x^j, j=0..n);
```

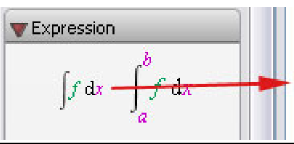
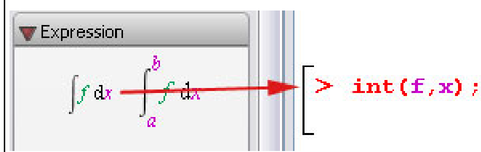
$$\sum_{k=0}^{m} a_k\,x^k = \prod_{j=0}^{n} b_j\,x^j$$

In Document Mode, to enter input using Maple Input mode, insert a Maple prompt by clicking >_ in the toolbar, and then click the **Text** button in the context bar. In Worksheet Mode, simply click the **Text** button. See **Figure 1.2**.

Figure 1.2: Text and Math Buttons on the Toolbar

Table 1.5: Text Mode vs. Math Mode

Math Mode	**Text Mode**
Maple's default setting. Executable standard math notation. This is also referred to as **2-D Math Input**. > $\int x^2 + 2\,x + 6\,dx$ $\frac{1}{3}\,x^3 + x^2 + 6\,x$	Executable Maple notation. This is referred to as **1-D Math Input** or **Maple Input**. `> int(x^2+2*x+6, x);` $\frac{1}{3}\,x^3 + x^2 + 6\,x$
Access from the **Insert** → **2-D Math** menu.	Access from the **Insert** → **Maple Input** menu.
When using 2-D Math, the **Math** mode icon is highlighted in the toolbar, Text Nonexecutable Math Math.	When entering Maple Input or text in a text region, the **Text** mode icon is highlighted in the toolbar, Text Nonexecutable Math Math.

Math Mode	**Text Mode**
In Document Mode (or a document block), input is entered in a document block with a slanted cursor, .	In Document Mode (or a document block), text is entered with a vertical cursor, as plain text, Enter some text.
In Worksheet Mode, input is made at an input prompt with a slanted cursor, .	In Worksheet Mode, input is made at an input prompt with a vertical cursor, .
To convert a 2-D Math expression to 1-D Math, click the expression, then from the context panel select **2-D Math** → **Convert To** → **1-D Math Input**.	To convert a 1-D Math expression to 2-D Math, click the expression, then from the context panel select **Convert To** → **2-D Math Input**.
Palettes make entering expressions in familiar notation easier than entering foreign syntax and reduces the possibility of introducing typing errors. 	Using palettes while in 1-D Math teaches you the related Maple command syntax. 

If you prefer 1-D Math input in Worksheet mode, you can change the default math input notation.

To change math input notation for a session or globally across all documents:

1. From the **Tools** menu, select **Options**. The **Options Dialog** opens.
2. Click the **Display** tab.
3. In the **Input Display** drop-down list, select **Maple Notation**.
4. Click the **Apply to Session** or **Apply Globally** button.

This changes the default input math notation at the prompt (>).

Important: The new input display becomes the default setting *after* pressing the **Enter** key.

Palettes

Palettes are collections of related items that you can insert into a document by clicking or drag-and-dropping. The Maple environment provides access to over 30 palettes containing items such as symbols (∞), layouts (A^b), mathematical operations $\left(\int_a^b f\,\mathrm{d}x\right)$, and much more.

By default, palettes are displayed in the left pane of the Maple environment when you launch Maple. If the palettes are not displayed,

1. From the **View** menu, select **Palettes**.
2. Select **Expand Dock**.
3. Right-click (**Control**-click, Mac) the palette dock. From the context menu, select **Show Palette**. Alternatively, from the main menu, select **View** → **Palettes** → **Show Palette**.

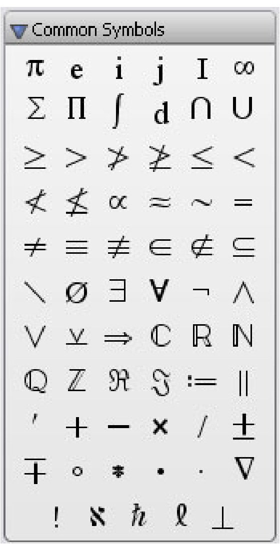
   A list of potential palettes is displayed, with currently displayed palettes dimmed.
4. Select a palette.

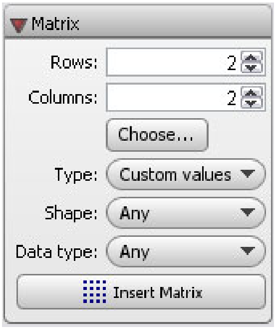
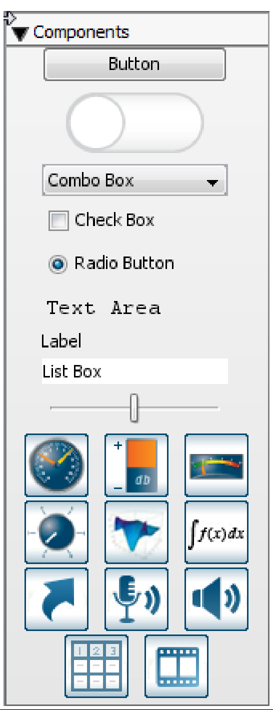
Alternatively, select **View** → **Palettes** → **Show All Palettes**.

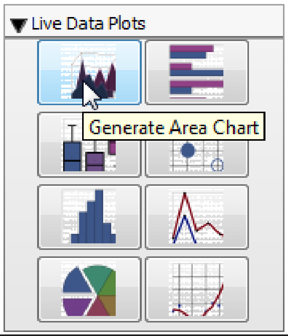
You can create a **Favorites** palette of the expressions and entities you use often by right-clicking (**Control**-click, Mac) the palette template you want to add and selecting **Add To Favorites Palette** from the context menu.

Table 1.6: Palette Categories

<table>
<tr><th>Palette Category</th><th>Palette Description</th></tr>
<tr><td>Alphabetical Palettes

</td>
<td>Greek β ,
Script 𝒜,
Fraktur 𝔄,
Open Face ℂ ,
Cyrillic Ж,
Diacritical Marks ´ ,
Roman Extended Upper Case Æ,
Roman Extended Lower Case æ .</td></tr>
<tr><td>Mathematical Palettes

</td>
<td>Palettes for constructing expressions
Common Symbols,
Relational ≥,
Relational Round ≳,
Operators ÷,
Large Operators ∯ ,
Negated ≠,
Fenced ⟪ ,
Arrows ↛,
Constants and Symbols ∞.
Punctuation - insert punctuation symbols, such as inserting the registered trademark and copyright symbols © into text regions
Miscellaneous - insert miscellaneous math and other symbols outside the above categories ▱.</td></tr>
</table>

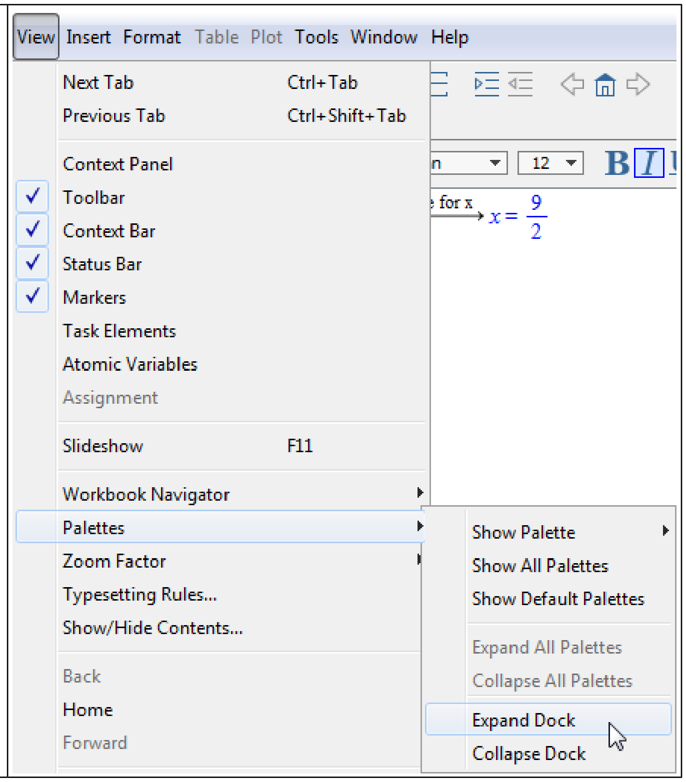
Palette Category	Palette Description
Expression Palettes 	**Expression -** construct expressions such as logarithms $\log_b(a)$. **Matrix** - enter the number of rows and columns required, designate type, such as zero-filled, and designate shape, such as diagonal. **Layout** - add math content that has specific layout, such as expressions with one or more superscripts and subscripts A^b. **Calculus** - construct expressions such as integrals $\int_a^b f\,dx$. **Handwriting** - an easy way to find a desired symbol. **Units** - select a unit and insert into document, such as $[[ft]]$ or $[[kg]]$. **Accents** - insert decorated names, such as an x with an arrow over it to denote a vector $\vec{A}$. **Trigonometric & Hyperbolic** - a palette for constructing expressions containing trigonometric and hyperbolic functions **Student Random Variables** - a palette for constructing random variables based on distributions in the **Student Statistics** package **Group Constructors** - a palette for constructing groups based on the **Group Theory** package
Other Palettes 	**Components** - embed graphical interface components such as a button into your document or worksheet. Components can be programmed to perform an action when selected such as executing a command when a button is clicked. **Favorites** - add templates that you use most often from other palettes.**Variables** - manage all of your assigned variables in your current Maple session.

Palette Category	Palette Description
Task Palettes 	**Live Data Plots** - templates for visual representation of your data. **eBook Metadata** - markup tags for use when creating eBooks from Maple worksheets **Tasks** - a palette where you can store tasks that you have created

Viewing and Arranging Palettes

By default, palettes display in palette dock in the left pane of the Maple window. To view and manage palettes and palette docks, see **Table 1.7**.

Table 1.7: Managing Palettes

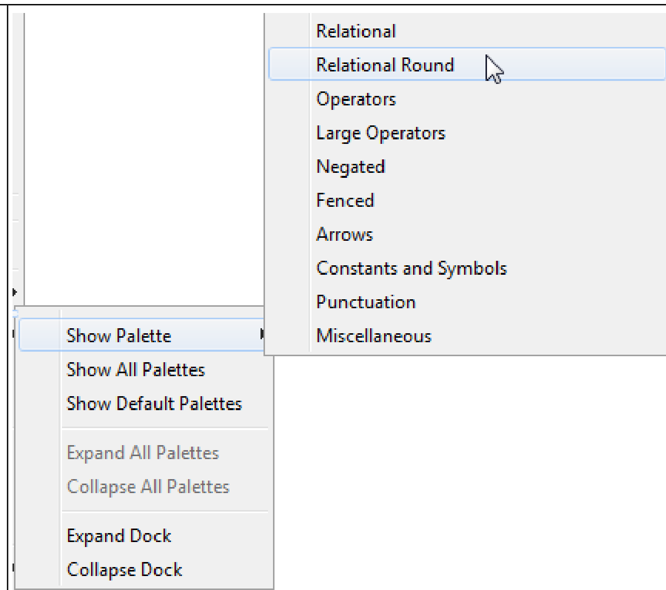
To view the palette dock: • From the **View** menu, select **Palettes**, and then **Expand Dock**. The dock is in the left pant of the window.	

<table>
<tr><td>

To add a palette:

1. Right-click the palette dock. Maple displays a context menu near the palette.
2. From the context menu, select **Show Palette** and then select the palette.

</td><td>

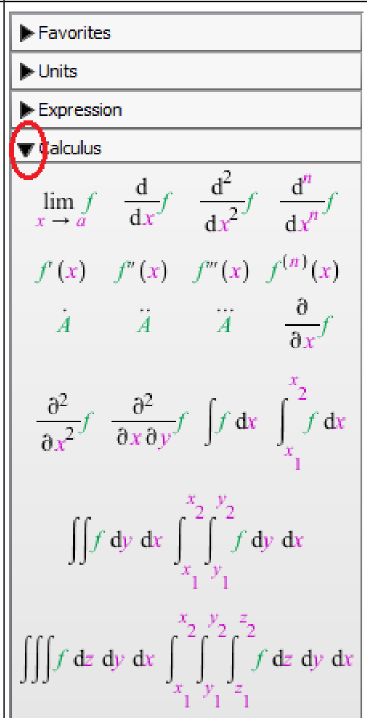


</td></tr>
<tr><td>

To expand or collapse a palette in the palette dock:

- Click the triangle at the left of the palette title.

</td><td>

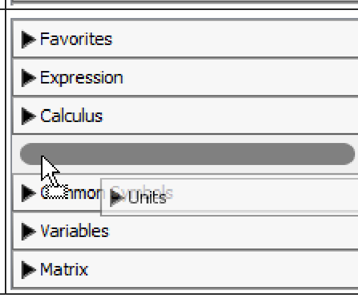


</td></tr>
<tr><td>

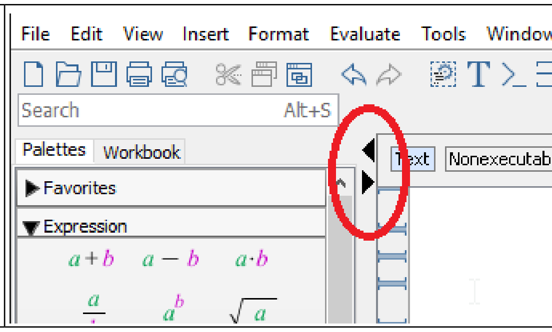
To move a palette in the palette dock:

- Move the palette by clicking the title and dragging the palette to the new location.

</td><td>



</td></tr>
</table>

To expand or collapse the pane containing the palette dock: • Select the appropriate triangle at the top right side of the palette region.	

Example 3 - Enter an Expression Using Palettes

Review the following example:

$$\sum_{i=1}^{10} (7\,i^2 - 5\,i) = 2420$$

In this example, we will enter $\sum_{i=0}^{10} (7\,i^2 - 5\,i)$ and evaluate the expression.

Action	Result in Document
1. Place the cursor in a new document block. In the **Expression** palette, click the summation template $\sum_{i=k}^{n} f$. Maple inserts the summation symbol with the range variable placeholder highlighted.	$\sum_{i=k}^{n} f$
2. Enter **i** and then press **Tab**. The left endpoint placeholder is selected. Notice that the color of the range placeholder has changed to black. Each placeholder must have an assigned value before you execute the expression. The **Tab** key advances you through the placeholders of an inserted palette item.	$\sum_{i=k}^{n} f$
3. Enter **1** and then press **Tab**. The right endpoint placeholder is selected.	$\sum_{i=1}^{n} f$
4. Enter **10** and then press **Tab**. The expression placeholder is selected.	$\sum_{i=1}^{10} f$
5. Enter $(\mathbf{7\,i^2 - 5\,i})$. For instructions on entering this type of expression, see *Example 1 - Enter and Evaluate an Expression Using Keystrokes (page 6)*.	$\sum_{i=1}^{10} (7\,i^2 - 5\,i)$
6. Press **Ctrl** + = (**Command** + = for Mac) to evaluate the summation.	$\sum_{i=1}^{10} (7\,i^2 - 5\,i) = 2420$

Favorites Palette

You can add to the **Favorites** palette any expressions that you use most often.

- To add an entry from another palette to the Favorites palette, simply drag the entry to the Favorites palette.

- To add a custom expression, select the expression in the worksheet and select **Add Selection to Favorites Palette** from the Context Panel.

Symbol Names

Each symbol has a name and some have aliases. By entering its name (or an alias) in Math mode, you can insert the symbol in your document. All common mathematical symbols, including all Greek characters, π, and the square root symbol ($\sqrt{\ }$), are recognized by Maple.

Note: If you hover the mouse pointer over a palette item, a tooltip displays the symbol's name.

To insert a symbol, enter the first few characters of a symbol name using a keyword that is familiar to you and then press the completion shortcut key, **Esc** (see *Shortcut Keys by Platform (page xiii)*). Symbol completion works in the same way as command completion (see *Command Completion (page 33)*).

- If a unique symbol name matches the characters entered, Maple inserts the corresponding symbol.
- If multiple symbol names match the characters entered, Maple displays the completion list, which lists all matches, including commands. To select an item, click its name or symbol.

Example 4 - Square Root

To find the square root of 603729**:**

Action	Result in Document
1. In a new document block, enter *sqrt*.	sqrt
2. Press the symbol completion shortcut key, **Esc**. Maple displays a popup list of exact matches.	sqrt sqrt $\sqrt{x}$ sqrt sqrt
3. In the completion list, select sqrt $\sqrt{x}$. Maple inserts the symbol with the x placeholder selected.	$\sqrt{x}$
4. Enter 603729 into the placeholder.	$\sqrt{603729}$
5. Press **Ctrl** + = (**Command** + =, Mac).	$\sqrt{603729} = 777$

Example 5 - Complex Numbers

In Maple, the default display for imaginary i is a capital I. When you simply type the letter i in Math mode, it is in italics. This letter is just a variable, and is not the same as the imaginary unit $\sqrt{-1}$, denoted by I or i in Maple.

$$sum(i, i = 1..4) = 10$$

Multiply two complex numbers, $-0.123 + 0.745\,\mathrm{i}$ **and** $4.2 - \mathrm{i}$**:**

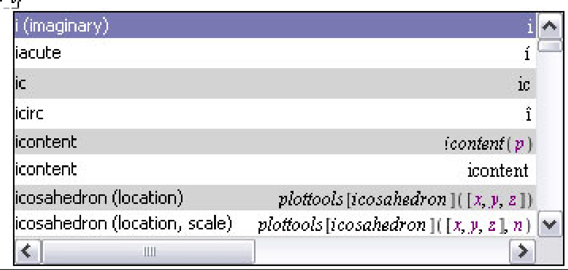
We will compute the result two ways, using I and then using i. The first way is the quickest to enter.

Table 1.8: Complex Numbers using I

Action	Result in Document
1. In a new document block, enter $(-0.123 + 0.745\,\mathrm{I})\cdot(4.2 - \mathrm{I})$ Use * to enter multiplication between the two complex numbers.	$(-0.123 + 0.745\,\mathrm{I})\cdot(4.2 - \mathrm{I})$
2. Press **Ctrl** + = (**Command** + =, Mac) to evaluate the product.	$(-0.123 + 0.745\,\mathrm{I})\cdot(4.2 - \mathrm{I}) = 0.2284 + 3.2520\,\mathrm{I}$

The next method, while not as quick to enter, displays the computation using lowercase i.

Table 1.9: Complex Numbers using i

Action	Result in Document
1. In a new document block, enter $(-0.123 + 0.745\,i$.	$(-0.123 + 0.745\,i$
2. Press the symbol completion shortcut key, **Esc**. Maple displays a popup list of partial and exact matches, including symbols and commands.	$(-0.123 + 0.745\,i$ 
3. Select the imaginary unit, i (imaginary) i.	$(-0.123 + 0.745\,\mathrm{i}$
4. Close the parentheses, enter * (for multiplication), and type the second expression in parentheses, using symbol completion for the second imaginary number.	$(-0.123 + 0.745\,\mathrm{i})\cdot(4.2 - \mathrm{i})$
5. Press **Ctrl** + = (**Command** + =, Mac) to evaluate the product.	$(-0.123 + 0.745\,\mathrm{i})\cdot(4.2 - \mathrm{i}) = 0.2284 + 3.2520\,\mathrm{I}$

For more information on entering complex numbers, refer to the **HowDoI/EnterAComplexNumber** help page.

Toolbar Icons

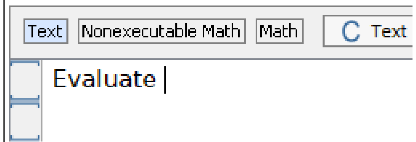
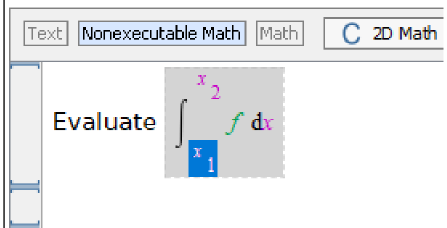
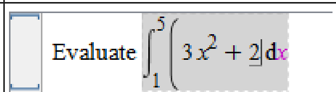
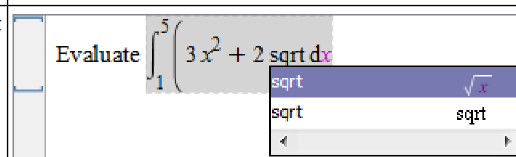
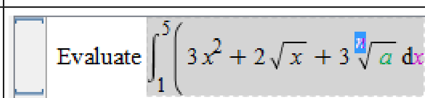
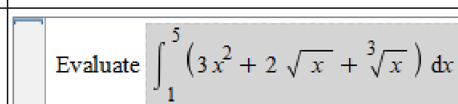
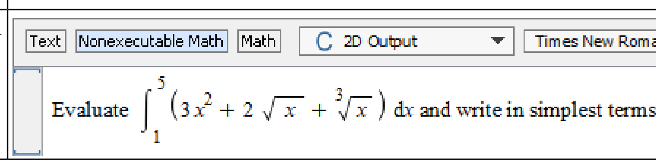
In the introduction section, you learned about the toolbar icons and context toolbars available in Maple (see *Toolbar Options (page 7)*). The toolbar can be used to format your document, alter plots and animations, draw in a canvas, write in both Math and Text modes in one line and much more. The last of these is demonstrated in the next example.

Example 6 - Enter Text and 2-D Math in the Same Line Using Toolbar Icons

Enter the following sentence:

Evaluate

$\int_1^5 \left(3\,x^2 + 2\,\sqrt{x} + 3\sqrt[3]{x}\right)\,dx$ and write in simplest terms.

Action	Result in Document
To enter this sentence: 1. Select the **Text** icon and enter "Evaluate ".	
2. Select the **Nonexecutable Math** icon. 3. From the **Calculus** palette, select the definite integration template, $\int_{x_1}^{x_2} f \, dx$. The expression is displayed with the first placeholder highlighted.	
4. With the first placeholder highlighted, enter **1**, then press **Tab**. 5. Enter **5** and press **Tab** to highlight the integrand region. 6. Enter **(3x^2** and press the right arrow to leave the superscript position. 7. Enter + **2**.	
8. Press the **Space** bar for implicit multiplication. Enter **sqrt** and press **Esc** to show the command completion options. Maple displays a popup list of exact matches. Select the square root symbol, $\sqrt{x}$. Maple inserts the symbol with the x placeholder selected. (Alternatively, select the square root symbol from the **Expression** palette.)	
9. Enter **x**, then press the right arrow to leave the square root region. 10. Enter + **3**, and then press the **Space** bar. 11. Select the **n-th root** symbol from the Expression palette, $\sqrt[n]{a}$.	
12. Enter **3**, then press **Tab**. 13. Enter **x**, then press the right arrow to leave the root region. 14. Enter **)**, then press **Tab**. 15. Enter **x** for the integration variable.	
16. Put the cursor after the expression and click the **Text** icon in the toolbar, then enter the rest of the sentence: " and write in simplest terms."	

Note: When an expression is intended for display purposes only, as in this example, it can be displayed in nonexecutable math. This is indicated by the gray background. For more information on executable and nonexecutable math, see 2DMathDetails.

1.4 Point-and-Click Interaction

Maple contains many built-in features that allow you to solve problems quickly without having to know any commands.

Assistants

Maple offers a set of assistants in the form of graphical user interfaces to perform many tasks without the need to use any syntax.

Using the **Tools** → **Assistants** menu, you can access tools to help you accomplish various tasks. See **Figure 1.3**. In some cases, you can launch an assistant by entering an expression and selecting the assistant from the options in the Context Panel.

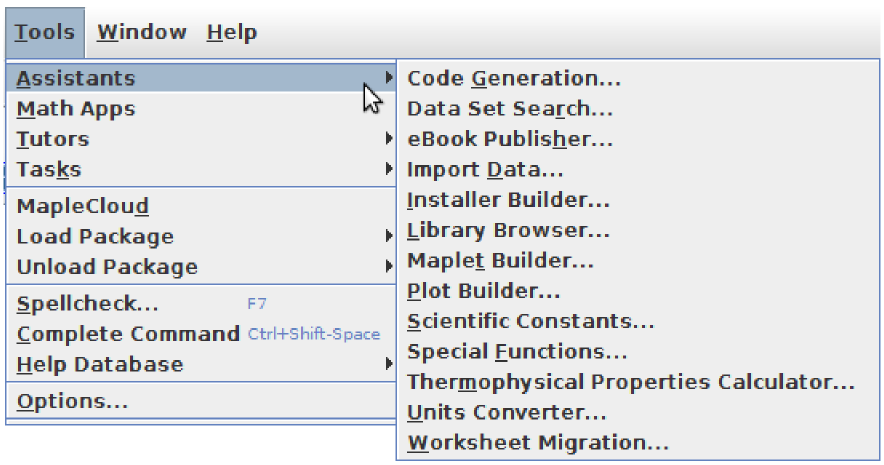


Figure 1.3: Accessing the Assistants from the Tools Menu

Descriptions of Assistants

The available assistants are described below. Some of the assistants are interfaces to package commands. For more information on package commands, see *Package Commands (page 33)*.

- **CAD Link -** an interface to explore the properties of models from supported CAD applications (available on Microsoft Windows only).
- **Code Generation** - an interface to automatically transform Maple expressions and programs to other languages.
- **Data Set Search** - an interface for searching built-in and online data sources.
- **eBook Publisher** - an interface to the eBook Publisher tools.
- **Import Data** - an interface to read data from an external file into Maple.
- **Installer Builder** - an interface to the **InstallerBuilder** package in which you can create installers for your Maple toolboxes.
- **Library Browser -** an interface to manipulate the libraries in a specified directory.
- **Maplet Builder** - an interface to the **Maplets** package. The **Maplets** package contains commands for creating and displaying Maplet applications (point-and-click interfaces). Using the Maplet Builder, you can define the layout of a Maplet, drag-and-drop elements (visual and functional components of Maplets), set actions associated with elements, and directly run a Maplet application. The Maplet Builder is available in the Standard interface only.
- **Plot Builder** - an interface for creating two and three-dimensional plots, animations, and interactive plots.
- **Scientific Constants** - an interface to over 20 000 values of physical constants and properties of chemical elements. All of these constants come with the corresponding unit and, if applicable, with the uncertainty or error, that is, how precisely the value of this constant is known.

- **Special Functions** - an interface to the properties of over 200 special functions, including the Hypergeometric, Bessel, Mathieu, Heun and Legendre families of functions.
- **Thermophysical Properties Calculator -** Calculate state-dependent and independent thermophysical properties.
- **Units Converter -** an interface to convert between 500 units of measurement.
- **Worksheet Migration -** an interface to convert worksheets from Classic Maple (.mws files) to Standard Maple (.mw files).

Tutors

Maple provides over 50 interactive tutors and assistants to aid in the learning of

- Basics
- Precalculus
- Calculus
- Multivariate Calculus
- Vector Calculus
- Complex Variables
- Differential Equations
- Linear Algebra
- Numerical Analysis
- Optimization
- Statistics

These tutors are easily accessible in the **Tools** menu by selecting **Tutors**. See **Figure 1.4**.

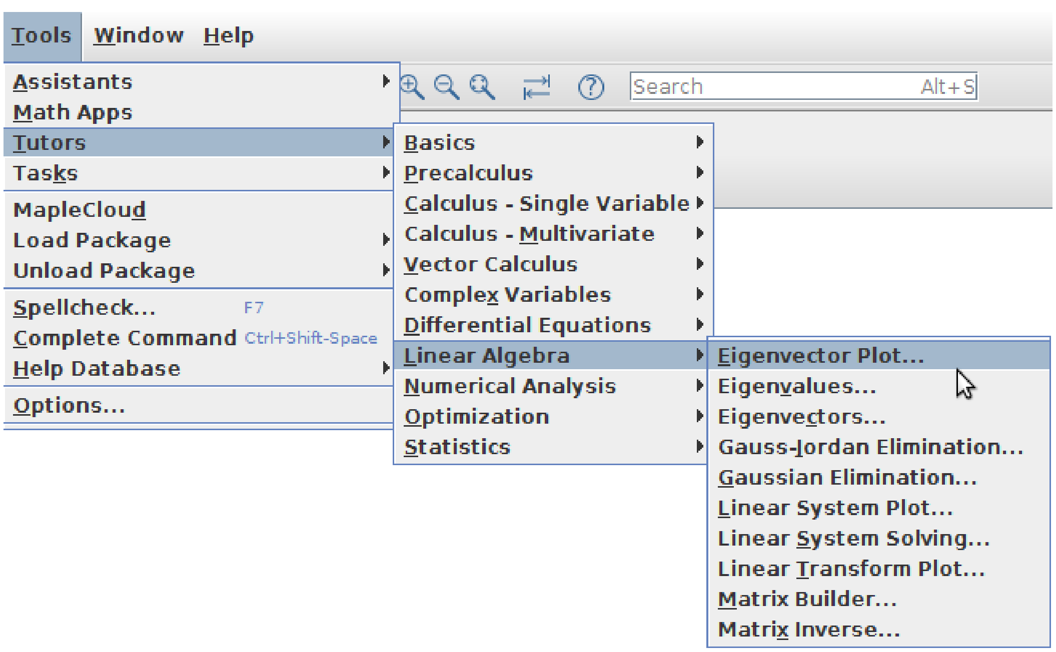


Figure 1.4: Accessing Tutors from the Tools Menu

Some of the tutors can also be accessed through the **Student** package. The Differential Equations tutor, **DE Plots**, is accessible through the **DEtools** package. For a definition of the term *package*, see *Package Commands (page 33)*.

The **Student** package is a collection of subpackages designed to assist with the teaching and learning of standard undergraduate mathematics. The subpackages contain many commands for displaying functions, computations, and theorems in various ways, and include support for stepping through important computations.

The **interactive** commands help you explore concepts and solve problems using a point-and-click interface. These commands launch tutors that provide a graphical interface to some of the visualization and computation commands described above. See **Figure 1.5** for an example of one of the tutors.

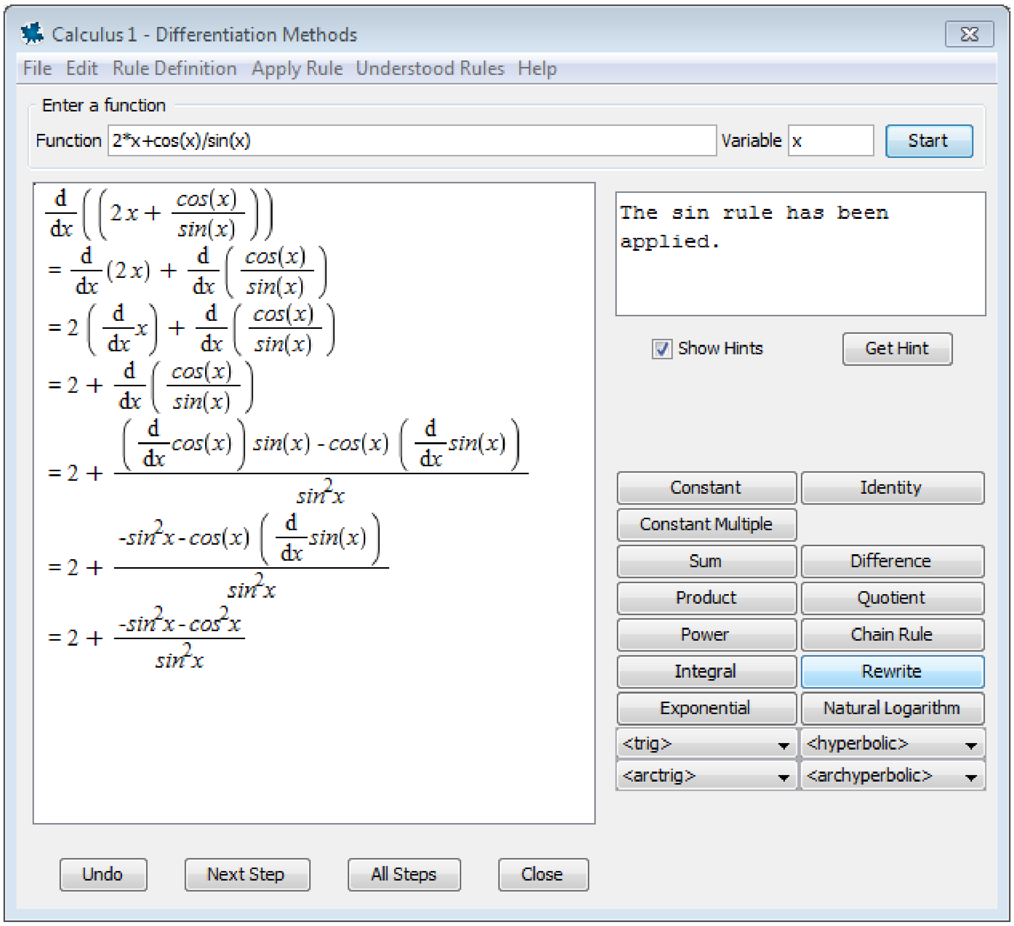


Figure 1.5: Calculus - Single Variable → Differentiation Methods Tutor

The Practice Sheets Assistant, found under **Tools → Tutors→ Basics**, lets you construct a practice sheet of math problems. It's easy to interactively construct a randomized set of problems arithmetic, algebra, calculus, factorization and more. Students can then do the problems, check their solutions, and even generate another set of problems.

For more information on the tutors and related resources for mathematics education, see *Teaching and Learning with Maple (page 145)*.

Math Apps

Maple provides Math Apps that offer interactive, entertaining ways to explore mathematical concepts, ranging from Precalculus to Physics to Economics. A guide to these demonstrations is accessible in the **Tools** menu by selecting **Math Apps**.

The Context Panel

The context panel is a dynamically generated list of tools and actions that are applicable for the region on which it is invoked. These tools and actions are further organized into menus. Use the context panel to perform calculations and manipulations on expressions without using Maple syntax. To display the context panel, select an object, expression, or region. Context panel options are available for many input regions, including:

- **expressions** to perform calculations, manipulations, or plotting
- **plot regions** to apply plot options and manipulate the plot
- **tables** to modify the table properties
- **text regions** to add annotations and format text

When performing calculations or manipulations on an expression, a self-documenting arrow or equal sign connects the input and output, indicating the action that took place. See Figures 1.8 and 1.9 for two examples of context-sensitive operations.

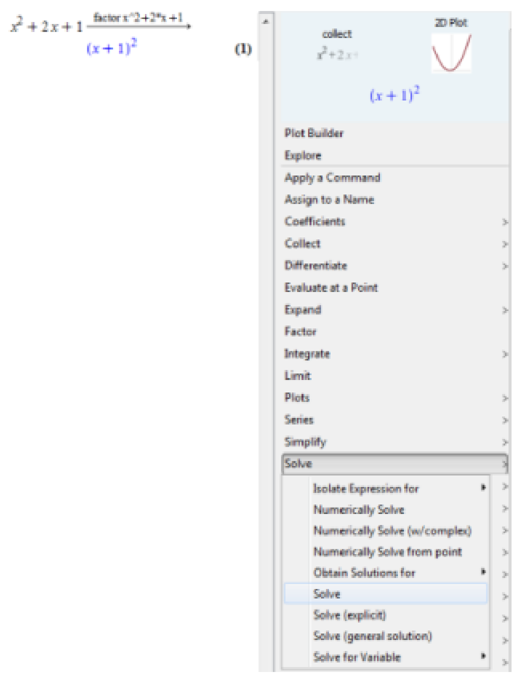


Figure 1.6: Click the expression to see applicable operations in the context panel

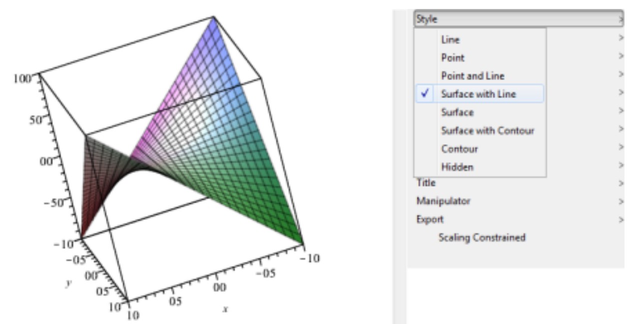


Figure 1.7: Click the plot to see plot options in the context panel

Task Templates

Task templates help you perform specific tasks in Maple, such as:

- performing a mathematical computation such as solving an equation symbolically or numerically, or determining the Taylor approximation of a function of one variable
- constructing a Maple object such as a function
- creating a document such as an application

Each task contains a description along with a collection of content that you can insert directly into your document. Content consists of 2-D mathematics, commands, embedded components (for example, buttons), and plots. You specify the parameters of your problem and then execute the commands in the document. See **Figure 1.8** for an example of a Task Template.

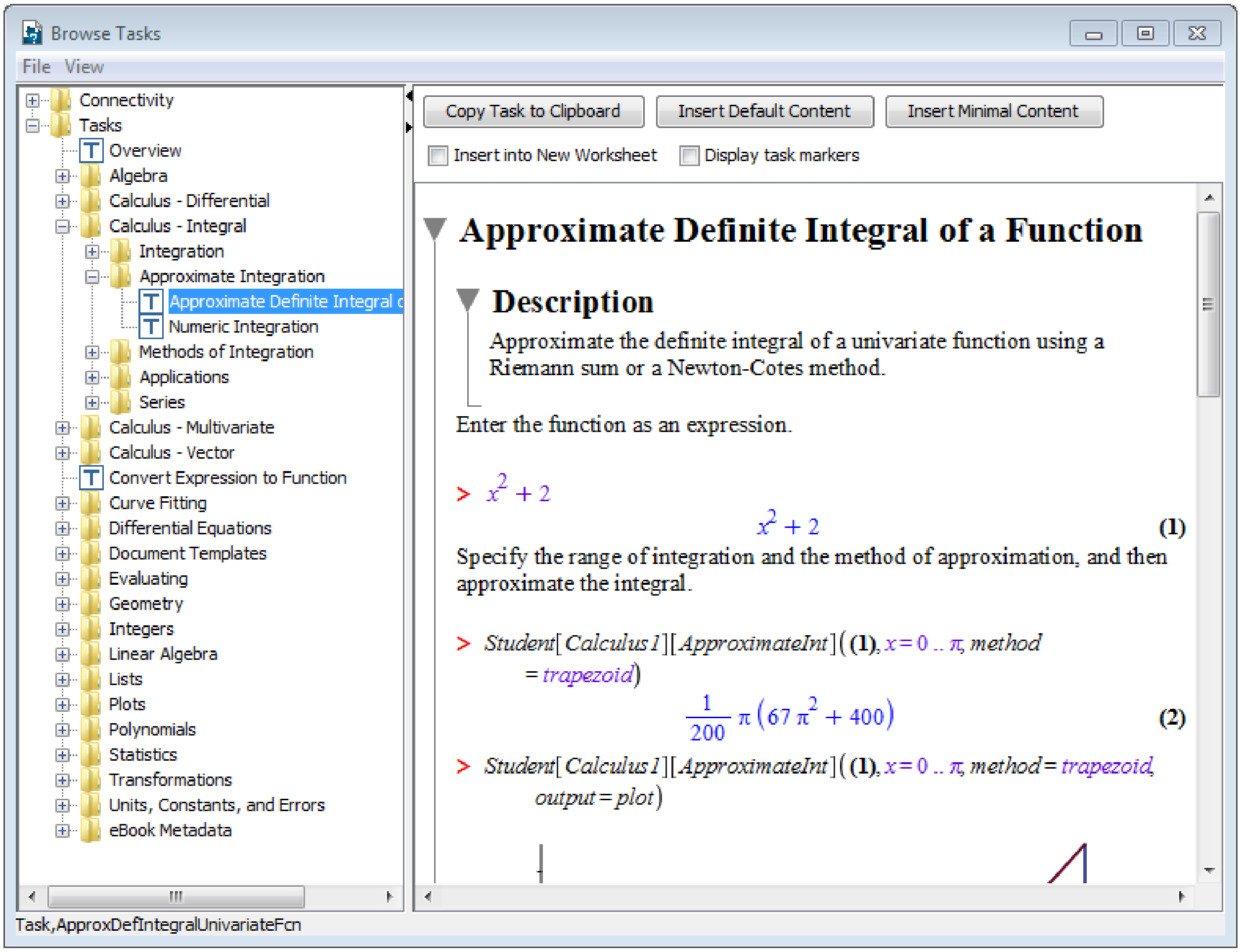


Figure 1.8: Browse Tasks Dialog

Previewing Tasks

To preview Maple tasks,

- From the **Tools** menu, select **Tasks**, and then **Browse**. The **Browse Tasks** dialog opens and displays the list of tasks.

The tasks are sorted by subject to help you quickly find the desired task. In the **Browse Tasks** dialog, you can view tasks without inserting them into your document.

Inserting a Task into the Document

To insert a task into your document,

1. Select the **Insert into New Worksheet** check box to insert the task into a new document.
2. Click one of the insert buttons.

- Click the **Insert Default Content** button. Maple inserts the *default content*. The default content level is set using the **Options** dialog. For instructions, see the **usingtasks** help page.
- Click the **Insert Minimal Content** button. Maple inserts only the commands and embedded components, for example, a button to launch the related assistant or tutor.
- Click the **Copy Task to Clipboard** button. Place the cursor where you want to insert the task, and then paste the task. Maple inserts the default content. Use this method to quickly insert a task multiple times.

Note: You can view the history of previously inserted tasks. From the **Tools** menu, select **Tasks.** Previously selected task names are displayed below the **Browse** menu item.

Before inserting a task, Maple checks whether the task variables have assigned values in your document. If any task variable is assigned, the **Task Variables** dialog opens to allow you to modify the names. Maple uses the edited variable names for all variable instances in the inserted task.

By default, the **Task Variables** dialog is displayed only if there is a naming conflict. You can set it to display every time you insert a task.

To specify that the Task Variables dialog be displayed every time you insert a task:

1. From the **Tools** menu, select **Options**.
2. Click the **Display** tab.
3. In the **Show task variables on insert** drop-down list, select **Always**.
4. Click **Apply to Session** or **Apply Globally**, as necessary.

Updating Parameters and Executing the Commands

In inserted Task Templates, parameters are marked as placeholders (in purple text) or specified using sliders or other embedded components.

1. Specify values for the parameters in placeholders or using graphical interface components. You can move to the next placeholder by pressing **Tab**.
2. Execute all commands in the task by:

- Placing the cursor in the first task command, and then pressing **Enter** repeatedly to execute each command.
- Selecting all the template commands, and then clicking the execute toolbar icon **!**.

3. If the template contains a button that computes the result, click it.

For more information on task templates, refer to the **tasks** help page.

Exploration Assistant

The Exploration Assistant allows you to interactively make parameter changes to expressions and view the result. The assistant can be used with almost any Maple expression or command that has at least one variable or parameter.

To launch the Exploration Assistant:

1. Enter an expression or command.
2. Click the expression or command. From the context panel, select **Explore**.
3. The **Explore** parameter selection dialog appears, where you can select the parameters to explore and the range for each parameter.

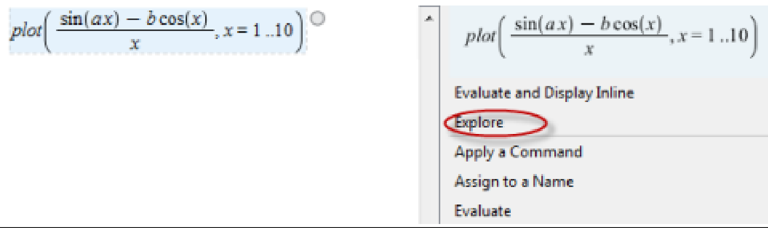
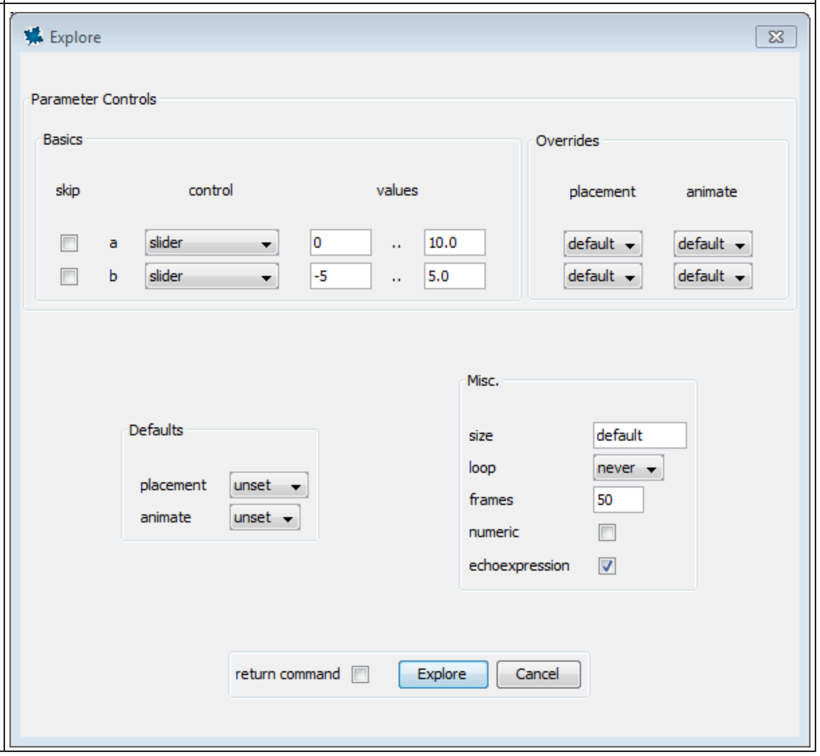
If you enter integer ranges, only integer values are allowed for parameters. To allow floating-point values, enter floating-point ranges.

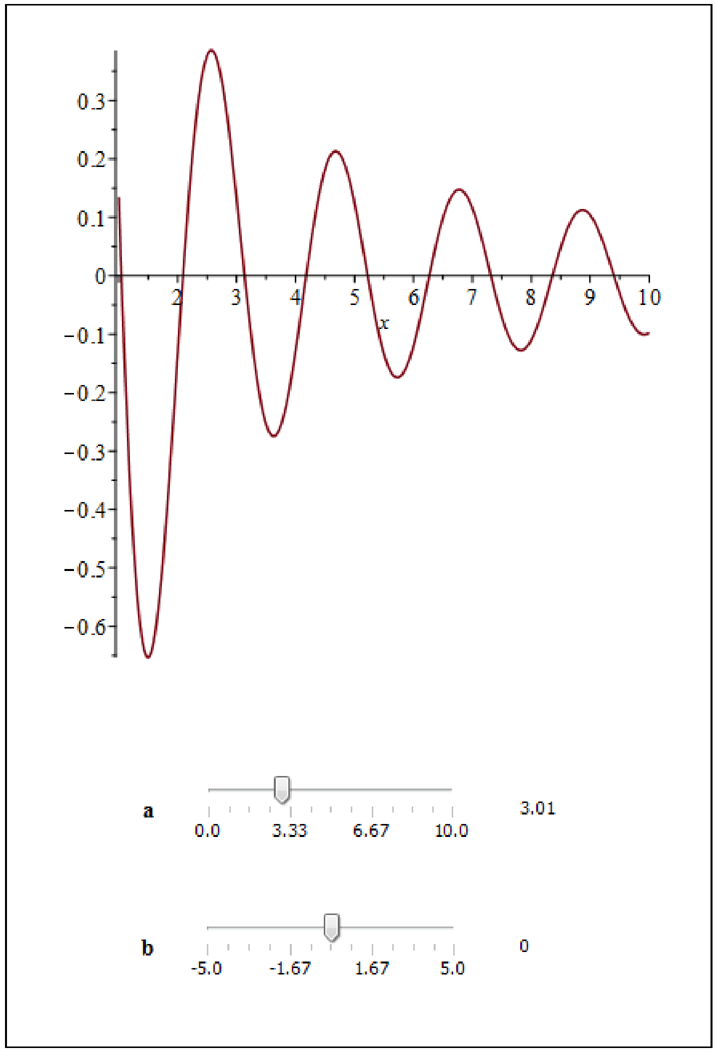
Select **skip** for any of the parameters to leave that parameter as a variable.

4. Click **Explore** to continue to the **Exploration Assistant**. The assistant creates a table in the document. You can use the slider or sliders to vary the parameters and see your changes as the expression output is updated.
5. Once you are finished interacting with the assistant, you can copy and paste the results into your document, or save the interactive document for later use.

Example 7 - Use the Exploration Assistant to Explore a Plot

In this example, we will explore how the plot of $\frac{\sin(a\,x) - b\cos(x)}{x}$ changes as we vary the parameters a and b.

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Enter the plot command shown.</td><td>$plot\left(\frac{\sin(a\,x) - b\cos(x)}{x}, x = 1\,..10\right)$</td></tr>
<tr><td>2. Click the expression and select Explore from the context panel.</td><td>
</td></tr>
<tr><td>3. In the Explore parameter selection dialog, set the ranges a = 0..10.0 and b = -5..5.0.</td><td>
</td></tr>
</table>

Action	Result in Document
4. Click **Explore**. The **Exploration Assistant** creates a table in the document. Move the sliders to see the plot as the parameters change. **Tip:** Since decimal numbers were used for the ranges in step 3, the slider uses decimal numbers. If you use integers for the ranges, the slider uses integer numbers.	

1.5 Commands

Even though Maple comes with many features to solve problems and manipulate results without entering any commands, you may find that you prefer greater control and flexibility by using the set of commands and programming language that Maple offers.

The Maple Library

Commands are contained in the Maple library, which is divided into two groups: the *main library* and *packages*.

The main library contains the most frequently used Maple commands.

Packages contain related commands for performing tasks from disciplines such as Student Calculus, Statistics, or Differential Geometry. For example, the **Optimization** package contains commands for numerically solving optimization problems.

For details on top-level and package commands, see *Commands (page 57)*.

Entering Commands

If you want to interact with Maple using commands, simply enter the command using 2-D math. Notice that commands and variable names display in italics. Maple commands are constructed in a format similar to *command***(***arguments***)**, based on the command you are using.

For example, to factor an expression, enter:

$$factor\left(x^2 + 2x + 1\right)$$

$$(x + 1)^2$$

To differentiate an expression, enter:

$$diff(\sin(x), x)$$

$$\cos(x)$$

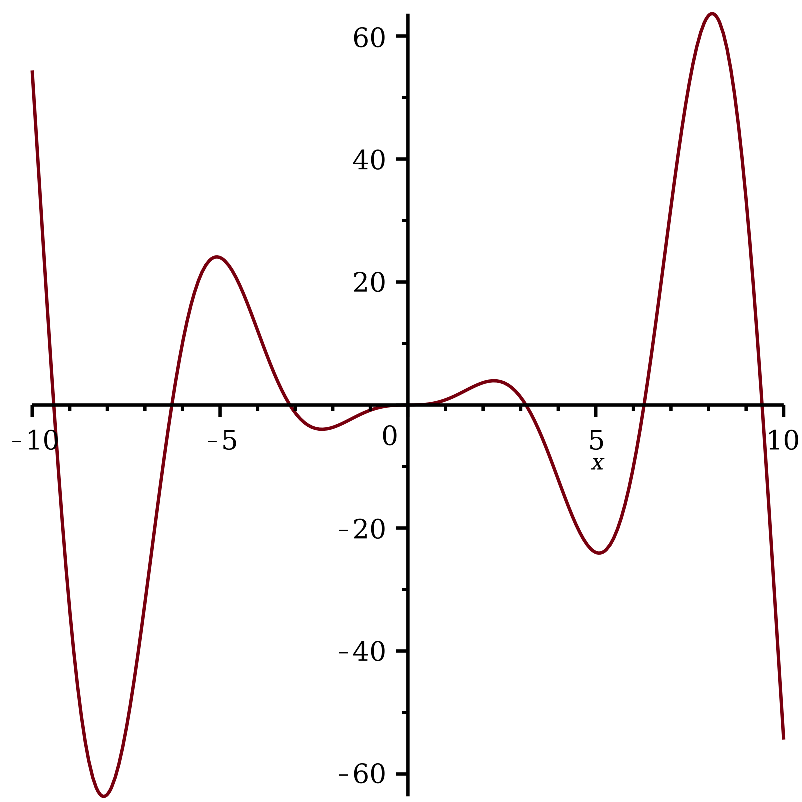
To integrate an expression on the interval $[0, 2\pi]$, enter:

$$int(2x + \cos(x), x = 0..2\pi)$$

$$4\pi^2$$

To plot an expression, enter:

$$plot\left(\sin(x)\,x^2, x = -10..10\right)$$



For a list of the top commands in Maple, see *Top Commands (page 59)*.

Package Commands

There are two ways to access commands within a package, using the long form of the package command or the short form.

Long Form of Accessing Package Commands:

The long form specifies both the package and command names using the syntax *package*[*command*](*arguments*).

$$LinearAlgebra[RandomMatrix](2)$$

$$\begin{bmatrix} 44 & -31 \\ 92 & 67 \end{bmatrix}$$

Short Form of Accessing Package Commands:

The short form makes all of the commands in the package available using the **with** command, *with(package)*. If you are using a number of commands in a package, loading the entire package is recommended. When you execute the **with** command, a list of all commands in the package displays. To suppress the display of all command names, end the *with(package)* command with a colon. Alternatively, you can load packages through the **Tools** menu, by selecting **Load Package**, and then the package name.

$$with(Optimization)$$

$$[ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]$$

After loading a package, you can use the short-form names, that is, the command names, without the package name.

$$LSSolve([x-2, x-6, x-9])$$

$$[12.3333333333333321, [x = 5.66666666666667]]$$

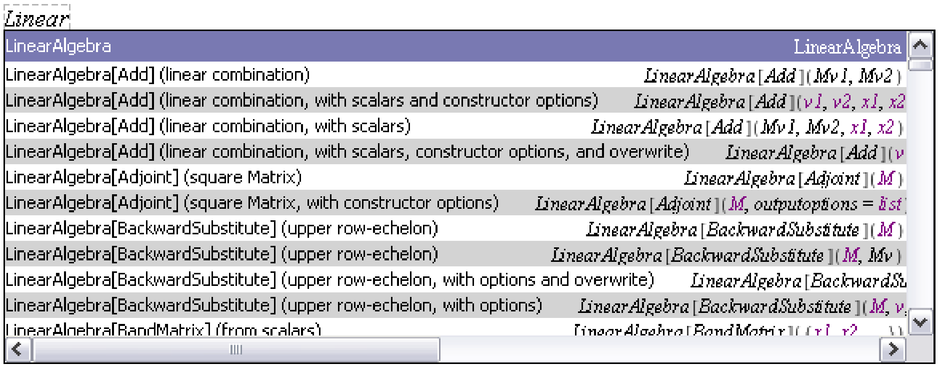
For a list of the top packages in Maple, see *Top Packages (page 60)*.

Command Completion

To help with syntax and reduce the amount of typing when entering Maple commands, you can use *command completion*. Command completion displays a list of all Maple packages, commands, and functions that match the entered text. If there are multiple ways to call a command, then the command completion list contains each one, with appropriate placeholders.

To use command completion:

1. Begin entering a command or package name.
2. Select **Tools** → **Complete Command** or use the shortcut key **Esc** (see *Shortcut Keys by Platform (page xiii)*). If there is a unique completion, it is inserted. Otherwise, a list of possible matches is displayed.
3. Select the correct completion from the list.



4. Some inserted commands have placeholders, denoted by purple text. The first placeholder is highlighted after you insert it into the document. Replace it with your parameter, then move to the next placeholder by pressing the **Tab** key.

Equation Labels

Equation labels help to save time entering expressions by referencing Maple output. See **Figure 1.9**.

By default, equation labels are displayed. If equation labels are not displayed,

1. From the **Tools** menu, select **Options**, and click the **Display** tab. Ensure that the **Show equation labels** check box is selected.
2. From the **Format** menu, select **Equation Labels**. Ensure that both **Execution Group** and **Worksheet** are selected.

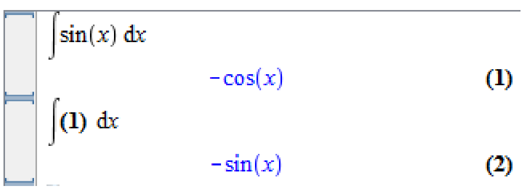


Figure 1.9: Equation Label

To apply equation labels:

1. Enter an expression and press **Enter**. Note that the equation label is displayed to the right of the answer in the document.
2. In a new execution group, enter another expression that will reference the output of the previous execution group.
3. From the **Insert** menu, select **Label**. Alternatively, press **Ctrl**+**L** (**Command**+**L**, for Mac) to open the **Insert Label** dialog. Enter the label number in the **Insert Label** dialog and click **OK**. The item is now a label. See **Figure 1.10**.

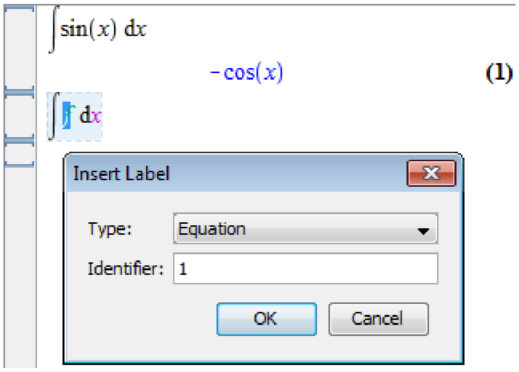


Figure 1.10: Inserting an Equation Label

4. Press **Enter** to obtain the result.

To change the format of equation labels:

- Select **Format** → **Equation Labels** → **Label Display**. In the **Format Labels** dialog, select one of the numbering schemes. Numbering can be flat numbers for the entire worksheet, or by section, so that equations in section 1 are labeled **(1.1)**, **(1.2)**, and so on.
- Optionally, enter an appropriate numbering prefix.

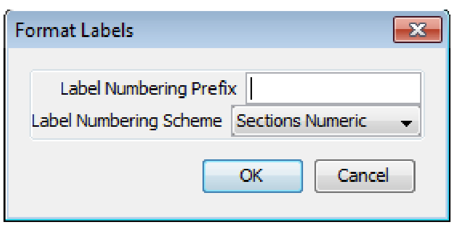


Figure 1.11: Controlling Equation Label Format

The **Label Reference** menu item allows you to switch between the label name and its reference content. Place the cursor on the referenced equation label and select **Format** → **Equation Labels** → **Label Reference**.

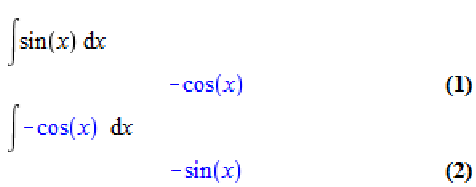


Figure 1.12: Label Reference

The label is associated with the last output within an execution group.

You cannot apply equation labels to the following:

- Error, warning, and information messages
- Tables, images, plots, or sketches

Document Blocks

In Document mode, content is created as a series of document blocks. Document blocks allow you to hide the syntax used to perform calculations, which in turn lets you focus on the concept presented instead of the command used to manipulate or solve the problem. You can also create document blocks in Worksheet mode to perform the same function.

Document blocks are typically collapsed to hide the Maple code, but these regions can also be expanded to reveal this code.

To create a document block:

From the **Edit** menu, select **Document Blocks** → **Create Document Block**. If text or math in one or more execution groups is selected, then a document block is created that contains those execution groups. If not, a new document block is created after the current execution group. For more information, see the next example.

Document block regions are identified using markers that are located in a vertical bar along the left pane of the document. See **Figure 1.13**. In addition to document block boundaries, these markers (icons) indicate the presence of hidden attributes in the document such as annotations, bookmarks, and numeric formatting.

To activate markers:

From the **View** menu, select **Markers**. See **Figure 1.13**.

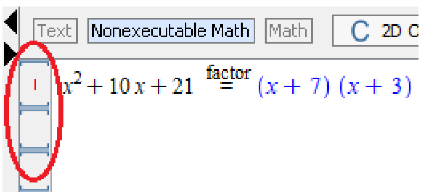


Figure 1.13: Document Block Markers

To view code in a document block:

1. Place the cursor in a document block to be expanded.
2. From the **Edit**→**Document Blocks** menu, select **Show Command**.

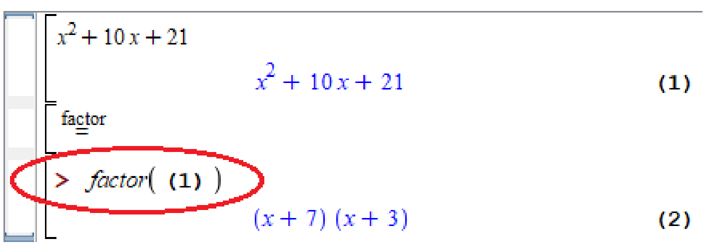


Figure 1.14: Expanded Document Block

With the Document Block expanded, you can see the Maple command that was used to perform this calculation. In **Figure 1.14**, the *solve* command was used.

Also notice a red prompt (>) before the original expression and the *solve* command. Entering commands outside of a document block region is done at this input region. To insert an input region, click the button in the toolbar menu.

In **Figure 1.14**, an equation label was used to refer to the expression. For more information, see *Equation Labels (page 34)*.

To collapse a Document Block:

- With your cursor inside the document block, from the **Edit** → **Document Blocks** menu, clear the check box for **Show Command**.

You can use this process of expanding document blocks to view and edit Maple commands within a document block.

Changing the Display:

You can specify which parts of the input and output are displayed when the document block is collapsed. For each execution group in the block, you can choose to display either the input or the output.

- Place the cursor in the execution group.
- From the **Edit** → **Document Blocks** menu, select **Toggle Input/Output Display**.

Also, you can choose to display output either inline or centered on a new line.

- From the **Edit** → **Document Blocks** menu, select **Inline Document Output**.

1.6 The Maple Help System

The Maple program provides a custom help system consisting of almost 5000 reference pages. The help system is a convenient resource for determining the syntax of Maple commands and for learning about Maple features.

Accessing the Help System

There are several ways to access the Maple help system:

- From the **Help** menu, select **Maple Help**.
- Enter a search term in the search box in the worksheet toolbar.
- Click  in the toolbar.

To get help on a specific word:

- In a document, place the insertion point in a word for which you want to obtain help. From the **Help** menu, select **Help on ...**. Alternatively, press **F2** (**Control** + **?**, for Mac) to access context-sensitive help.
- In a document, execute the command **?topic**, for example, enter **?LinearAlgebra** and press **Enter**.

The Maple help system opens in a separate window with two panes. The left pane contains the Help Navigator where you initiate searches and browse the table of contents, and the right pane displays the final search result, such as a specific help page.

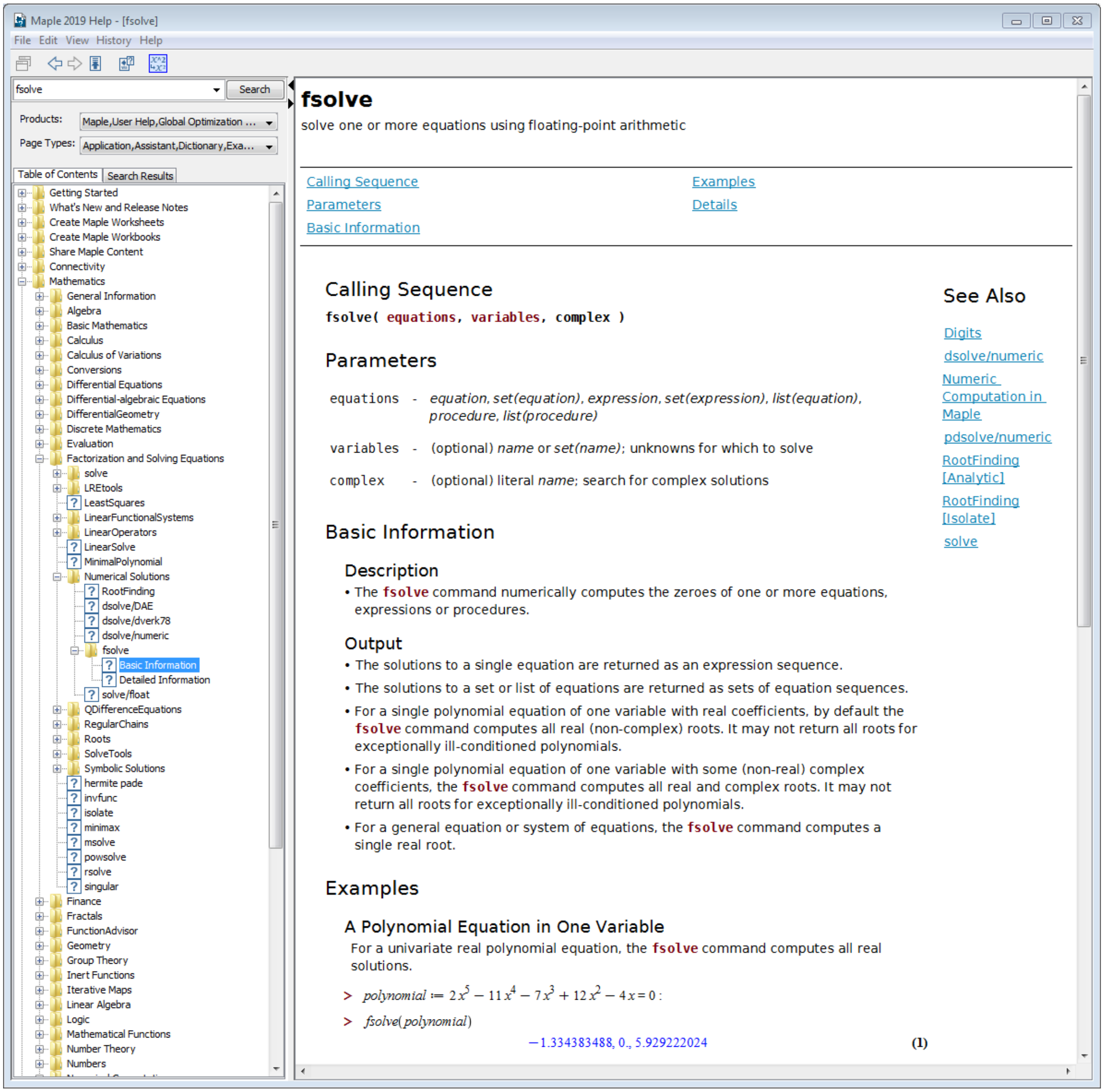


Figure 1.15: Sample Help Page

Every help page in Maple lists the command's calling sequence, parameters, and a description, with examples of the command at the end of the page. Some help pages also contain hyperlinks to related help pages and hyperlinks to dictionary definitions. Hyperlinks to dictionary definitions display in red to distinguish them from hyperlinks to help pages.

Using the Help Navigator

Use the search field in the Help Navigator to find information on Maple commands and features.

- **Search** for a known help topic, a command name, or a keyword or phrase.
- If you have any add-on products, you can restrict the search to Maple, MapleSim, or any combination of those products and their add-ons.

- You can search all of the help system or specific Resources such as Help Pages, Tasks, Math Apps, and Manuals by selecting the **Page Types** drop-down menu.

Search results are displayed as a list in the **Search Results** tab of the left pane. Click the **Table of Contents** tab to view a structured list of all topics in the help system.

Note that some tutorials open in a Maple window instead of in the Help window.

In the left pane, the type of resource is indicated by an icon. **Table 1.10** describes the icons.

Table 1.10: Help Page Icons

Icon	Description
	A folder icon in the **Table of Contents** tab indicates that a topic can be expanded into subtopics.
	Question mark icon indicates a help page and displays the associated help page in the right pane when selected.
	WS icon indicates an example worksheet or tutorial. These worksheets open in a new tab in the Maple document.
	D icon indicates a definition and displays the associated dictionary definition in the right pane when selected.
	T icon indicates a Task template and displays the associated Task Template in the right pane when selected.

Viewing Help Pages as Documents

In the help system, examples are not executable.

The Maple help system allows you to open help pages as documents that you can execute.

To open a help page as a worksheet:

- With the help page displayed in the right pane of the help system, from the **View** menu, select **Open Page as Worksheet.** A new worksheet tab opens and displays the help page as an executable document.

	Alternatively, in the help system toolbar, click the *open current help page in a worksheet window* icon.

Viewing Examples in 2-D Math

You can choose to view the examples in most help pages in either 2-D math or 1-D Math (Maple input) mode. The default is 2-D Math.

To change the math mode:

In the Maple help system:

- From the **View** menu, select or clear the **Display Examples with 2D math input** check box.
- Click the **2-D Math** icon, x^2 ↳ x^2.

Note: Some input in help pages displays as 1-D Math, no matter which option you have chosen. This is for Maple procedures and other code that is best input in 1-D Math. For more information, see the **helpnavigator** help page.

Copying Examples

Instead of opening the entire page as a document, you can copy the **Examples** section only.

To copy examples:

1. With the help page displayed in the right pane of the help system, from the **Edit** menu, select **Copy Examples.**
2. Close or minimize the Help Navigator and return to your document.
3. In your document, place the cursor at the location where you want to paste the examples.

4. From the **Edit** menu, select **Paste**. The **Examples** section of the help page is inserted as executable content in your document.

1.7 Available Resources

Your work with Maple is supported by numerous resources.

Resources Available through the Maple Help System

Help Pages

Use the help system to find information about a specific topic, command, package, or feature. For more information, see *The Maple Help System (page 37)*.

Dictionary

More than 5000 mathematical and engineering terms with over 300 figures and plots.

1. From the **Help** menu, select **Maple Help**.
2. Enter a search term. Dictionary entries that match your query are displayed in the left pane with a D icon.

Tutorials and the Maple Portal

The **Maple Portal** includes material designed for all Maple users, from new users to users who want more advanced tutorials. The Maple Portal also includes specific sections for students and engineers. The Maple Portal includes:

- How Do I... topics that give quick answers to essential questions.
- Tutorials that provide an overview of topics from getting started to plotting, data manipulation, and interactive application development.
- Navigation to portals with specialized information for students and engineers.

Access the portal in the Table of Contents under **Getting Started**.

Applications and Example Worksheets

Applications

Sample applications demonstrate how Maple can be used to find and document a solution to a specific problem. Some applications allow for input or contain animations that you can run; however, their primary use is for demonstrations. Topics include Bouncing Ball, Digital Filter Design, Frequency Domain System Identification, Harmonic Oscillator, Image Processing, Radiator Design with CAD Systems, and Sunspot Periodicity.

Examples

Example worksheets are executable documents covering topics that demonstrate syntax or invoke a user interface to make complex problems easy to solve and visualize. You can copy and modify the examples as needed. Topics include Algebra, Calculus, Connectivity, Discrete Mathematics, General Numerics and Symbolics, and Integral Transforms.

- Explore the available topics in the Table of Contents under **Applications and Example Worksheets**.

Manuals

You can access all of Maple's manuals from within Maple, including the *Maple Programming Guide* and this manual. You can execute examples, copy content into other documents, and search the contents using the Maple Help System.

- Access the manuals in the Table of Contents under **Manuals**.

Maple's manuals are also available as PDFs on the Maplesoft website.

http://www.maplesoft.com/documentation_center

Task Templates

Set of commands with placeholders that you can use to quickly perform a task. For details, see *Task Templates (page 27)*.

- From the **Tools** menu, select **Tasks**, and then **Browse**.

Quick Reference Card

The Quick Reference Card is a table of commands and information for new users that opens in a new window. It contains hyperlinks to help pages for more information.

- From the **Help** menu, select **Quick Reference**. Alternatively, press **Ctrl** + **F2** (**Command** + **F2**, for Mac).

Website Resources

Student Help Center

The Student Help Center offers a Maple student forum, online math Oracles, training videos, and a math homework resource guide.

http://www.maplesoft.com/studentcenter

Teacher Resource Center

The Teacher Resource Center is designed to ensure you get the most out of your Maple teaching experience. It provides sample applications, course material, training videos, white papers, and tips.

http://www.maplesoft.com/TeacherResource

Application Center

Maple website resource for free applications related to mathematics, education, science, engineering, computer science, statistics and data analysis, finance, communications, and graphics. Many applications are available in translations (French, Spanish, and German).

http://www.maplesoft.com/applications

Training

Maplesoft offers a comprehensive set of complementary training materials. From complete training videos to recorded training seminars to downloadable documentation, you have many options to get familiar with Maplesoft products. In addition, custom training sessions can be created to meet your needs.

http://www.maplesoft.com/support/training

MaplePrimes

A web community dedicated to sharing experiences, techniques, and opinions about Maple and related products, as well as general interest topics in math and computing.

http://www.mapleprimes.com

Online Help

All of Maple's help pages are available online.

http://www.maplesoft.com/support/help

Technical Support

A Maple website containing FAQs, downloads and service packs, links to discussion groups, and a form for requesting technical support.

http://www.maplesoft.com/support

For a complete list of resources, refer to the **MapleResources** help page.

2 Document Mode

Using the Maple software, you can create powerful interactive documents. You can visualize and animate problems in two and three dimensions. You can solve complex problems with simple point-and-click interfaces or easy-to-modify interactive documents. You can also devise custom solutions using the Maple programming language. While you work, you can document your process, providing text descriptions.

2.1 In This Chapter

Section	Topics
Introduction (page 43)	• Comparison of Document and Worksheet Modes
Entering Expressions (page 44) - Overview of tools for creating complex mathematical expressions	• Palettes • Symbol Names • Mathematical Functions
Evaluating Expressions (page 46) - How to evaluate expressions	• Displaying the Value Inline • Displaying the Value on the Following Line
Editing Expressions and Updating Output (page 47) - How to update expressions and regenerate results	• Updating a Single Computation • Updating a Group of Computations • Updating All Computations in a Document
Performing Computations (page 48)- Overview of tools for performing computations and solving problems	• Computing with Palettes • Context Panel • Assistants and Tutors

2.2 Introduction

Maple has two modes: *Document* mode and *Worksheet* mode.

Document mode is designed for quickly performing calculations. You can enter a mathematical expression, and then evaluate, manipulate, solve, or plot it with a few keystrokes or mouse clicks. This chapter provides an overview of Document mode.

Document mode sample:

Find the value of the derivative of $\ln(x^2 + 1)$ at $x = 4$.

$$\ln(x^2 + 1) \xrightarrow{\text{differentiate w.r.t. x}} \frac{2\,x}{x^2 + 1} \xrightarrow{\text{evaluate at point}} \frac{8}{17}$$

Integrate $\sin\left(\frac{1}{x}\right)$ over the interval $[0, \pi]$.

$$\int_0^{\pi} \sin\left(\frac{1}{x}\right)\,\mathrm{d}x = \sin\left(\frac{1}{\pi}\right)\pi - \mathrm{Ci}\left(\frac{1}{\pi}\right)$$

Worksheet mode is designed for interactive use through commands and programming using the Maple language. The Worksheet mode supports the features available in Document mode described in this chapter. For information on using Worksheet mode, see Chapter 3, *Worksheet Mode (page 55)*. **Note**: To enter a Maple input prompt while in Document mode, click >_ in the Maple toolbar.

Important: In any Maple document, you can use Document mode and Worksheet mode.

Interactive document features include:

- Embedded graphical interface components, like buttons, sliders, and check boxes
- Automatic execution of marked regions when a file is opened
- Tables
- Character and paragraph formatting styles
- Hyperlinks

These features are described in Chapter 7, *Creating Mathematical Documents (page 227)*.

Note: This chapter and Chapter 1 were created using Document mode. All of the other chapters were created using Worksheet mode.

2.3 Entering Expressions

Chapter 1 provided an introduction to entering simple expressions in 2-D Math (see *Entering Expressions (page 14)*). It is also easy to enter mathematical expressions, such as:

- Piecewise-continuous functions: $|x| = \begin{cases} -x & x < 0 \\ 0 & x = 0 \\ x & 0 < x \end{cases}$
- Limits: $\delta(x) = \lim_{\epsilon \to 0} \epsilon |x|^{\epsilon - 1}$
- Continued fractions: $\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}$

and more complex expressions.

Mathematical expressions can contain the following objects.

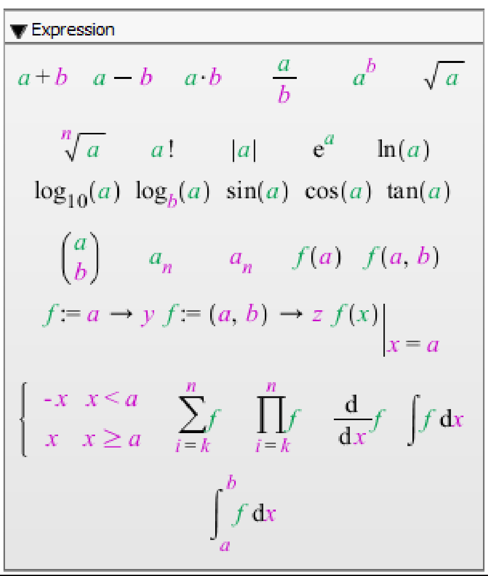
- Numbers: integers, rational numbers, complex numbers, floating-point values, finite field elements, i, ∞ , ...
- Operators: $+$, $-$, $!$, $/$, $\cdot$, $\int$, $\lim_{x \to a}$, $\frac{\partial}{\partial x}$, ...
- Constants: π, e, ...
- Mathematical functions: $\sin(x)$, $\cos\left(\frac{\pi}{3}\right)$, $\Gamma(2)$, ...
- Names (variables): x, y, z, α, β, ...
- Data structures: sets, lists, Arrays, Vectors, Matrices, ...

Maple contains over a thousand symbols. For some numbers, operators, and names, you can press the corresponding key, for example, **9**, =, >, or **x**. Most symbols are not available on the keyboard, but you can insert them easily using two methods, palettes and symbol names.

Example 1 - Enter a Partial Derivative

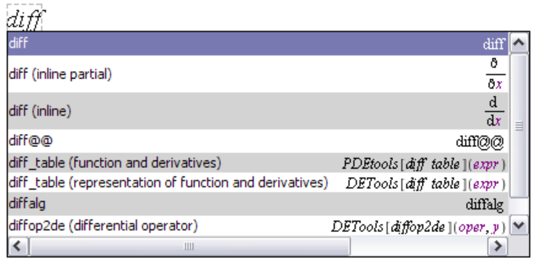
To insert a symbol, you can use palettes or symbol names.

Enter the partial derivative $\frac{\partial}{\partial t} e^{-t^2}$ using palettes.

Action	Result in Document
1. In the **Calculus** palette, click the partial differentiation item $\frac{\partial}{\partial x} f$. Maple inserts the partial derivative. The variable placeholder is selected.	$\frac{\partial}{\partial x} f$
2. Enter **t**, and then press **Tab**. The expression placeholder is selected.	$\frac{\partial}{\partial t} f$
3. Enter e^{-t^2}. **Note**: To enter the exponential e, use the **Expression** palette or the **Common Symbols** palette. Here, we use the e^a template from the Expression palette. Click the template. The exponent placeholder is selected. Enter the exponent. 	$\frac{\partial}{\partial t} e^{-t^2}$

To evaluate the derivative and display the result inline, press **Ctrl+=** (**Command**+=, for Macintosh) or **Enter**. For more information, see *Computing with Palettes (page 48)*.

You can enter any expression using symbol names and the symbol completion list.

Action	Result in Document
1. Begin typing the name of the symbol, **diff**, and press the symbol completion key (see *Shortcut Keys by Platform (page xiii)*).	
2. Select the partial differentiation item, diff (inline partial) $\frac{\partial}{\partial x}$	$\frac{\partial}{\partial x}$
3. Replace the placeholder with **t**. Use the right arrow to move out of the denominator. Enter e^{-t^2} as in the previous example.	$\frac{\partial}{\partial t} e^{-t^2}$

Example 2 - Define a Mathematical Function

Define the function *twice*, which doubles its input.

Action	Result in Document
1. In the **Expression** palette, click the single variable function definition item, $f := a \rightarrow y$.	$f := a \rightarrow y$
2. Replace the placeholder **f** with the function name, *twice*. Press **Tab** to move to the next placeholder.	$twice := a \rightarrow y$
3. Replace the parameter placeholder, **a**, with the independent variable x. Press **Tab**.	$twice := x \rightarrow y$
4. Replace the output placeholder, **y**, with the desired output, $2\,x$.	$twice := x \rightarrow 2\,x$ $twice := x \mapsto 2\,x$

$twice(1342) = 2684$

$twice(y - z) = 2\,y - 2\,z$

Note: To insert the right arrow symbol $\rightarrow$, you can also enter the characters -> in Math mode. In this case, symbol completion is automatic.

Important: The expression $2\,x$ is different from the function $x \rightarrow 2\,x$.

For more information on functions, see *Functional Operators (page 274)*.

2.4 Evaluating Expressions

To evaluate a mathematical expression, place the cursor in the expression and press **Ctrl** + = (**Command** + =, for Macintosh). That is, *press and hold* the **Ctrl** (or **Command**) key, and then press the equal sign (=) key.

To the right of the expression, Maple inserts an equal sign and then the value of the expression.

$$\frac{2}{9} + \frac{7}{11} = \frac{85}{99}$$

In mathematical content, pressing **Enter** evaluates the expression and displays it centered on the following line. The cursor moves to a new line below the output.

$$\frac{2}{9} + \frac{7}{11}$$

$$\frac{85}{99} \tag{2.1}$$

By default, Maple labels output that is generated by pressing **Enter**. For information on equation labels, see *Equation Labels (page 68)*. In this manual, labels are generally not displayed.

You can use the basic algebraic operators, such as $+$ and $-$, with most expressions, including polynomials—see *Polynomial Algebra (page 110)* and matrices and vectors—see *Matrix Arithmetic (page 124)*.

$$\left(2\,x^2 - x + 1\right) - \left(x^2 + 2\,x + 12\right) = x^2 - 3\,x - 11$$

$$3 \cdot \begin{bmatrix} -4 & 8 & 99 \\ 27 & 69 & 29 \end{bmatrix} = \begin{bmatrix} -12 & 24 & 297 \\ 81 & 207 & 87 \end{bmatrix}$$

Tip: The **Enter** key has a number of uses.

- Pressing **Enter** with your cursor in a math expression executes it and evaluates it on a new line.
- Pressing **Enter** in text inserts a line break.
- Pressing **Enter** when at the beginning of a line (the **Home** position) inserts a new line above the current line.

For more information on these concepts, refer to the Using Document Blocks help page.

Tip: If you want to change an input/output expression from evaluate and display inline to evaluate and display on a new line, with your cursor in the input expression, from the **Evaluate** menu or the Context Panel, clear the check box beside **Evaluate and Display Inline**.

2.5 Editing Expressions and Updating Output

One important feature of Maple is that your documents are *live*. That is, you can edit expressions and quickly recalculate results.

To update one computation:

1. Edit the expression.
2. Press **Ctrl** + **=** (**Command** + **=**, for Macintosh) or **Enter**.

The result is updated.

To update a group of computations:

1. Edit the expressions.
2. Select all edited expressions and the results to recalculate.
3. Click the Execute toolbar icon ! .

All selected results are updated.

To update all output in a Maple document:

- Click the Execute All toolbar icon .

All results in the document are updated.

Note: Be careful when you revisit a document and make changes, as it's possible to produce a document with worksheet commands out of order (i.e. where a certain command won't work properly without a later one).

2.6 Performing Computations

Using the Document mode, you can access the power of the advanced Maple mathematical engine without learning Maple syntax. In addition to solving problems, you can also easily plot expressions.

The primary tools for syntax-free computation are:

- Palettes
- The Context Panel
- Assistants and tutors

Note: The Document mode is designed for quick calculations, but it also supports Maple commands. For information on commands, see *Commands (page 57)* in Chapter 3, *Worksheet Mode (page 55)*.

Important: In Document mode, you can execute a statement *only if* you enter it in Math mode. To use a Maple command, you must enter it in Math mode.

Computing with Palettes

As discussed in *Entering Expressions (page 44)*, some palettes contain mathematical operations.

To perform a computation using a palette mathematical operation:

1. In a palette that contains operators, such as the **Expression** or **Calculus** palettes, click an operator item.
2. In the inserted item, specify values in the placeholders.
3. To execute the operation and display the result, press **Ctrl+=** (**Command**+=, for Macintosh) or **Enter**.

For example, to evaluate $\frac{\partial}{\partial t} e^{-t^2}$ **inline:**

1. Using the **Calculus** palette, enter the partial derivative. See *Example 1 - Enter a Partial Derivative (page 45)*.
2. Press **Ctrl+=** (**Command**+=, for Macintosh).

$$\frac{\partial}{\partial t} e^{-t^2} = -2\, t\, e^{-t^2}$$

Computing with the Context Panel

The context panel is a collection of tools and operations that are appropriate for a particular expression. The context panel changes according to the expression, table, or region that you click on. See **Figure 2.1**.

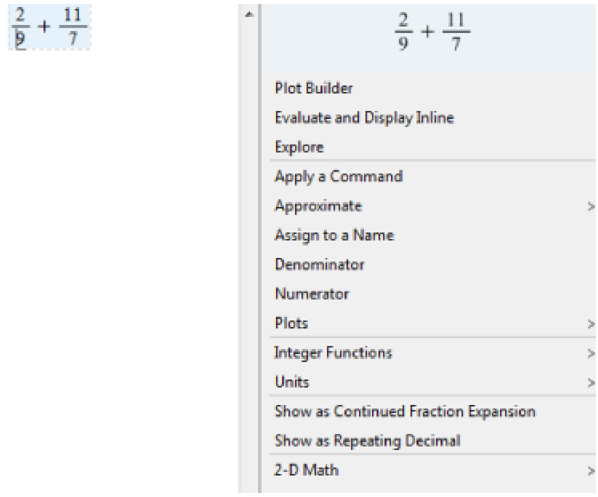
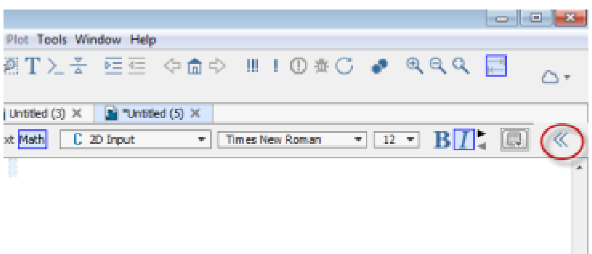


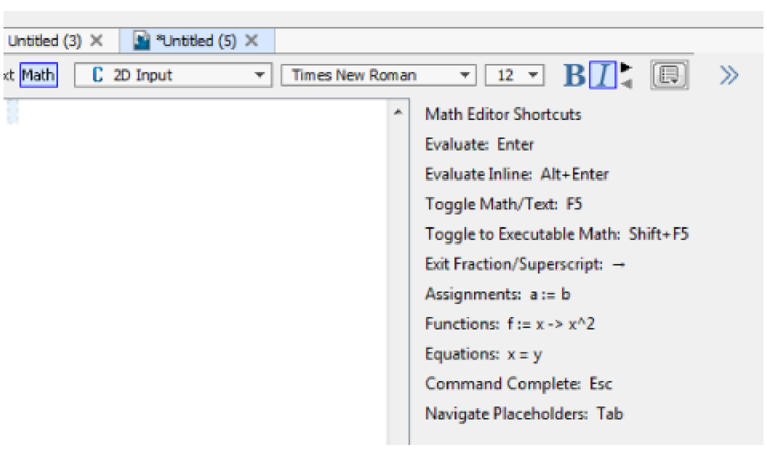
Figure 2.1: Context Panel

To display the context panel:

1. Click the expression.
2. Move your mouse cursor over Pin Open Context Panel («), or click it to fix the context panel in place.



The context panel is displayed.



You can evaluate expressions using context panel options. The **Evaluate and Display Inline** operation (see **Figure 2.1**) is equivalent to pressing **Ctrl+=** (**Command+=**, for Macintosh). That is, it inserts an equal sign (=) and then the value of the expression.

Alternatively, press **Enter** to evaluate the expression and display the result centered on the following line.

For more information on evaluation, see *Evaluating Expressions (page 46)*.

From the context panel, you can also select operations different from evaluation. To the right of the expression, Maple inserts a right arrow symbol (→) and then the result.

For example, use the **Approximate** operation to approximate a fraction: $\frac{2}{3} \xrightarrow{\text{at 10 digits}} 0.6666666667$

You can perform a sequence of operations by repeatedly using context panel options. For example, to compute the derivative of $\cos(x^2)$, use the **Differentiate** operation on the expression, and then to evaluate the result at a point, use the **Evaluate at a Point** operation on the output and enter 10:

$$\cos(x^2) \xrightarrow{\text{differentiate w.r.t. x}} -2\sin(x^2)\,x \xrightarrow{\text{evaluate at point}} -20\sin(100)$$

Note: For the sequence of operations to make sense when being read from left to right, stale results are deleted before new operations are performed.

For example:

Enter the expression x^2.	x^2
Click on the expression and use the **Differentiate** option from the context panel to differentiate with respect to x.	$x^2 \xrightarrow{\text{differentiate w.r.t. x}} 2\,x$
Click on 2x and use the **Integrate** option from the context panel to integrate with respect to x.	$x^2 \xrightarrow{\text{differentiate w.r.t. x}} 2\,x \xrightarrow{\text{integrate w.r.t. x}} x^2$
Click on 2x again and use the **Differentiate** option to differentiate with respect to x. Notice how the result of integration with respect to x has been replaced with the result of differentiation with respect to x so that the sequence of operations makes sense.	$x^2 \xrightarrow{\text{differentiate w.r.t. x}} 2\,x \xrightarrow{\text{differentiate w.r.t. x}} 2$

The following subsections provide detailed instructions on performing a few of the numerous operations available using context-sensitive operations in the context panel.

Approximating the Value of an Expression

To approximate a fraction numerically:

1. Enter a fraction.
2. Click the fraction to display the context panel. See **Figure 2.2**.
3. From the context panel, select **Approximate**, and then the number of significant digits to use: **5**, **10**, **20**, **50**, or **100**.

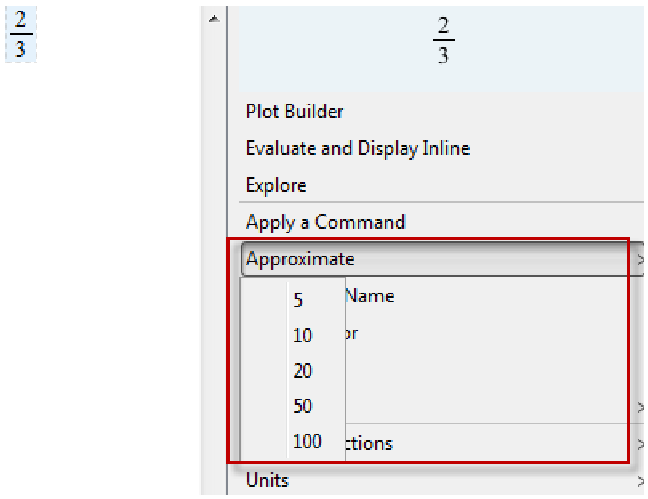


Figure 2.2: Approximating the Value of a Fraction

$$\frac{2}{3} \xrightarrow{\text{at 10 digits}} 0.6666666667$$

You can replace the inserted right arrow with text or mathematical content.

To replace the right arrow (→):

1. Select the arrow and text. Press **Delete**.
2. Enter the replacement text or mathematical content.

Note: To replace the right arrow with text, you must first press **F5** to switch to Text mode.

For example, you can replace the arrow with the text "is approximately equal to" or the symbol ≈.

$$\frac{2}{3} \xrightarrow{\text{is approximately equal to}} 0.6666666667$$

$$\frac{2}{3} \approx 0.6666666667$$

Solving an Equation

You can find an exact (*symbolic*) solution or an approximate (*numeric*) solution of an equation. For more information on symbolic and numeric computations, see *Symbolic and Numeric Computation (page 74)*.

To solve an equation:

1. Enter an equation.
2. Display the context panel. See **Figure 2.3**.
3. From the context panel, select **Solve** or **Numerically Solve** in the **Solve** menu item.

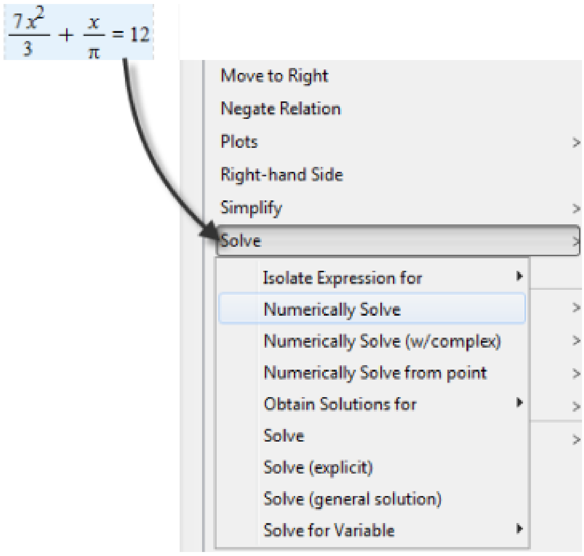


Figure 2.3: Finding the Approximate Solution to an Equation

$$\frac{7\,x^2}{3} - \frac{x}{\pi} = 12 \xrightarrow{\text{solve}} \left\{x = \frac{3}{14}\,\frac{1+\sqrt{112\,\pi^2+1}}{\pi}\right\}, \left\{x = -\frac{3}{14}\,\frac{-1+\sqrt{112\,\pi^2+1}}{\pi}\right\}$$

$$\frac{7\,x^2}{3} - \frac{x}{\pi} = 12 \xrightarrow{\text{solve}} -2.200603126, 2.337021648$$

For more information on solving equations, including solving inequations, differential equations, and other types of equations, see *Solving Equations (page 80)*.

Using Units

You can create expressions with units. To specify a unit for an expression, use the **Units** palette. See **Figure 2.4**.

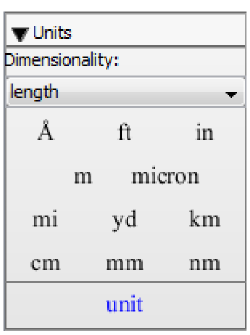


Figure 2.4: Units Palette

To insert an expression with a unit:

1. Enter the expression.
2. In a unit palette, click a unit symbol.

Note: To include a reciprocal unit, divide by the unit.

To evaluate an expression that contains units:

1. Enter the expression using the units palettes to insert units.
2. Click the expression.
3. From the context panel, select the **Simplify** menu and then **Simplify**.

For example, compute the electric current passing through a wire that conducts 590 coulombs in 2.9 seconds.

$$\frac{590\mathrm{C}}{2.9\mathrm{s}} \xrightarrow{\text{simplify units}} 203.4482759\ \mathrm{A}$$

For more information on using units, see *Units (page 93)*.

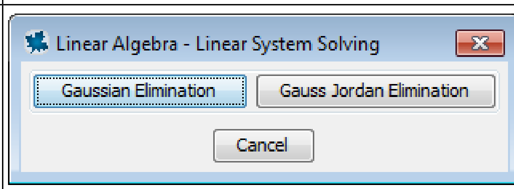
Assistants and Tutors

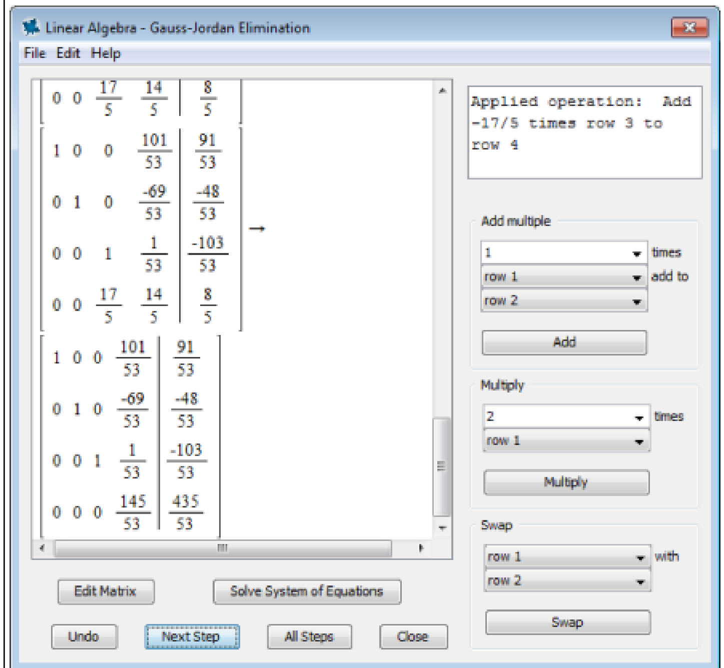
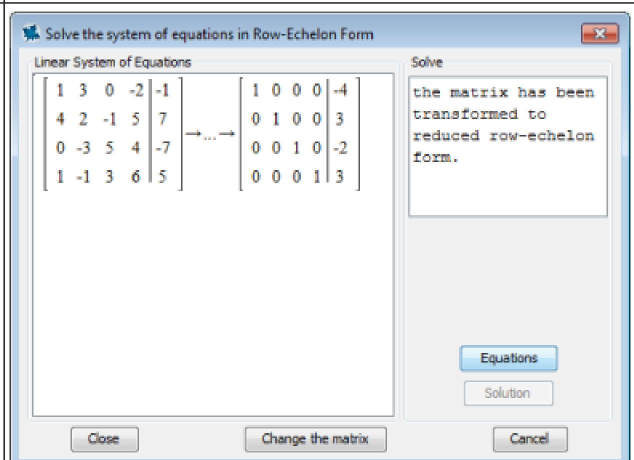
Assistants and tutors provide point-and-click interfaces with buttons, text input regions, and sliders. For details on assistants and tutors, see *Point-and-Click Interaction (page 24)*.

Assistants and tutors can be launched from the **Tools** menu or the Context Panel for an expression. For example, you can use the **Linear System Solving** tutor to solve a linear system, specified by a matrix or a set of equations.

Example 3 - Using the Context Panel to Open the Linear System Solving Tutor

Use the **Linear System Solving** tutor to solve the following system of linear equations, written in matrix form:

Action	Result in Document
1. In a new document block, create the matrix or set of linear equations to be solved.	$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 \\ 4 & 2 & -1 & 5 & 7 \\ 0 & -3 & 5 & 4 & -7 \\ 1 & -1 & 3 & 6 & 5 \end{bmatrix}$
2. Load the **Student[LinearAlgebra]** package. From the **Tools** menu, select **Load Package** → **Student Linear Algebra**. This makes the tutors in that package available. For details, see *Package Commands (page 33)*.	Loading **Student:-LinearAlgebra**
3. Click the matrix and from the context panel select **Student Linear Algebra** → **Tutors** → **Linear System Solving...**. The **Linear System Solving** dialog appears, where you can choose the solving method. **Gaussian Elimination** reduces the matrix to row-echelon form, then performs back-substitution to solve the system. **Gauss Jordan Elimination** reduces the matrix to reduced row-echelon form, where the equations are already solved. For this example, choose **Gaussian Elimination**.	

Action	**Result in Document**
4. The **Gaussian Elimination** dialog opens. You can specify the Gaussian elimination step-by-step, or you can use the **Next Step** or **All Steps** buttons to have Maple perform the steps for you. 5. Once the matrix is in row-echelon (upper-triangular) form, click the **Solve System** button to move to the next step.	
6. The **Solve the system of equations in Row-Echelon Form** dialog appears. Click the buttons on the right to calculate the solution: first find the **Equations**, then solve for each variable. Click the **Solution** button to display the solution in the tutor.	
7. Click the **Close** button to return the solution to your document.	$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 \\ 4 & 2 & -1 & 5 & 7 \\ 0 & -3 & 5 & 4 & -7 \\ 1 & -1 & 3 & 6 & 5 \end{bmatrix} \xrightarrow{\text{linear solve tutor}} \begin{bmatrix} -4 \\ 3 \\ -2 \\ 3 \end{bmatrix}$

For more information on linear systems and matrices, see *Linear Algebra (page 116)*.

3 Worksheet Mode

The *Worksheet* mode of the Standard Worksheet interface is designed for:

- Interactive use through Maple commands, which offers advanced functionality and customized control not available using the context panel or other syntax-free methods
- Programming using the powerful Maple language

Using Worksheet mode, you have access to all of the Maple features described in Chapter 1, and most of those described in Chapter 2, including:

- Math and Text modes
- Palettes
- The context panel
- Assistants and tutors

For information on these features, see Chapter 1, *Getting Started (page 1)* and Chapter 2, *Document Mode (page 43)*.

Note: Using a document block, you can use all Document mode features in Worksheet mode. For information on document blocks, see *Document Blocks (page 35)*.

Note: This chapter and the following chapters except Chapter 7 were created using Worksheet mode.

3.1 In This Chapter

Section	Topics
Input Prompt (page 56) - Where you enter input	• The Input Prompt (>) • Suppressing Output • 2-D and 1-D Math Input • Input Separators
Commands (page 57)- Thousands of routines for performing computations and other operations	• The Maple Library • Top-Level Commands • Mathematical Functions • Package Commands • Lists of Common Commands and Packages
Palettes (page 61) - Items that you can insert by clicking or dragging	• Using Palettes
The Context Panel (page 63)- Clickable access to common operations	• Using the Context Panel
Assistants and Tutors (page 64)- Graphical interfaces with buttons and sliders	• Launching Assistants and Tutors
Task Templates (page 64) - Sets of commands with placeholders that you can insert and use to perform a task	• Viewing Task Templates • Inserting a Task Template • Performing the Task
Text Regions (page 65) - Areas in the document in which you can enter text	• Inserting a Text Region • Formatting Text

Section	Topics
Names (page 66) - References to the expressions you assign to them	• Assigning to Names • Unassigning Names • Valid Names
Equation Labels (page 68) - Automatically generated labels that you can use to refer to expressions	• Displaying Equation Labels • Referring to a Previous Result • Execution Groups with Multiple Outputs • Label Numbering Schemes • Features of Equation Labels

3.2 Input Prompt

In Worksheet mode, you enter input at the Maple *input prompt* (>). The default mode for input is Math mode (*2-D Math*).

To evaluate input:

- Press **Enter**.

Maple displays the result (output) below the input.

For example, to find the value of $\sin^3\left(\frac{\pi}{3}\right)$, enter the expression, and then press **Enter**.

> $\sin^3\left(\frac{\pi}{3}\right)$

$$\frac{3\sqrt{3}}{8} \tag{3.1}$$

For example, compute the sum of two fractions.

> $\frac{2}{9}+\frac{7}{11}$

$$\frac{85}{99} \tag{3.2}$$

A set of Maple input and its output are referred to as an *execution group*.

In the worksheet, the semicolon as a statement terminator is optional.

Suppressing Output

To suppress the output, enter a colon (**:**) at the end of the input.

> $a := \frac{2}{9}+\frac{7}{11}:$

1-D Math Input

You can also insert input using 1-D Math mode. The input is entered as a one-dimensional sequence of characters. 1-D Math input is red.

To enter input using 1-D Math:

- At the input prompt, press **F5** or click the **Text** button in the context bar, Text , to switch from 2-D Math to 1-D Math.

```
> 123^2 - 29857/120;
```

$$\frac{1785623}{120}$$

As with 2-D math, in 1-D math, if you use a colon, Maple suppresses the output.

$$> 123^2 - \frac{29857}{120} :$$

To set the default input mode at a prompt to 1-D Math:

1. From the **Tools** menu, select **Options**. The **Options** dialog is displayed.
2. On the **Display** tab, in the **Input display** drop-down list, select **Maple Notation**.
3. Click **Apply to Session** (to set for only the current session) or **Apply Globally** (to set for all Maple sessions).

To convert between 2-D Math input and 1-D Math input:

1. Select the 2-D (or 1-D) Math input.
2. From the **Format** menu, select **Convert To**, and then **1-D Math Input** (or 2-D Math Input).

Input Separators

In 1-D and 2-D Math input, you can use a semicolon or colon to separate multiple inputs in the same input line.

```
> sqrt(4.4); tan(3.2);
```

$$2.097617696$$

$$0.05847385446$$

If you do not specify a semicolon or colon, Maple interprets it as a single input. This can either give unexpected results or an error. Notice that the following example gives an error in 1-D math but in 2-D math this is interpreted as multiplication.

```
> sqrt(4.4) tan(3.2)
Error, missing operator or `;`
```

$$> \sqrt{4.4}\ \tan(3.2)$$

$$0.1226557919$$

3.3 Commands

Maple contains a large set of commands and a powerful programming language. Most Maple commands are written using the Maple programming language.

You can enter commands using 1-D or 2-D Math. 1-D Math input is recommended when programming in Maple. *Basic Programming (page 295)* provides an introduction to Maple programming.

To learn how to use Maple commands, see the appropriate help page, or use task templates. For more information, see *The Maple Help System (page 37)* and *Task Templates (page 64)*.

The Maple Library

Maple's commands are contained in the Maple library. There are two types of commands: *top-level commands* and *package commands*.

- The top-level commands include many of the most frequently used Maple commands, as well as an extensive list of mathematical functions.
- Packages contain related specialized commands in areas such as calculus, linear algebra, vector calculus, and code generation.

For a complete list of packages and commands, refer to the **index/help** help pages. For information on the Maple Help System, see *The Maple Help System (page 37)*.

Top-Level Commands

To use a top-level command, enter its name followed by parentheses (**( )**) containing any parameters. This is referred to as a *calling sequence* for the command.

```
command(arguments)
```

Note: In 1-D Math input, include a semicolon or colon at the end of the calling sequence.

For example, to differentiate an expression, use the **diff** command. The required parameters are the expression to differentiate, which must be specified first, and the independent variable.

> $diff(\tan(x)\sin(x), x)$

$$\left(1+\tan(x)^2\right)\sin(x)+\tan(x)\cos(x)$$

Mathematical Functions

For a complete list of commands that implement mathematical functions, such as **BesselI** and **AiryAi**, available in the library, refer to the **initialfunctions** help page.

> $\frac{\mathrm{BesselI}(0.1, 1)}{\mathrm{AiryAi}(2.2)}$

$$47.53037086$$

For detailed information on the properties of a function, use the **FunctionAdvisor** command.

> $FunctionAdvisor('definition', \mathrm{BesselI})$

$$\left[I_a(z)=\frac{z^a\,{}_0F_1\left(;\,a+1;\,\frac{z^2}{4}\right)}{\Gamma(a+1)\,2^a},\ \textit{with no restrictions on}\ (a, z)\right]$$

This definition is displayed using the typeset form of the BesselI function, the hypergeometric function (hypergeom) and the Gamma function (GAMMA). To see the function names rather than the typeset form, use **lprint**:

> $lprint(\%)$

```
[BesselI(a, z) = z^a*hypergeom([], [a+1], (1/4)*z^2)/(GAMMA(a+1)*2^a), `with no restrictions
 on `(a, z)]
```

For detailed information on how to use a function in Maple, refer to its help page.

For example:

> ?*Bessel*

Another way to access help is to select the word for which you want help and use the shortcut key for context help, **F2** (**Control + Shift + ?**, for Mac).

Top Commands

Here are a few of the most frequently used Maple commands. A complete list of top-level commands is available on the **index/function** help page.

Table 3.1: Top Commands

Command Name	Description
plot and **plot3d**	Create a two-dimensional and three-dimensional plot of functions.
solve	Solve one or more equations or inequalities for their unknowns.
fsolve	Solve one or more equations using floating-point arithmetic.
eval	Evaluate an expression at a given point.
evalf	Numerically evaluate expressions.
dsolve	Solve ordinary differential equations (ODEs).
int	Compute an indefinite or definite integral.
diff	Compute an ordinary or partial derivative, as the context dictates.
limit	Calculate the limiting value of a function.
sum	For symbolic summation. It is used to compute a closed form for an indefinite or definite sum.
assume/is	Set variable properties and relationships between variables. Similar functionality is provided by the **assuming** command.
assuming	Compute the value of an expression under assumptions.
simplify	Apply simplification rules to an expression.
factor	Factor a polynomial.
expand	Distribute products over sums.
normal	Normalize a rational expression.
convert	Convert an expression to a different type or form.
type	Type-checking command. In many contexts, it is not necessary to know the exact value of an expression; it suffices to know that an expression belongs to a broad class, or group, of expressions that share some common properties. These classes or groups are known as *types*.
series	Generalized series expansion.
map	Apply a procedure to each operand of an expression.

Package Commands

To use a package command, the calling sequence must include the package name, and the command name enclosed in square brackets (**[ ]**).

```
package[command](arguments)
```

If you are frequently using the commands in a package, load the package.

To load a package:

- Use the **with** command, specifying the package as an argument.

The **with** command displays a list of the package commands loaded (unless you suppress the output by entering a colon at the end of the calling sequence).

After loading a package, you can use the short form names of its commands. That is, you can enter the commands without specifying the package name.

For example, use the **NLPSolve** command from the **Optimization** package to find a local minimum of an expression and the value of the independent variable at which the minimum occurs.

> $Optimization[NLPSolve]\left(\frac{\sin(x)}{x}, x = 1..15\right)$

$$[-0.0913252028230577, [x = 10.9041216489198]]$$

> $with(Optimization);$

$$[ImportMPS, Interactive, LPSolve, LSSolve, Maximize, Minimize, NLPSolve, QPSolve]$$

> $NLPSolve\left(\frac{\sin(x)}{x}, x = 1..15\right)$

$$[-0.0913252028230576718, [x = 10.9041216700744900]]$$

For more information on optimization, see *Optimization (page 138)*.

To unload a package:

- Use the **unwith** command, specifying the package as an argument.

> $unwith(Optimization)$

Alternatively, use the **restart** command. The restart command clears Maple's internal memory. The effects include unassigning all names and unloading all packages. For more information, refer to the **restart** help page.

Note: To execute the examples in this manual, you may be required to use the unassign or restart command between examples.

Some packages contain commands that have the same name as a top-level command. For example, the **plots** package contains a **changecoords** command. Maple also contains a top-level **changecoords** command.

> $with(plots):$

After the plots package is loaded, the name **changecoords** refers to the **plots[changecoords]** command. To use the top-level **changecoords** command, unload the package or use the restart command. (For alternative methods of accessing the top-level command, see the **rebound** help page.)

Top Packages

Here are a few of the most frequently used Maple packages. A complete list of packages is available on the **index/package** help page.

Table 3.2: Top Packages

Package Name	Description
CodeGeneration	The **Code Generation** package is a collection of commands and subpackages that enable the translation of Maple code to other programming languages, such as C, C#, Fortran, MATLAB®, Visual Basic®, Java™, Julia, Perl, and Python®.
LinearAlgebra	The **Linear Algebra** package contains commands to construct and manipulate Matrices and Vectors, and solve linear algebra problems. **LinearAlgebra** routines operate on three principal data structures: Matrices, Vectors, and scalars.

Package Name	Description
Optimization	The **Optimization** package is a collection of commands for numerically solving optimization problems, which involve finding the minimum or maximum of an objective function possibly subject to constraints.
Physics	The **Physics** package implements computational representations and related operations for most of the objects used in mathematical physics computations.
RealDomain	The **Real Domain** package provides an environment in which Maple assumes that the basic underlying number system is the field of real numbers instead of the complex number field.
ScientificConstants	The **Scientific Constants** package provides access to the values of various physical constants, for example, the velocity of light and the atomic weight of sodium. This package provides the units for each of the constant values, allowing for greater understanding of an equation. The package also provides units-matching for error checking of the solution.
ScientificErrorAnalysis	The **Scientific Error Analysis** package provides representation and construction of numerical quantities that have a central value and an associated uncertainty (or error), which is a measure of the degree of precision to which the quantity's value is known. Various first-order calculations of error analysis can be performed with these quantities.
Statistics	The **Statistics** package is a collection of tools for mathematical statistics and data analysis. The package supports a wide range of common statistical tasks such as quantitative and graphical data analysis, simulation, and curve fitting.
Student	The **Student** package is a collection of subpackages designed to assist with teaching and learning standard undergraduate mathematics. The many commands display functions, computations, and theorems in various ways, including stepping through important computations. The **Student** package contains the following subpackages: • **Basics** - fundamental math concepts • **Calculus1** - single-variable calculus • **LinearAlgebra** - linear algebra • **MultivariateCalculus** - multivariate calculus • **NumericalAnalysis** - numerical analysis • **Precalculus** - precalculus • **Statistics** - statistics • **VectorCalculus** - multivariate vector calculus
Units	The **Units** package contains commands for unit conversion and provides environments for performing calculations with units. It accepts approximately 300 distinct unit names (for example, meters and grams) and over 550 units with various contexts (for example, standard miles and U.S. survey miles). Maple also contains two **Units** palettes that allow you to enter the unit for an expression quickly.
VectorCalculus	The **Vector Calculus** package is a collection of commands that perform multivariate and vector calculus operations. A large set of predefined orthogonal coordinate systems is available. All computations in the package can be performed in any of these coordinate systems. It contains a facility for adding a custom but orthogonal coordinate system and using that new coordinate system for your computations.

3.4 Palettes

Palettes are collections of related items that you can insert by clicking or dragging. For example, see **Figure 3.1**.

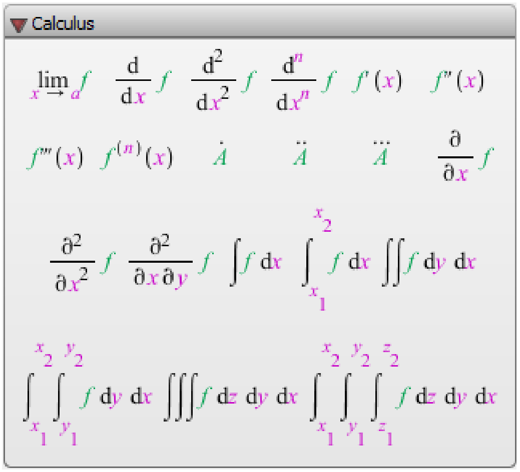


Figure 3.1: Calculus Palette

You can use palettes to enter input.

For example, evaluate a definite integral using the definite integration item $\int_{x_1}^{x_2} f \, dx$ in the **Calculus** palette.

In 2-D Math, clicking the definite integration item inserts:

$$> \int_{x_1}^{x_2} f \, dx$$

1. Enter values in the placeholders. To move to the next placeholder, press **Tab**.
2. To evaluate the integral, press **Enter**.

$$> \int_{0}^{1} \tanh(x) \, dx$$

$$-\ln(2) + \ln\left(e^{-1} + e\right)$$

In 1-D Math, clicking the definite integration item inserts the corresponding command calling sequence.

```
> int(f,x=x[1]..x[2]);
```

Specify the problem values (using the **Tab** to move to the next placeholder), and then press **Enter**. **Note:** If pressing the **Tab** key inserts a tab, under the **Format** menu, toggle **Tab Navigation**. Then the **Tab** key will move the cursor to the next placeholder.

```
> int(tanh(x), x = 0..1);
```

$$-\ln(2) + \ln\left(e^{-1} + e\right)$$

Note: Some palette items cannot be inserted into 1-D Math because they are not defined in the Maple language. When the cursor is in 1-D Math input, unavailable palette items are dimmed.

For more information on viewing and using palettes, see *Palettes (page 15)* in Chapter 1.

3.5 The Context Panel

The context panel is a collection of tools and operations that are appropriate for a particular expression. The context panel changes according to the expression, table, or region that you click on. See **Figure 3.2**.

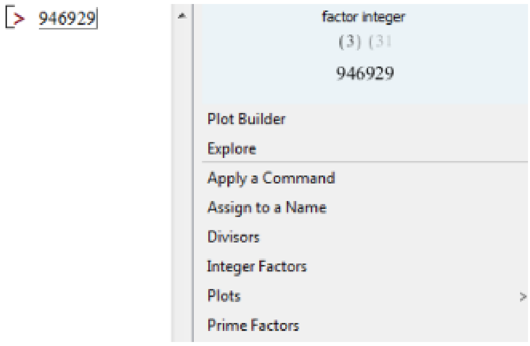


Figure 3.2: Integer Context Panel

You can use the context panel to perform operations on 2-D Math, including output.

To use the context panel:

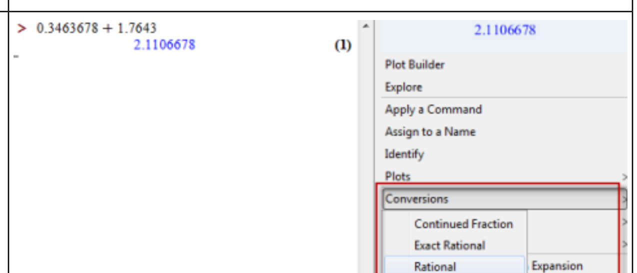
1. Click the expression.
2. Move your mouse cursor over Pin Open Context Panel («), or click it to fix the context panel in place.
3. From the context panel, select a tool or operation.

Maple inserts a new execution group containing:

- The calling sequence that performs the operation
- The result of the operation

Example - Using the Context Panel

Determine the rational expression (fraction) that approximates the floating-point number $0.3463678 + 1.7643$.

Action	Result in Document
1. Enter and execute the expression.	> $0.3463678 + 1.7643$ 2.1106678 (3.3)
2. Click the output floating-point number.	
3. From the context panel, select **Conversions** → **Rational**. The inserted calling sequence includes an equation label reference to the number you are converting.	> $convert(\,(3.3), 'rational'\,)$ $\frac{32270}{15289}$

Notice that an equation label reference has been used. For information on equation labels and equation label references, see *Equation Labels (page 68)*.

For more information on the Context Panel, see *Computing with the Context Panel (page 48)* in Chapter 2.

3.6 Assistants and Tutors

Assistants and tutors provide point-and-click interfaces with buttons, text input regions, and sliders. See **Figure 3.3**.

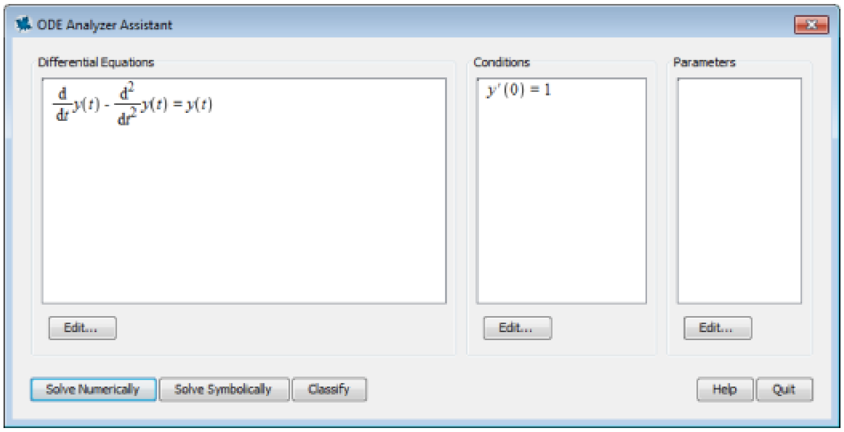


Figure 3.3: ODE Analyzer Assistant

Launching an Assistant or Tutor

To launch an assistant or tutor:

1. Open the **Tools** menu.
2. Select **Assistants** or **Tutors**.
3. Navigate to and select one of the assistants or tutors.

For more information on assistants and tutors, see *Assistants (page 24)* in Chapter 1.

3.7 Task Templates

Maple can solve a diverse set of problems. The task template facility helps you quickly find and use the commands required to perform common tasks.

After inserting a task template, specify the parameters of your problem in the placeholders, and then execute the commands, or click a button.

The **Task Browser** (**Figure 3.4**) organizes task templates by subject.

To launch the Task Browser:

- From the **Tools** menu, select **Tasks**, and then **Browse**.

You can also browse the task templates in the Table of Contents of the Maple Help System.

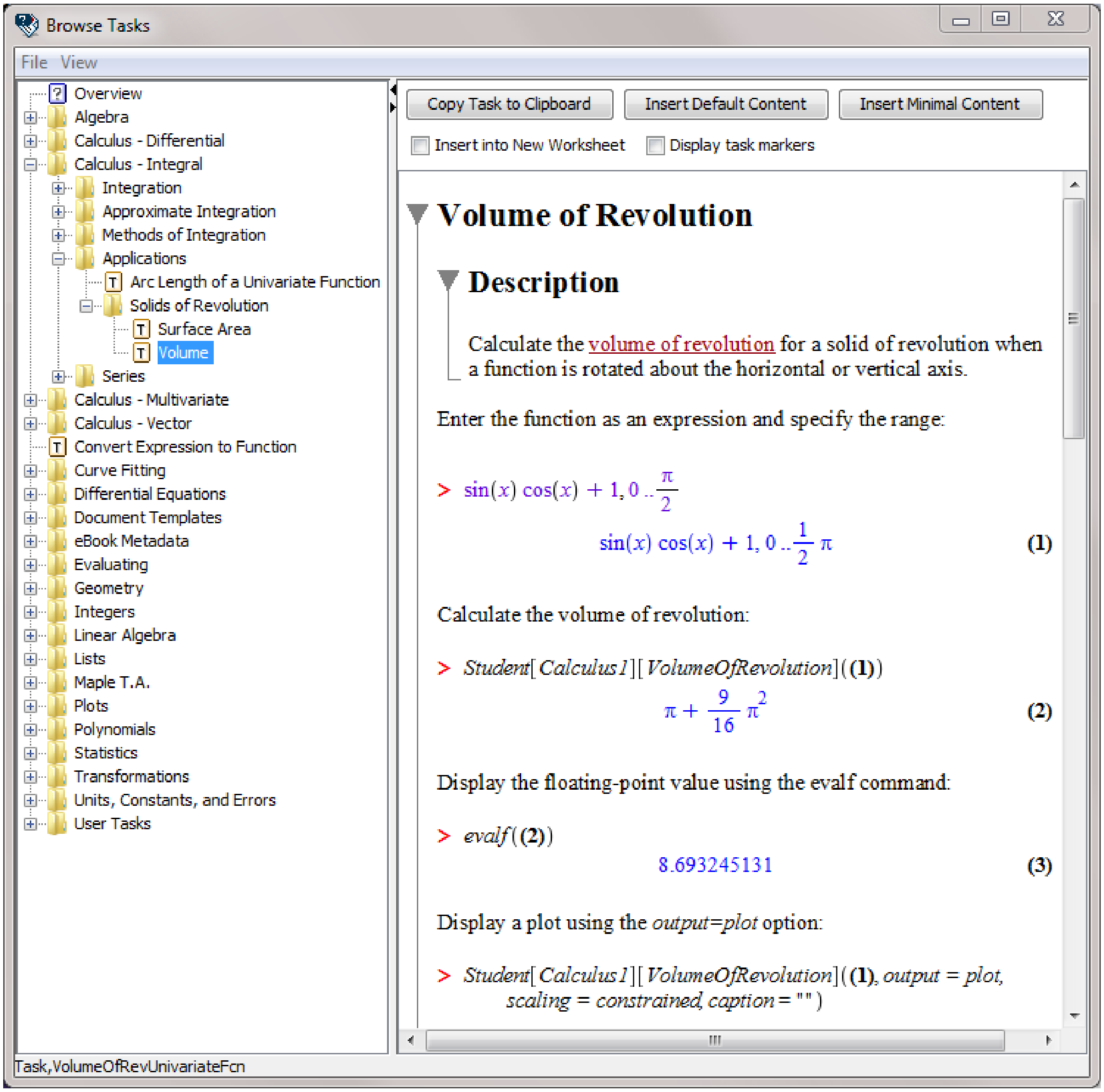


Figure 3.4: Task Browser

For details on inserting and using task templates, see *Task Templates (page 27)*. You can also create your own task templates for performing common tasks. For details, refer to the **creatingtasks** help page.

3.8 Text Regions

To add descriptive text in Worksheet mode, use a *text region*.

To insert a text region:

- In the toolbar, click the Text region icon .

The default mode in a text region is Text mode.

In a text region, you can:

- Enter text with inline mathematical content by switching between Text and Math modes. To toggle between Text mode and Math mode, press **F5** or click the Math and Text toolbar icons, Text Nonexecutable Math Math

  Note: To change a math region to nonexecutable mathematical content, use **Shift** + **F5**.
- Insert any palette item. Palette items are inserted in Math mode (2-D Math). **Note**: After you insert a palette item, you must press **F5** or click the toolbar icon to return to Text mode.

You can format text in a text region. Features include:

- Character styles
- Paragraph styles
- Sections and subsections
- Tables

For more information on formatting documents, see *Creating Mathematical Documents (page 227)*.

3.9 Names

Instead of re-entering an expression every time you need it, you can assign it to a *name* or add an *equation label* to it. Then you can quickly refer to the expression using the name or an equation label reference. For information on labels, see the following section, *Equation Labels (page 68)*.

Note: Through the Variable Manager you can manage the top-level assigned variables currently active in your Maple Session. For more information about the Variable Manager, see the **Variable Manager** help page. For Maple workbooks, you can use the Variable Manager palette to return to the saved state of your variables.

Assigning to Names

You can assign any Maple expression to a name: numeric values, data structures, procedures (a type of Maple program), and other Maple objects.

Initially, the value of a name is itself.

> a

$$a$$

The assignment operator (**:=**) associates an expression with a name.

> $a := \pi$

$$a := \pi$$

Recall that you can enter π using the following two methods.

- Use the **Common Symbols** palette.
- In 2-D Math enter *pi*, and then press the symbol completion shortcut key. See *Shortcuts for Entering Mathematical Expressions (page 5)*.

When Maple evaluates an expression that contains a name, it replaces the name with its value. For example:

> cos(*a*)

$$-1$$

For information on Maple evaluation rules, see *Evaluating Expressions (page 285)*.

Mathematical Functions

To define a function, assign it to a name.

For example, define a function that computes the cube of its argument.

> $cube := x \rightarrow x^3$:

For information on creating functions, see *Example 2 - Define a Mathematical Function (page 46)*.

> $cube(3)$; $cube(1.666)$

$$27$$

$$4.624076296$$

Note: To insert the right arrow, enter the characters **->**. In 2-D Math, Maple replaces -> with the right arrow symbol **→.** In 1-D Math, the characters are not replaced.

For example, define a function that squares its argument.

```
> square := x -> x^2:
> square(32);
```

$$1024$$

For more information on functions, see *Functional Operators (page 274)*.

Protected Names

Protected names are valid names that are predefined or reserved.

If you attempt to assign to a protected name, Maple returns an error.

> sin := 2

```
Error, attempting to assign to `sin` which is protected.  Try declaring `local sin`; see
?protect for details.
```

For more information, refer to the **type/protected** and **protect** help pages.

Unassigning Names right single

To unassign a name, reset the value of a name to itsef. Right single quotes (*unevaluation quotes*) prevent Maple from evaluating the name.

> *a* :='*a*'

> *a*

You can also use the **unassign** command. You must enclose the name in right single quotes (**' '**).

```
> unassign('a')
> a
```

$$a$$

For more information on the uses of unevaluation quotes, see *Delaying Evaluation (page 291)* or refer to the **uneval** help page.

Unassigning all names:

The **restart** command clears Maple's internal memory. The effects include unassigning all names. For more information, refer to the **restart** help page.

Note: To execute the examples in this manual, you may be required to use the unassign or restart command between examples.

Valid Names

A Maple name must be one of the following.

- A sequence of alphanumeric and underscore (_) characters that begins with an alphabetical character.
- A sequence of characters enclosed in left single quotes (\` \`).

Important: Do not begin a name with an underscore character. Maple reserves names that begin with an underscore for use by the Maple library.

Examples of valid names:

- **a**
- **a1**
- **polynomial**
- **polynomial1_divided_by_polynomial2**
- **\`2a\`**
- **\`x y\`**

3.10 Equation Labels

Maple marks the output of each execution group with a unique equation label.

Note: The equation label is displayed to the right of the output.

> $\int \sin(x)\, dx$

$$-\cos(x) \tag{3.4}$$

Using equation labels, you can refer to the result in other computations.

> $\int \textbf{(3.4)}\, dx$

$$-\sin(x) \tag{3.5}$$

Displaying Equation Labels

Important: By default, equation labels are displayed. If equation label display is turned off, complete **both** of the following operations.

- From the **Format** menu, select **Equation Labels**, and then ensure that **Worksheet** is selected.
- In the **Options** dialog (**Tools→Options**), on the **Display** tab, ensure that **Show equation labels** is selected.

Referring to a Previous Result

Instead of re-entering previous results in computations, you can use equation label references. Each time you need to refer to a previous result, insert an equation label reference.

To insert an equation label reference:

1. From the **Insert** menu, select **Label**. Alternatively, double-click on the equation label (to the far right side of the output) or press **Ctrl+L**; **Command+L**, Macintosh.
2. In the **Insert Label** dialog (see **Figure 3.5**), enter the label value, and then click **OK**.

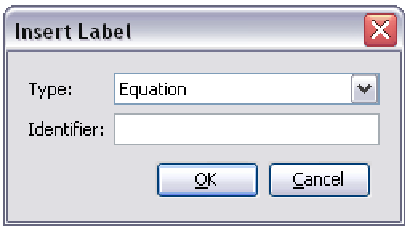
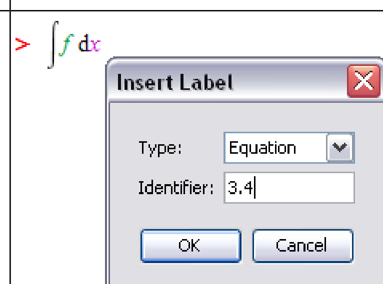
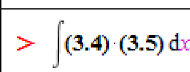


Figure 3.5: Insert Label Dialog

Maple inserts the reference.

For example:

To integrate the product of `(3.4)` and `(3.5)`:

Action	Result in Document
1. In the **Expression** palette, click the indefinite integration item  . The item is inserted and the integrand placeholder is highlighted.	
2. Press **Ctrl+L** (**Command+L**, for Macintosh). 3. In the **Insert Label** dialog, enter **3.4**. Click **OK**.	
4. Press *. 5. Press **Ctrl+L** (**Command+L**, for Macintosh). 6. In the **Insert Label** dialog, enter **3.5**. Click **OK**.	

Action	**Result in Document**
7. To move to the variable of integration placeholder, press **Tab**. 8. Enter **x**. 9. To evaluate the integral, press **Enter**.	$> \int (3.4) \cdot (3.5)\, dx$ $\frac{\sin(x)^2}{2}$ (3.6)

Execution Groups with Multiple Outputs

An equation label is associated with the *last output* within an execution group.

$$> \left(\frac{2}{3.5}\right)^2;\ \cos\left(\frac{\pi}{6}\right)$$

$$0.3265306122$$

$$\frac{1}{2}\sqrt{3} \qquad (3.7)$$

$$> (3.7)^2$$

$$\frac{3}{4} \qquad (3.8)$$

Label Numbering Schemes

You can number equation labels in two ways:

- **Flat** - Each label is a single number, for example, 1, 2, or 3.
- **Sections** - Each label is numbered according to the section in which it occurs. For example, 2.1 is the first equation in the second section, and 1.3.2 is the second equation in the third subsection of the first section.

To change the equation label numbering scheme:

- From the **Format** menu, select **Equation Labels** → **Label Display**. In the **Format Labels** dialog (Figure 3.6), select one of the formats.
- Optionally, enter a prefix.

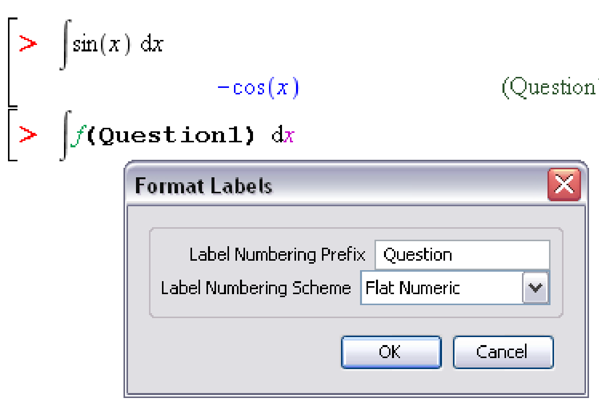


Figure 3.6: Format Labels Dialog: Adding a Prefix

Features of Equation Labels

Although equation labels are not descriptive names, labels offer other important features.

- Each label is unique, whereas a name may be inadvertently assigned more than once for different purposes.
- Maple labels the output values sequentially. If you remove or insert an output, Maple automatically re-numbers all equation labels and updates the label references.
- If you change the equation label format (see *Label Numbering Schemes (page 70)*), Maple automatically updates all equation labels and label references.

For information on assigning to, using, and unassigning names, see *Names (page 66)*.

For more information on equation labels, refer to the **equationlabels** help page.

The following chapters describe how to use Maple to perform tasks such as solving equations, producing plots and animations, and creating mathematical documents. The chapters were created using Worksheet mode. Except where noted, all features are available in both Worksheet mode and Document mode.

4 Basic Computations

This chapter discusses key concepts related to performing basic computations with Maple. It discusses important features that are relevant to all Maple users. After learning about these concepts, you will learn how to use Maple to solve problems in specific mathematical disciplines in the following chapter.

4.1 In This Chapter

Section	Topics
Symbolic and Numeric Computation (page 74)- An overview of exact and floating-point computation	• Exact Computations • Floating-Point Computations • Converting Exact Quantities to Floating-Point Values • Sources of Error
Integer Operations (page 76) - How to perform integer computations	• Important Integer Commands • Non-Base 10 Numbers • Finite Rings and Fields • Gaussian Integers
Solving Equations (page 80) - How to solve standard mathematical equations	• Equations and Inequations • Ordinary Differential Equations • Partial Differential Equations • Integer Equations • Integer Equations in a Finite Field • Linear Systems • Recurrence Relations
Units, Scientific Constants, and Uncertainty (page 93) - How to construct and compute with expressions that have units, scientific constants, or uncertainty	**Units** • Conversions • Applying Units to an Expression • Performing Computations with Units • Changing the Current System of Units • Extensibility **Scientific Constants** • Scientific Constants • Element and Isotope Properties • Value, Units, and Uncertainty • Performing Computations • Modification and Extensibility **Uncertainty Propagation** • Quantities with Uncertainty • Performing Computations with Quantities with Uncertainty

Section	Topics
Restricting the Domain (page 103) - How to restrict the domain for computations	• Real Number Domain • Assumptions on Variables

4.2 Symbolic and Numeric Computation

Symbolic computation is the mathematical manipulation of expressions involving symbolic or abstract quantities, such as variables, functions, and operators; and exact numbers, such as integers, rationals, π, and e^2. The goal of such manipulations may be to transform an expression to a simpler form or to relate the expression to other, better understood formulas.

Numeric computation is the manipulation of expressions in the context of finite-precision arithmetic. Expressions involving exact numbers, for example, $\sqrt{2}$, are replaced by close approximations using floating-point numbers, for example 1.41421. These computations generally involve some error. Understanding and controlling this error is often of as much importance as the computed result.

In Maple, numeric computation is normally performed if you use floating-point numbers (numbers containing a decimal point) or the **evalf** command. The **plot** command (see *Plots and Animations (page 183)*) uses numeric computation, while commands such as **int**, **limit**, and **gcd** (see *Integer Operations (page 76)* and *Mathematical Problem Solving (page 109)*) generally use only symbolic computation to achieve their results.

Exact Computations

In Maple, integers, rational numbers, mathematical constants such as π and ∞, and mathematical structures such as matrices with these as entries are treated as exact quantities. Names, such as x, y, my_variable, and mathematical functions, such as $\sin(x)$ and $\mathrm{LambertW}(k, z)$, are *symbolic* objects. Names can be assigned exact quantities as their values, and functions can be evaluated at symbolic or exact arguments.

> $\frac{3}{2} + \frac{1}{3}, 1 + \frac{\pi}{2}$

$$\frac{11}{6}, 1 + \frac{\pi}{2}$$

Important: Unless requested to do otherwise (see the following section), Maple evaluates expressions containing exact quantities to exact results, as you would do if you were performing the calculation by hand, and not to numeric approximations, as you normally obtain from a standard hand-held calculator.

> $\sin(1), \sin(\pi), \sin(x)$

$$\sin(1), 0, \sin(x)$$

> $\int \tan(t)\, \mathrm{d}t$

$$-\ln(\cos(t))$$

> $\sqrt{32}$

$$4\sqrt{2}$$

Floating-Point Computations

In some situations, a numeric approximation of an exact quantity is required. For example, the **plot** command requires the expression it is plotting to evaluate to numeric values that can be rendered on the screen: π cannot be so rendered, but 3.14159 can be. Maple distinguishes *approximate* from *exact* quantities by the presence or absence of a decimal point: 1.9 is approximate, while $\frac{19}{10}$ is exact.

Note: An alternative representation of floating-point numbers, called *e-notation*, may not include an explicit decimal point: *1e5* $= 100000.$, *3e-2* $= .03$.

In the presence of a floating-point (approximate) quantity in an expression, Maple generally computes using numeric approximations. Arithmetic involving mixed exact and floating-point quantities results in a floating-point result.

$$> 1.5 + \frac{2}{3}, 1 + 0.5 \cdot \pi$$

$$2.166666667, 2.570796327$$

If a mathematical function is passed a floating-point argument, it normally attempts to produce a floating-point approximation of the result.

$$> \sin(1.5), \int_{0.0}^{1.0} e^x \, dx$$

$$0.9974949866, 1.718281828$$

Converting Exact Quantities to Floating-Point Values

To convert an exact quantity to a numeric approximation of that quantity, use the **evalf** command or the **Approximate** context panel operation (see *Approximating the Value of an Expression (page 50)*).

$$> \mathit{evalf}(\pi), \mathit{evalf}(\sin(3)), \mathit{evalf}\left(\frac{3}{2} + \frac{1}{3}\right)$$

$$3.141592654, 0.1411200081, 1.833333333$$

By default, Maple computes such approximations using 10 digit arithmetic. You can modify this in one of two ways:

- *Locally*, you can pass the precision as an index to the **evalf** call.

$$> \mathit{evalf}[20](\exp(2)), \mathit{evalf}\left(\Gamma\left(\frac{2}{3}\right)\right)$$

$$7.3890560989306502272, 1.354117939$$

- *Globally*, you can set the value of the **Digits** environment variable.

$$> \mathit{Digits} := 25 :$$

$$> \mathit{evalf}\left(\tan\left(\frac{\pi}{3}\right)\right)$$

$$1.732050807568877293527446$$

- Set the value back to the default Digits level:

> $Digits := 10:$

For more information, see the evalf and Digits help pages.

Note: When appropriate, Maple performs floating-point computations directly using your computer's underlying hardware.

Sources of Error

By its nature, floating-point computation normally involves some error. Controlling the effect of this error is the subject of active research in *Numerical Analysis*. Some sources of error are:

- An exact quantity may not be exactly representable in decimal form: $\frac{1}{3}$ and π are examples.
- Small errors can accumulate after many arithmetic operations.
- Subtraction of nearly equal quantities can result in essentially no useful information. For example, consider the computation $x - \sin(x)$ for $x \approx 0.$

> $(x-\sin(x))\Big|_{x=.00001}$

$$0.$$

No correct digits remain. If, however, you use Maple to analyze this expression, and replace this form with a representation that is more accurate for small values of $x,$ a fully accurate 10-digit result can be obtained.

> $t := taylor(x-\sin(x), x)$

$$t := \frac{1}{6}x^3 - \frac{1}{120}x^5 + O(x^7)$$

> $t\Big|_{x=0.00001}$

$$1.666666667 \times 10^{-16}$$

For information on evaluating an expression at a point, see *Substituting a Value for a Subexpression (page 285)*. For information on creating a series approximation, see *Series (page 133)*. For more information on floating-point numbers, refer to the **float** and **type/float** help pages.

4.3 Integer Operations

In addition to the basic arithmetic operators, Maple has many specialized commands for performing more complicated integer computations, such as factoring an integer, testing whether an integer is a prime number, and determining the greatest common divisor (GCD) of a pair of integers.

Note: Many integer operations are available as task templates (**Tools→Tasks→Browse**, under **Integers**).

You can quickly perform many integer operations using the context panel. Clicking on an integer displays the context panel with integer commands. For example, the context-sensitive operation **Integer Factors** applies the **ifactor** command to compute the prime factorization of the given integer. See **Figure 4.1**.

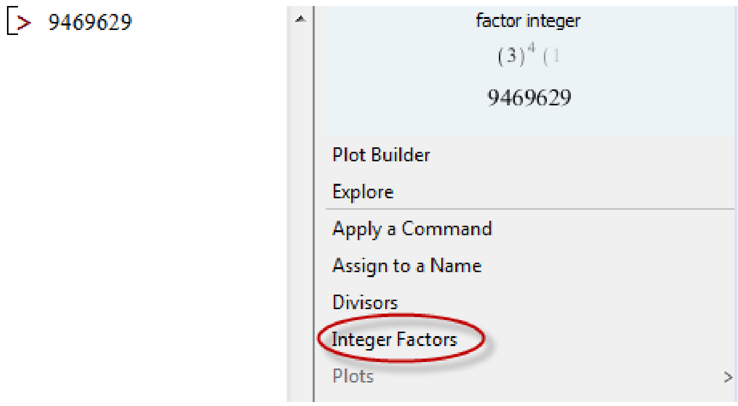


Figure 4.1: Context Panel for an Integer

The result of applying **Integer Factors** is shown:

> 9469629

$$9469629 \tag{4.1}$$

> *ifactor*(**(4.1)**)

$$(3)^4 (13) (17) (23)^2 \tag{4.2}$$

Maple inserts the command **ifactor**, using an equation label reference to the integer 946929. For more information on equation labels, see *Equation Labels (page 68)*.

For more information on using context-sensitive operations in Worksheet mode, see *The Context Panel (page 63)*. For information on using context-sensitive operations in Document mode, see *Computing with the Context Panel (page 48)*.

Maple has many other integer commands, including those listed in **Table 4.1**.

Table 4.1: Select Integer Commands

Command	Description
abs	absolute value (displays in 2-D math as $\|a\|$)
factorial	factorial (displays in 2-D math as $a!$)
ifactor	prime factorization
igcd	greatest common divisor
iquo	quotient of integer division
irem	remainder of integer division
iroot	integer approximation of nth root
isprime	test primality
isqrt	integer approximation of square root
max, min	maximum and minimum of a set
mod	modular arithmetic (See *Finite Rings and Fields (page 78)*.)
NumberTheory[Divisors]	set of positive divisors

> $iquo(209, 17)$

$$12$$

> $irem(209, 17)$

$$5$$

> $igcd(2024, 4862)$

$$22$$

> $iroot(982523, 4)$

$$31$$

For information on finding integer solutions to equations, see *Integer Equations (page 92)*.

Non-Base 10 Numbers and Other Number Systems

Maple supports:

- Non-base 10 numbers
- Finite ring and field arithmetic
- Gaussian integers

Non-Base 10 Numbers

To represent an expression in another base, use the **convert** command.

> $convert(6000, 'binary')$

$$1011101110000$$

> $convert(34271, 'hex')$

$$85DF$$

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation (page 291)*.

You can also use the **convert/base** command.

> $convert(34271, 'base', 16)$

$$[15, 13, 5, 8]$$

Note: The **convert/base** command returns a list of digit values in order of *increasing significance*.

Finite Rings and Fields

Maple supports computations over the integers modulo m.

The **mod** operator evaluates an expression over the integers modulo m.

> 27 **mod** 4

$$3$$

By default, the **mod** operator uses positive representation (**modp** command). Symmetric representation is available using the **mods** command.

> *modp*(27, 4)

$$3$$

> *mods*(27, 4)

$$-1$$

For information on setting symmetric representation as the default, refer to the **mod** help page.

The modular arithmetic operators are listed in **Table 4.2**.

Table 4.2: Modular Arithmetic Operators

Operation	Operator	Example
Addition	+	> 7 + 6 **mod** 5 3
Subtraction	-	> *mods*(3 − 16, 11) −2
Multiplication (displays in 2-D Math as ·)	*	> 13·5 **mod** 3 2
Multiplicative inverse (displays in 2-D Math as a superscript)	**^(-1)**	> $3^{(-1)}$**mod** 5 2
Division (displays in 2-D Math as $\frac{a}{b}$)	/	> $\frac{2}{3}$ **mod** 5 4
Exponentiation[1]	**&^**	> (100&^100)**mod** 7 2

[1]To enter a caret (^) in 2-D Math, enter a backslash character followed by a caret, that is, \^.

For information on solving an equation modulo an integer, see *Integer Equations in a Finite Field (page 92)*.

The **mod** operator also supports polynomial and matrix arithmetic over finite rings and fields. For more information, refer to the **mod** help page.

Gaussian Integers

Gaussian integers are complex numbers in which the real and imaginary parts are integers.

The **GaussInt** package contains commands that perform Gaussian integer operations.

The **GIfactor** command returns the Gaussian integer factorization.

> *GaussInt*[*GIfactor*](173 + 16 *I*)

$$(1+2\,\mathrm{I})\,(41-66\,\mathrm{I})$$

In Maple, complex numbers are represented as **a+b*I**, where the uppercase I represents the imaginary unit $\sqrt{-1}$.

You can also enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the I, i or j item. See *Palettes (page 15)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 21)*.

Note that the output will still be displayed with I, no matter what symbol was used for input. You can customize Maple's settings to use a different symbol for $\sqrt{-1}$. For more information on entering complex numbers, including how to customize this setting, refer to the **HowDoI/EnterAComplexNumber** help page.

The **GIsqrt** command approximates the square root in the Gaussian integers.

> *GaussInt*[*GIsqrt*](9 − 5 j)

$$3-\mathrm{I}$$

For more information on Gaussian integers including a list of **GaussInt** package commands, refer to the **GaussInt** help page.

4.4 Solving Equations

You can solve a variety of equation types, including those described in **Table 4.3**.

Table 4.3: Overview of Solution Methods for Important Equation Types

Equation Type	Solution Method
Equations and inequations	**solve** and **fsolve** commands
Ordinary differential equations	**ODE Analyzer Assistant** (and **dsolve** command)
Partial differential equations	**pdsolve** command
Integer equations	**isolve** command
Integer equations in a finite field	**msolve** command
Linear integral equations	**intsolve** command
Linear systems	**LinearAlgebra[LinearSolve]** command
Recurrence relations	**rsolve** command

Note: Many solve operations are available in the context panel and as task templates (**Tools→Tasks→Browse**). Most of this section focuses on other methods.

Solving Equations and Inequations

Using Maple, you can symbolically solve equations and inequations. You can also solve equations numerically.

To solve an equation or set of equations using the context panel:

1. Click the equation.
2. From the context panel, select **Solve** (or **Solve Numerically**). See **Figure 4.2**.

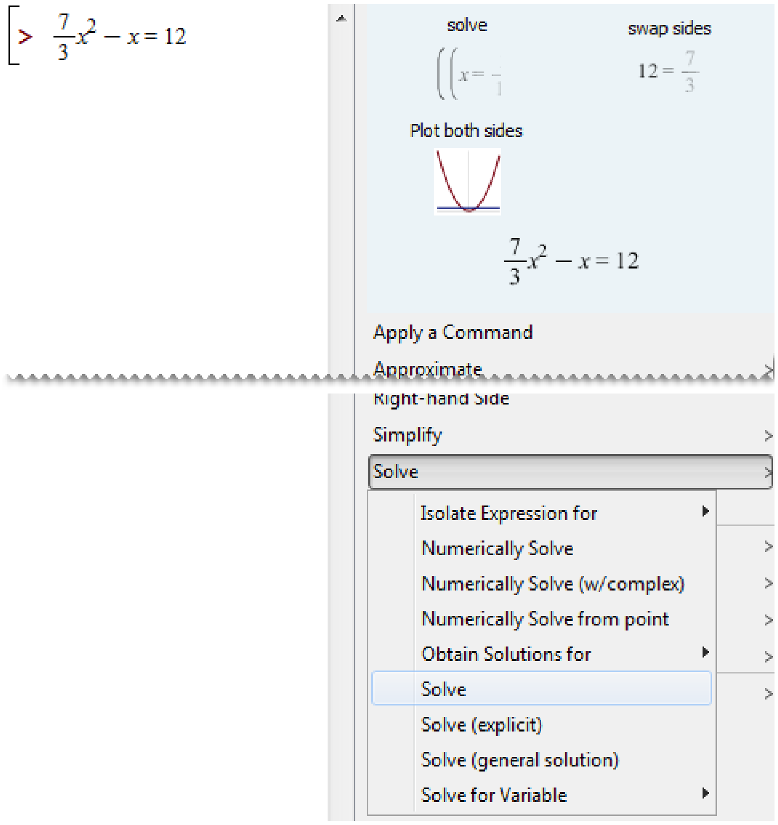


Figure 4.2: Context Panel for an Equation

In Worksheet mode, Maple inserts a calling sequence that solves the equation followed by the solutions.

If you select **Solve**, Maple computes exact solutions.

$$> \frac{7\,x^2}{3} - x = 12$$

$$\frac{7}{3}\,x^2 - x = 12 \tag{4.3}$$

> *solve*({**(4.3)**})

$$\left\{x = \frac{3}{14} + \frac{3\sqrt{113}}{14}\right\}, \left\{x = \frac{3}{14} - \frac{3\sqrt{113}}{14}\right\} \tag{4.4}$$

If you select **Solve Numerically**, Maple computes floating-point solutions.

$$> \frac{7\,x^2}{3} - x = 12$$

$$\frac{7}{3}\,x^2 - x = 12 \tag{4.5}$$

> *fsolve*({**(4.5)**})

$$\{x = -2.063602674\}, \{x = 2.492174103\} \tag{4.6}$$

For information on solving equations and inequations symbolically using the **solve** command, see the following section. For information on solving equations numerically using the **fsolve** command, see *Numerically Solving Equations (page 84)*.

Symbolically Solving Equations and Inequations

The **solve** command is a general solver that determines exact symbolic solutions to equations or inequations. The solutions to a single equation or inequation are returned as an expression sequence. For details, see *Creating and Using Data Structures (page 269)*. If Maple does not find any solutions, the **solve** command returns the empty expression sequence.

> $solve(x^2 + 3\,x + 14 = 0)$

$$-\frac{3}{2} + \frac{I\sqrt{47}}{2}, -\frac{3}{2} - \frac{I\sqrt{47}}{2}$$

In general, **solve** computes solutions in the field of complex numbers. To restrict the problem to only real solutions, see *Restricting the Domain (page 103)*.

It is recommended that you verify the solutions returned by the **solve** command. For details, see *Working with Solutions (page 85)*.

To return the solutions as a list, enclose the calling sequence in brackets (**[ ]**).

> $[solve(x^2 + x = 256\,y, x)]$

$$\left[-\frac{1}{2} + \frac{\sqrt{1+1024\,y}}{2}, -\frac{1}{2} - \frac{\sqrt{1+1024\,y}}{2}\right]$$

Expressions: You can specify expressions instead of equations. The **solve** command automatically equates them to zero.

> $solve(e^z + z)$

$$-\mathrm{LambertW}(1)$$

(In this case, the solution involves the **LambertW** function.)

Multiple Equations: To solve multiple equations or inequations, specify them as a list or set. For an introduction to both lists and sets, see *Creating and Using Data Structures (page 269)*.

> $solve([x\,y^2 - y = 5, x > 0])$

$$\left\{x = \frac{y+5}{y^2}, -5 < y, y < 0\right\}, \left\{x = \frac{y+5}{y^2}, 0 < y\right\}$$

> $solve(\{x\,y^2 - y = 5, x < 0\})$

$$\left\{x = \frac{y+5}{y^2}, y < -5\right\}$$

Solving for Specific Unknowns: By default, the **solve** command returns solutions for all unknowns. You can specify the unknowns for which to solve.

> $solve\left(q^2 - r\,s + \frac{q}{r} = 5, q\right)$

$$\frac{-1 + \sqrt{4\,r^3\,s + 20\,r^2 + 1}}{2\,r}, -\frac{1 + \sqrt{4\,r^3\,s + 20\,r^2 + 1}}{2\,r}$$

To solve for multiple unknowns, specify them as a **list**.

> $solve\left(\left\{\frac{q}{s} - \frac{r}{s+1} + \frac{q}{r} = 5, r\,s = 1\right\}, [q, r]\right)$

$$\left[\left[q = \frac{5\,s^2 + 5\,s + 1}{s^3 + s^2 + s + 1}, r = \frac{1}{s}\right]\right]$$

Transcendental Equations: In general, the **solve** command returns one solution to transcendental equations.

> $equation1 := \sin(x) = \cos(x)$:

> $solve(equation1)$

$$\frac{\pi}{4}$$

To produce all solutions, use the **allsolutions** option.

> $solve(equation1, allsolutions = true)$

$$\frac{1}{4}\,\pi + \pi\,\_Z1\sim$$

Maple uses variables of the form **_ZN~**, where **N** is a positive integer, to represent arbitrary integers. The tilde (~) indicates that it is a quantity with an assumption. For information about names with assumptions, see *Assumptions on Variables (page 104)*.

RootOf Structure: The **solve** command may return solutions, for example, to higher order polynomial equations, in an implicit form using **RootOf** structures.

> $[solve(x^5 - 2\,x^4 + 3\,x^3 - 2)]$

$$[1, RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 1), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 2), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 3), RootOf(\_Z^4 - \_Z^3 + 2\,\_Z^2 + 2\,\_Z + 2, index = 4)]$$

These **RootOf** structures are placeholders for the roots of the equation $z^4 - z^3 + 2\,z^2 + 2\,z + 2$. The **index** parameter numbers and orders the four solutions.

Like any symbolic expression, you can convert **RootOf** structures to a floating-point value using the **evalf** command.

> $evalf(\mathbf{(4.7)})$

$$[1., 0.9840010519 + 1.526590834\,I, -0.4840010519 + 0.6099471405\,I, -0.4840010519 - 0.6099471405\,I, 0.9840010519 - 1.526590834\,I]$$

Some equations are difficult to solve symbolically. For example, polynomial equations of order five and greater do not in general have a solution in terms of radicals. If the **solve** command does not find any solutions, it is recommended that you use the Maple numerical solver, **fsolve**. For information, see the following section, *Numerically Solving Equations*.

For more information on the **solve** command, including how to solve equations defined as procedures and how to find parametric solutions, refer to the **solve/details** help page.

For information on verifying and using solutions returned by the **solve** command, see *Working with Solutions (page 85)*.

Numerically Solving Equations

The **fsolve** command solves equations numerically. The behavior of the **fsolve** command is similar to that of the **solve** command.

> $equation2 := z\cos(z) = 2:$

> $fsolve(equation2)$

$$23.64662473 \qquad (4.8)$$

Note: You can also numerically solve equations using the context panel. See *Solving Equations and Inequations (page 80)*.

It is recommended that you verify the solutions returned by the **fsolve** command. For details, see *Working with Solutions (page 85)*.

Multiple Equations: To solve multiple equations, specify them as a set. For more information, see *Creating and Using Data Structures (page 269)*. The **fsolve** command solves for all unknowns.

> $fsolve(\{\ln(x) = y^2 + 1, x\,y = e^y\})$

$$\{x = 3.396618823, y = 0.4719962637\}$$

Univariate Polynomial Equations: In general, the **fsolve** command finds one real solution. However, for a univariate polynomial equation, the **fsolve** command returns all *real* roots.

> $equation3 := y^4 - 3\,y^2 - 2\,y + 1:$

> $fsolve(equation3, y)$

$$0.3365322739, 1.940392664$$

Controlling the Number of Solutions: To limit the number of roots returned, specify the **maxsols** option.

> $fsolve(equation3, y, 'maxsols' = 1)$

$$0.3365322739$$

To find additional solutions to a general equation, use the **avoid** option to ignore known solutions.

> $fsolve(equation2, z, 'avoid' = \{z = \mathbf{(4.8)}\})$

$$-2.498755763$$

Complex Solutions: To search for a complex solution or find all complex and real roots for a univariate polynomial, specify the **complex** option for the **fsolve** command.

> *fsolve*(*equation3*, *y*, '*complex*')

$-1.13846246879373 - 0.485062494059435\,I, -1.13846246879373 + 0.485062494059435\,I, 0.336532273926790, 1.94039266366067$

If the **fsolve** command does not find any solutions, it is recommended that you specify a range in which to search for solutions, or specify an initial value.

Range: To search for a solution in a range, specify the range in the calling sequence. The range can be real or complex.

> *fsolve*(*equation2*, *z*, {*z* = 100 ..200})

$$199.5011587$$

The syntax for specifying a region in the complex plane is **lower-left point..upper-right point**.

> *fsolve*(*equation3*, *y*, {*y* = −2 − *I*..0}, '*complex*');

$$-1.13846246879373 - 0.485062494059435\,I$$

Initial Values: You can specify a value for each unknown. The **fsolve** command uses these as initial values for the unknowns in the numerical method.

> *fsolve*(*equation2*, {*z* = 100})

$$\{z = 98.98037599\} \tag{4.9}$$

For more information and examples, refer to the **fsolve/details** help page.

For information on verifying and using solutions returned by the **fsolve** command, see the following section, *Working with Solutions*.

Working with Solutions

Verifying: It is recommended that you always verify solutions (that the **solve** and **fsolve** commands return) using the **eval** command.

> *equation4* := *sin*(*x*) = −*cos*(*x*):

> *solve*(*equation4*)

$$-\frac{\pi}{4} \tag{4.10}$$

> *eval*(*equation4*, *x* = **(4.10)**)

$$-\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \tag{4.11}$$

> $equation5 := cos(z) = \frac{2}{z}$:

> *fsolve*(*equation5*)

$$-2.498755763 \tag{4.12}$$

> *eval*(*equation5*, {*z* = **(4.12)**})

$$-0.8003983544 = -0.8003983540 \tag{4.13}$$

For more information, see *Substituting a Value for a Subexpression (page 285)*.

Assigning the Value of a Solution to a Variable: To assign the value of a solution to the corresponding variable as an *expression*, you can use the **assign** command, which can turn an equation such as the numeric solution in **(4.9)**, $\{z = 98.98037599\}$, into an assignment.

> *assign*(**(4.9)**)

> *z*

$$98.98037599$$

A word of caution, if you do this, z now has a value so you cannot afterwards treat it as an unknown unless you unassign it. The way to do that is

> z := 'z':

For more information on unassigning names, see *Unassigning Names right single (page 67)*.

Creating a Function from a Solution: The **assign** command assigns a value as an expression to a name. It does **not** define a function. To convert a solution to a function, use the **unapply** command.

Consider one of the solutions for **q** to the equation $q^2 - r\,s + \frac{q}{r} = 5$.

> $solutions := \left[solve\left(q^2 - r\,s + \frac{q}{r} = 5, q \right) \right]$

$$solutions := \left[\frac{-1 + \sqrt{4\,r^3\,s + 20\,r^2 + 1}}{2\,r}, -\frac{1 + \sqrt{4\,r^3\,s + 20\,r^2 + 1}}{2\,r} \right]$$

> *f* := *unapply*(*solutions*[1] , *r* , *s*)

$$f := (r, s) \mapsto \frac{-1 + \sqrt{4\,r^3\,s + 20\,r^2 + 1}}{2\,r}$$

Here, **solutions[1]** selects the first element of the list of solutions. For more information on selecting elements, see *Accessing Elements (page 269)*.

You can evaluate this function at symbolic or numeric values.

> $f(x, y)$

$$\frac{-1+\sqrt{4\,x{\sim}^3\,y+20\,x{\sim}^2+1}}{2\,x{\sim}}$$

> $f\left(\frac{1}{\sqrt{2}}, 1\right)$

$$\frac{\sqrt{2}\left(-1+\sqrt{\sqrt{2}+11}\right)}{2}$$

> $f(5.7, 2.1)$

$$4.032680521587978641060188$$

For more information on defining and using functions, see *Functional Operators (page 274)*.

Other Specialized Solvers

In addition to equations and inequations, Maple can solve other equations including:

- Ordinary differential equations (ODEs)
- Partial differential equations (PDEs)
- Integer equations
- Integer equations in a finite field
- Linear systems
- Recurrence relations

Ordinary Differential Equations (ODEs)

Maple can solve ODEs and ODE systems, including initial value and boundary value problems, symbolically and numerically.

ODE Analyzer Assistant The **ODE Analyzer Assistant** is a point-and-click interface to the Maple ODE solving routines.

To open the ODE Analyzer:

- From the **Tools** menu, select **Tutors**, **Differential Equations**, and then **ODE Analyzer**.

Maple inserts the *dsolve*[*interactive*]() calling sequence in the document. The **ODE Analyzer Assistant** (**Figure 4.3**) is displayed.

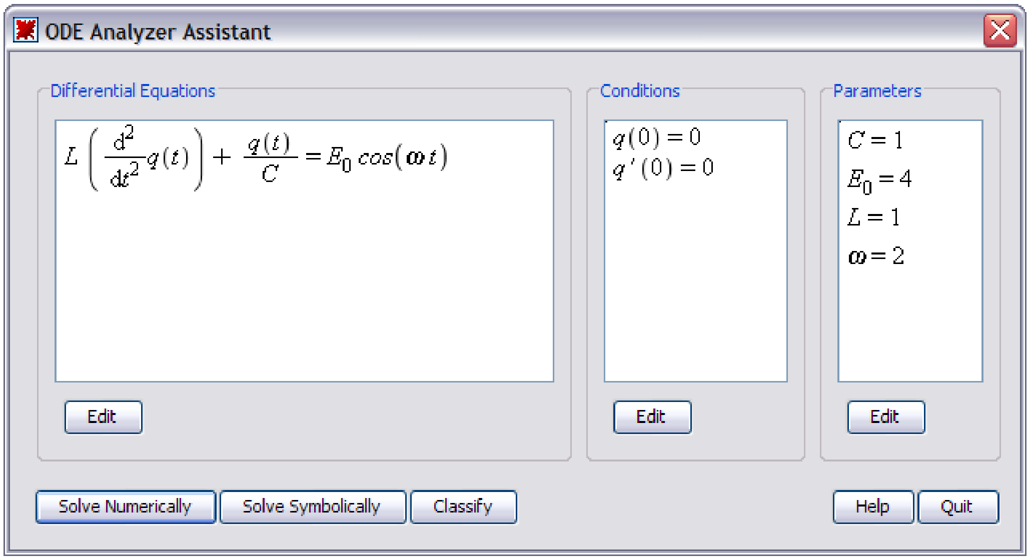


Figure 4.3: ODE Analyzer Assistant

In the main **ODE Analyzer Assistant** window, you can define ODEs, initial or boundary value conditions, and parameters. To define derivatives, use the **diff** command. For example, `diff(x(t), t)` corresponds to $\frac{df}{dx}$, and `diff(x(t), t, t)` corresponds to $\frac{d^2 f}{dx^2}$. For more information on the **diff** command, see *The diff Command (page 130)*.

After defining an ODE, you can solve it numerically or symbolically.

To solve a system numerically using the ODE Analyzer Assistant:

1. Ensure that the conditions guarantee uniqueness of the solution.
2. Ensure that all parameters have fixed values.
3. Click the **Solve Numerically** button.
4. In the **Solve Numerically** window (**Figure 4.4**), you can specify the numeric method and relevant parameters and error tolerances to use for solving the problem.
5. To compute solution values at a point, click the **Solve** button.

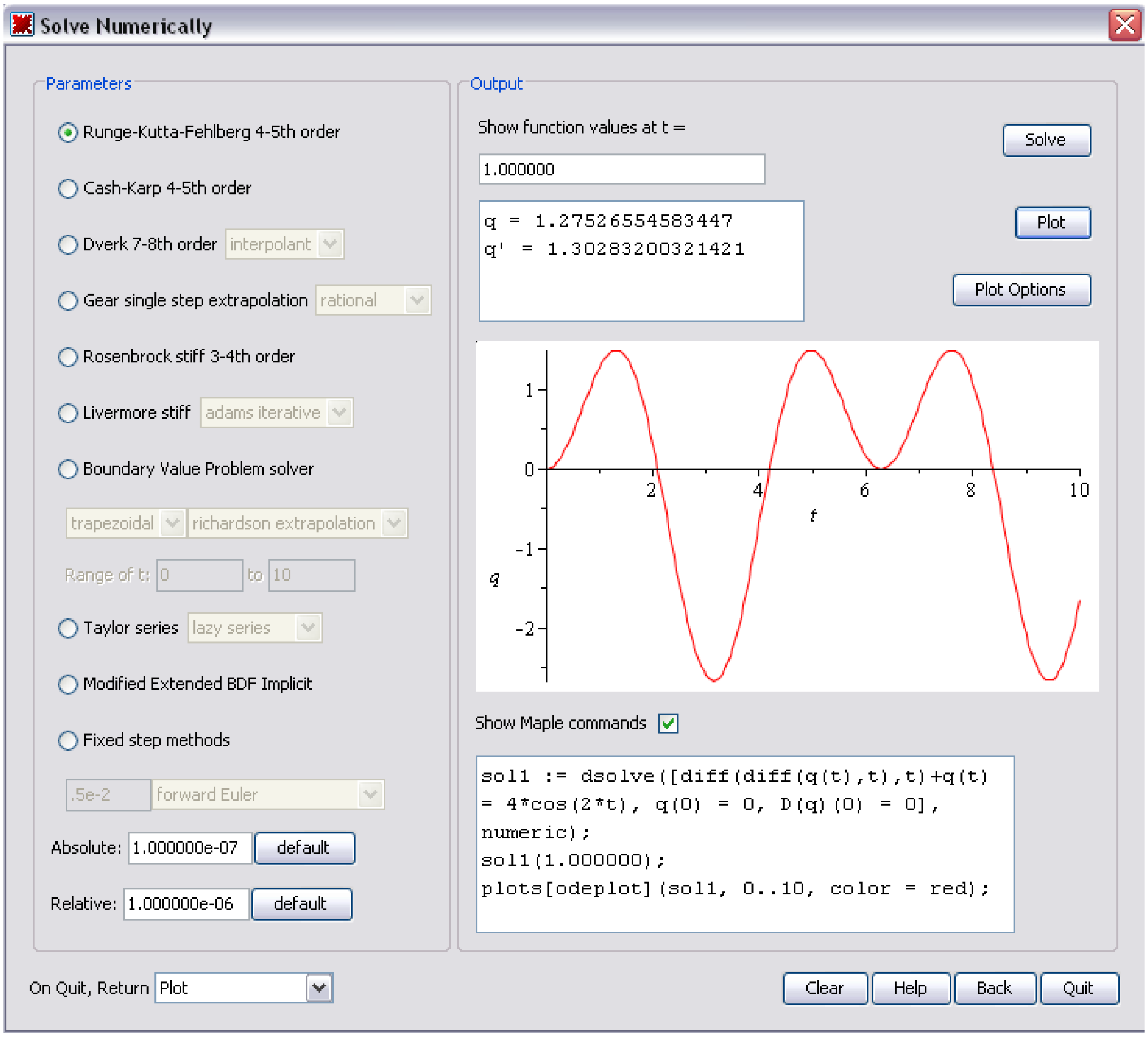


Figure 4.4: ODE Analyzer Assistant: Solve Numerically Dialog

To solve a system symbolically using the ODE Analyzer Assistant:

1. Click the **Solve Symbolically** button.
2. In the **Solve Symbolically** window (**Figure 4.5**), you can specify the method and relevant method-specific options to use for solving the problem.
3. To compute the solution, click the **Solve** button.

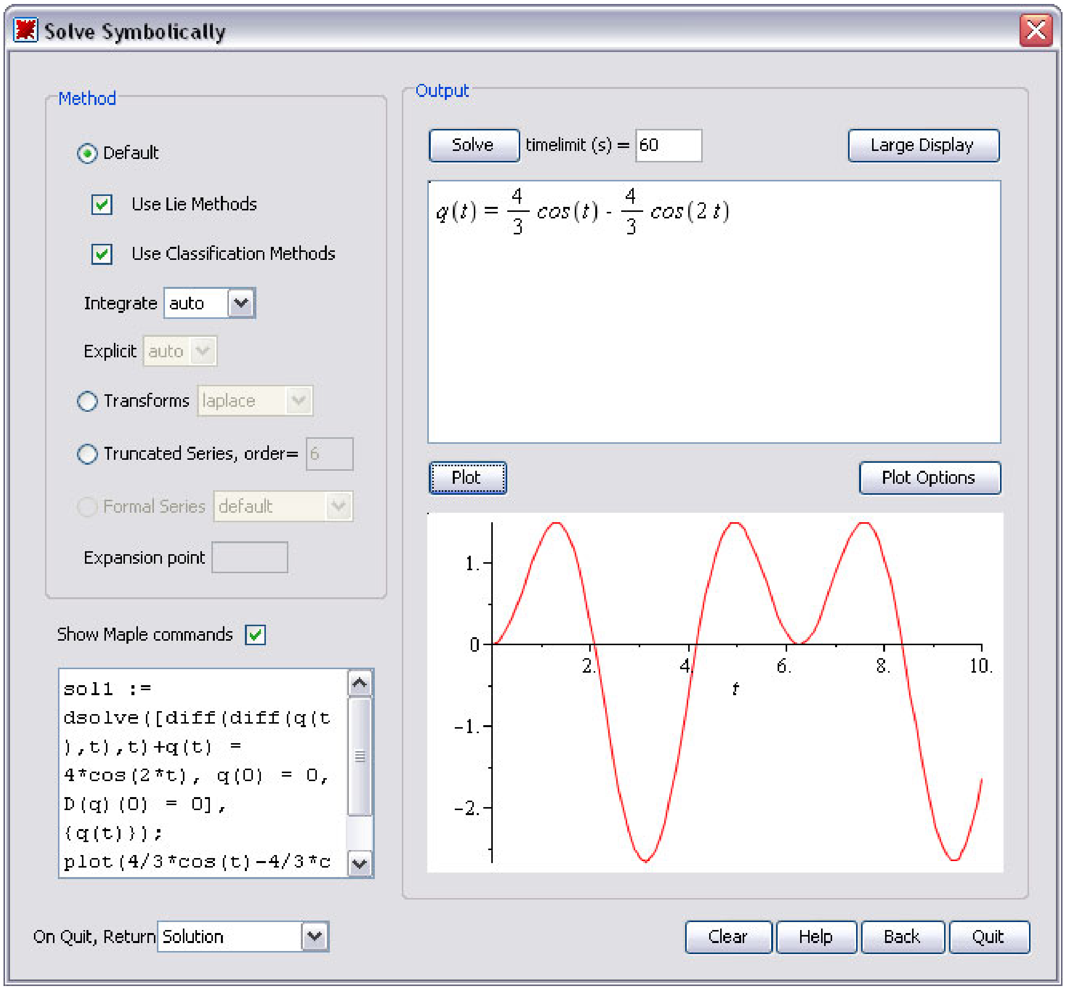


Figure 4.5: ODE Analyzer Assistant: Solve Symbolically Dialog

When solving numerically or symbolically, you can view a plot of the solution by clicking the **Plot** button.

- To plot the solution to a symbolic problem, all conditions and parameters must be set.
- To customize the plot, click the **Plot Options** button to open the **Plot Options** window.

To view the corresponding Maple commands as you solve the problem or plot the solution, select the **Show Maple** commands check box.

You can control the return value of the ODE Analyzer using the **On Quit, Return** drop-down list. You can select to return nothing, the displayed plot, the computed numeric procedure (for numeric solutions), the solution (for symbolic solutions), or the Maple commands needed to produce the solution values and the displayed plot.

For more information, refer to the **ODEAnalyzer** help page.

The dsolve Command

The ODE Analyzer provides a point-and-click interface to the Maple **dsolve** command.

For ODEs or systems of ODEs, the **dsolve** command can find:

- Closed form solutions
- Numerical solutions

- Series solutions

In addition, the **dsolve** command can find:

- Formal power series solutions to linear ODEs with polynomial coefficients
- Formal solutions to linear ODEs with polynomial coefficients

To access all available functionality, use the **dsolve** command directly. For more information, refer to the **dsolve** help page.

Partial Differential Equations (PDEs)

To solve a PDE or PDE system symbolically or numerically, use the **pdsolve** command. PDE systems can contain ODEs, algebraic equations, and inequations.

For example, solve the following PDE symbolically. For help entering a partial derivative, see *Example 1 - Enter a Partial Derivative (page 45)*.

> $restart:$

> $x\left(\frac{\partial}{\partial y}f(x,y)\right)-y\left(\frac{\partial}{\partial x}f(x,y)\right)=0$

$$x\left(\frac{\partial}{\partial y}f(x,y)\right)-y\left(\frac{\partial}{\partial x}f(x,y)\right)=0 \quad (4.14)$$

> $pdsolve(\mathbf{(4.14)})$

$$f(x,y)=\_F1\left(x^2+y^2\right)$$

The solution is an arbitrary univariate function applied to x^2+y^2.

Maple generally prints only the return value, errors, and warnings during a computation. To print information about the techniques Maple uses, increase the **infolevel** setting for the command.

To return all information, set **infolevel** to 5.

> $infolevel[pdsolve]:=5:$

> $pdsolve(\mathbf{(4.14)})$

```
Checking arguments ...
First set of solution methods (general or quasi general solution)
   -> trying characteristic strip method for first order PDEs
   <- characteristic strip method for first order PDEs successful
<- First set of solution methods successful
<- Returning a *general* solution
```

$$f(x,y)=\_F1\left(x^2+y^2\right)$$

For more information on solving PDEs, including numeric solutions and solving PDE systems, refer to the **pdsolve** help page.

Integer Equations

To find only integer solutions to an equation, use the **isolve** command. The isolve command finds solutions for all variables. For more information, refer to the **isolve** help page.

$> isolve(\{x^2 + y = 13\})$

$$\{x = \_Z1, y = -\_Z1^2 + 13\}$$

Integer Equations in a Finite Field

To solve an equation modulo an integer, use the **msolve** command. The msolve command finds solutions for all variables. For more information, refer to the **msolve** help page.

$> msolve(\{x^2 = 1\}, 13)$

$$\{x = 1\}, \{x = 12\}$$

Solving Linear Systems

To solve a linear system, use the **LinearAlgebra[LinearSolve]** command. The LinearSolve command returns the vector x that satisfies A . x = B. For more information, refer to the **LinearAlgebra[LinearSolve]** help page.

For example, construct an augmented matrix using the **Matrix** palette (see *Creating Matrices and Vectors (page 116)*) in which the first four columns contain the entries of **A** and the final column contains the entries of **B**.

$$> linearsystem := \begin{bmatrix} \frac{59}{10} & \frac{44}{25} & \frac{17}{2} & \frac{1}{100} & \frac{1}{2} \\ 1 & 0 & 7 & \frac{533}{100} & \frac{61}{50} \\ 98 & \frac{21}{10} & \frac{3}{10} & 7 & \frac{2178}{25} \\ 23 & 9 & 12 & \frac{51}{10} & \frac{786}{25} \end{bmatrix}:$$

$> LinearAlgebra[LinearSolve](linearsystem)$

$$\begin{bmatrix} \frac{31753441047}{41858667400} \\ \frac{16991806239}{8371733480} \\ -\frac{1489266217}{1674346696} \\ \frac{262603866}{209293337} \end{bmatrix}$$

For more information on using Maple to solve linear algebra problems, see *Linear Algebra (page 116)*.

Solving Recurrence Relations

To solve a recurrence relation, use the **rsolve** command. The rsolve command finds the general term of the function. For more information, refer to the **rsolve** help page.

> $rsolve(\{f(n) = f(n-1) + f(n-2), f(0) = 1, f(1) = 1\}, \{f(n)\})$

$$\left\{ f(n) = \left(\frac{1}{2} + \frac{\sqrt{5}}{10}\right)\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n + \left(\frac{1}{2} - \frac{\sqrt{5}}{10}\right)\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n \right\}$$

4.5 Units, Scientific Constants, and Uncertainty

In addition to manipulating exact symbolic and numeric quantities, Maple can perform computations with units and uncertainties.

Maple supports hundreds of units, for example, miles, coulombs, and bars, and provides facilities for adding custom units.

Maple has a library of hundreds of scientific constants with units, including element and isotope properties.

To support computations with uncertainties, Maple propagates errors through computations.

Units

The **Units** package in Maple provides a library of units, and facilities for using units in computations. It is fully extensible so that you can add units and unit systems as required.

Note: Some unit operations are available as task templates (see **Tools→Tasks→Browse**) and through the context panel.

Overview of Units

A *dimension* is a measurable quantity, for example, length or force. The set of dimensions that are fundamental and independent are known as *base dimensions*.

In Maple, the base dimensions include length, mass, time, electric current, thermodynamic temperature, amount of substance, luminous intensity, information, and currency. For a complete list, enter and execute *Units*[*GetDimensions*]().

Complex dimensions (or composite dimensions) measure other quantities in terms of a combination of base dimensions. For example, the complex dimension force is a measurement of $\frac{mass \cdot length}{time^2}$.

Each dimension, base or complex, has associated units. (Base units measure a base dimension. Complex units measure a complex dimension.) Maple supports over 40 units of length, including feet, miles, meters, angstroms, microns, and astronomical units. A length must be measured in terms of a unit, for example, a length of 2 parsecs.

Table 4.4 lists some dimensions, their corresponding base dimensions, and example units.

Table 4.4: Sample Dimensions

Dimension	Base Dimensions	Example Units
Time	time	second, minute, hour, day, week, month, year, millennium, blink, lune
Energy	$\frac{length^2 \cdot mass}{time^2}$	joule, electron volt, erg, watt hour, calorie, Calorie, British thermal unit
Electric potential	$\frac{length^2 \cdot mass}{time^3 \cdot electric\ current}$	volt, abvolt, statvolt

For the complete list of units (and their contexts and symbols) available for a dimension, refer to the corresponding help page, for example, the **Units/length** help page for the units of length.

Each unit has a *context*. The context differentiates between different definitions of the unit. For example, the standard and US survey miles are different units of length, and the second is a unit of time and of angle. You can specify the context for a unit by appending the context as an index to the unit, for example, **mile[US_survey]**. If you do not specify a context, Maple uses the default context.

Units are collected into systems, for example, the foot-pound-second (FPS) system and international system, or *système international*, (SI). Each system has a default set of units used for measurements. In the FPS system, the foot, pound, and second are used to measure the dimensions of length, mass, and time. The unit of speed is the foot/second. In SI, the meter, kilogram, and second are used to measure the dimensions of length, mass, and time. The units of speed, magnetic flux, and power are the meter/second, weber, and watt, respectively.

Unit Conversions

To convert a value measured in a unit to the corresponding value in a different unit, use the **Units Converter**.

- From the Tools→Assistants menu, select **Units Converter**.

The **Units Converter** application (**Figure 4.6**) opens.

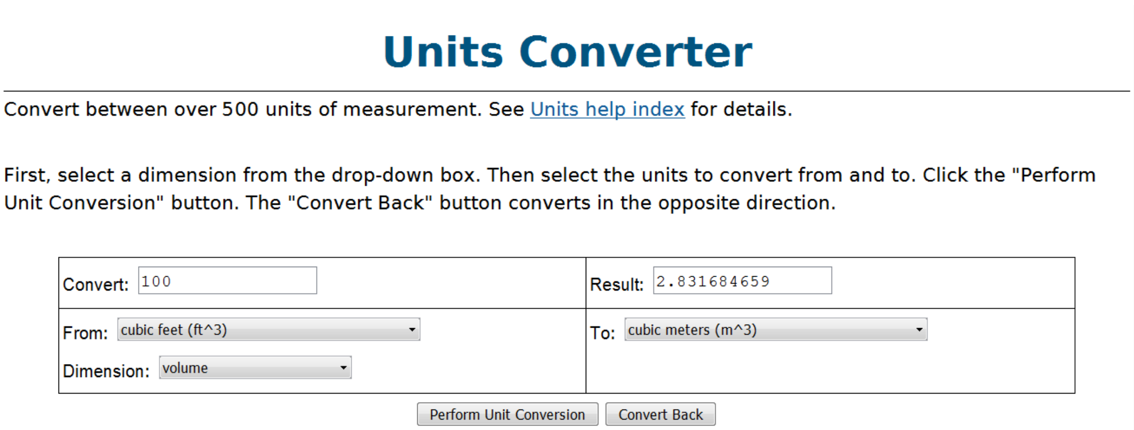


Figure 4.6: Units Converter Assistant

To perform a conversion:

1. In the **Convert** text field, enter the numeric value to convert.
2. In the **Dimension** drop-down list, select the dimensions of the unit.
3. In the **From** and **To** drop-down lists, select the original unit and the unit to which to convert.
4. Click **Perform Unit Conversion**.

The same conversion can be done with the **convert/units** command.

> *convert*(1.0, '*units*', '*lbf ft*(*radius*)', '*N m*(*radius*)')

$$1.355817948$$

Using the **Units Converter**, you can convert temperatures and temperature changes.

- To perform a *temperature* conversion, in the **Dimension** drop-down list, select **temperature(absolute)**.
- To perform a *temperature change* conversion, in the **Dimension** drop-down list, select **temperature(relative)**.

To convert temperature changes, the **Units Converter** uses the **convert/units** command. For example, an increase of 32 degrees Fahrenheit corresponds to an increase of almost 18 degrees Celsius.

> *convert*(32.0, '*units*', '*degF*', '*degC*')

17.77777778

To convert absolute temperatures, the **Unit Converter** uses the **convert/temperature** command. For example, 32 degrees Fahrenheit corresponds to 0 degrees Celsius.

> *convert*(32, '*temperature*', '*degF*', '*degC*')

0

Applying Units to an Expression

To insert a unit, use the **Units** palette . See **Figure 4.7**.

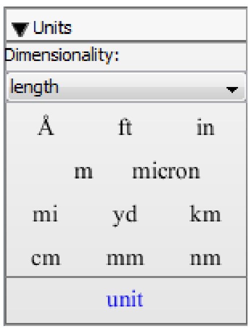


Figure 4.7: Units Palette

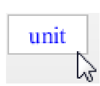
To insert a unit:

- In the **Units** palette, select a unit dimension, then click the desired unit symbol.

> 3ft

3 ft

To insert a unit that is unavailable in the palettes:

1. In the **Units** palette, click the **unit** symbol 

. Maple inserts a **Unit** object with the placeholder selected.
2. In the placeholder, enter the unit name (or symbol). Note that you see double brackets around the unit when you are editing it.

For example, to enter 0.01 standard (the default context) miles, you can specify the unit name, **mile**, or symbol, **mi**.

As you edit, the unit is enclosed in double brackets: 0.01⟦mile⟧

> 0.01mile

0.01 mi

The context of a unit is displayed only if it is not the default context.

Alternative ways to enter units:

- Enter *Unit*, and then press the symbol completion key (see *Symbol Names (page 21)*) and then enter the unit.
- Use the shortcut key **Ctrl** + **Shift** + **U** (Command + Shift + U, on Mac) and then enter the unit.

- Use the **Unit** command. Note that to write a quantity with a unit using this command, write multiplication between them; this is especially evident in 1-D math:

Important: In 1-D Math input, the quantity and unit (entered using the top-level **Unit** command) are a product, not a single entity. The following calling sequences define different expressions.

```
> 1*Unit(m)/(2*Unit(s));
```

$$\frac{1}{2\ \mathrm{s}}\ \mathrm{m}$$

```
> 1*Unit(m)/2*Unit(s);
```

$$\frac{1}{2}\ \mathrm{m\ s}$$

Some units support **prefixes**. For example, SI units support prefixes to names and symbols. You can specify 1000 meters using kilometer or km. For more information, refer to the **Units/prefixes** help page.

> $1.5[\![km_{SI}]\!]$

$$1.5\ \mathrm{km}$$

Performing Computations with Units

In the default Maple environment, you cannot perform computations with quantities that have units. You can perform only unit conversions. For more information about the default environment, refer to the **Units/Default** help page.

To compute with expressions that have units, you must load a **Units** environment: Simple, Standard, or Natural. It is recommended that you use the **Simple environment**, and that environment is loaded by default when you load the Units package:

> $with(Units):$

```
Automatically loading the Units[Simple] subpackage
```

In the **Simple Units** environment, commands that support expressions with units return results with the correct units.

> $area := 3\mathrm{ft}\cdot\frac{1}{8}\mathrm{mile}$

$$area := \frac{3}{8}\ \mathrm{ft\ mi}$$

> $(-12\sin(x)+x^2)\frac{\mathrm{m}}{\mathrm{s}}$

$$\left(-12\sin(x)+x^2\right)\frac{\mathrm{m}}{\mathrm{s}} \tag{4.15}$$

> $int(\mathbf{(4.15)}, x)$

$$\left(\frac{x^3}{3}+12\cos(x)\right)\frac{\mathrm{m}}{\mathrm{s}} \tag{4.16}$$

> $diff(\mathbf{(4.16)}, x\mathrm{s})$

$$\left(-12\sin(x)+x^2\right)\frac{\mathrm{m}}{\mathrm{s}^2}$$

For information on differentiation and integration, see *Calculus (page 128)*.

Changing the Current System of Units

If a computation includes multiple units, all units are expressed using units from the current system of units.

> 132.25mile

$$132.25 \text{ mi} \tag{4.17}$$

By default, Maple uses the SI system of units, in which length is measured in meters and time is measured in seconds.

> $\frac{\textbf{(4.17)}}{3\text{hour}}$

$$19.70701333 \ \frac{\text{m}}{\text{s}}$$

To view the name of the default system of units, use the **Units[UsingSystem]** command or view the current selection for **Choose System**, under the **Convert Output Units** section of the context panel.

> *with*(*Units*) :

```
Automatically loading the Units[Simple] subpackage
```

> *UsingSystem*( )

$$SI$$

To change the system of units, use the **Units[UseSystem]** command or click the output and then select the desired system of units in the **Choose System** list, under the **Convert Output Units** section of the context panel.

> *UseSystem*(*FPS*) :

> **(4.17)** · 3m · 1.1kg

$$1.666720741 \times 10^{7} \text{ ft}^{2} \text{ lb}$$

Extensibility

You can extend the set of:

- Base dimensions and units
- Complex dimensions
- Complex units
- Systems of units

For more information, refer to the **Units[AddBaseUnit]**, **Units[AddDimension]**, **Units[AddUnit]**, and **Units[AddSystem]** help pages.

For more information about units, refer to the **Units** help page.

Scientific Constants and Element Properties

Computations often require not only units (see *Units (page 93)*), but also the values of scientific constants, including properties of elements and their isotopes. Maple supports computations with scientific constants. You can use the built-in constants and add custom constants.

Overview of Scientific Constants and Element Properties

The **ScientificConstants** package provides the values of constant physical quantities, for example, the velocity of light and the atomic weight of sodium. The **ScientificConstants** package also provides the units for the constant values, allowing for greater understanding of the equation as well as unit-matching for error checking of the solution.

The quantities available in the **ScientificConstants** package are divided into two distinct categories.

- Physical constants
- Chemical element (and isotope) properties

Scientific Constants

List of Scientific Constants

You have access to scientific constants important in engineering, physics, chemistry, and other fields. **Table 4.5** lists some of the supported constants. For a complete list of scientific constants, refer to the **ScientificConstants/Physical-Constants** help page.

Table 4.5: Scientific Constants

Name	Symbol
Newtonian_constant_of_gravitation	G
Planck_constant	h
elementary_charge	e
Bohr_radius	a[0]
deuteron_magnetic_moment	mu[d]
Avogadro_constant	N[A]
Faraday_constant	F

You can specify a constant using either its name or symbol.

Accessing Constant Definition

The **GetConstant** command in the **ScientificConstants** package returns the complete definition of a constant.

To view the definition of the Newtonian gravitational constant, specify the symbol **G** (or its name) in a call to the **GetConstant** command.

```
> with(ScientificConstants):
> GetConstant('G')
```

$$Newtonian\_constant\_of\_gravitation, symbol = G, value = 6.67408 \times 10^{-11}, uncertainty = 3.1 \times 10^{-15}, units = \frac{m^3}{kg\, s^2}$$

For information on accessing a constant's value, units, or uncertainty, see *Value, Units, and Uncertainty (page 99)*.

Element Properties

Maple also contains element properties and isotope properties.

Elements

Maple supports all 117 elements of the periodic table. Each element has a unique name, atomic number, and chemical symbol. You can specify an element using any of these labels. For a complete list of supported elements, refer to the **ScientificConstants/elements** help page.

Maple supports key element properties, including atomic weight (**atomicweight**), electron affinity (**electronaffinity**), and density. For a complete list of element properties, refer to the **ScientificConstants/properties** help page.

Isotopes

Isotopes, variant forms of an element that contain the same number of protons but a different number of neutrons, exist for many elements.

To see the list of supported isotopes for an element, use the **GetIsotopes** command.

> *GetIsotopes*('*element*' = 'Li')

$$\mathrm{Li}_4, \mathrm{Li}_5, \mathrm{Li}_6, \mathrm{Li}_7, \mathrm{Li}_8, \mathrm{Li}_9, \mathrm{Li}_{10}, \mathrm{Li}_{11}, \mathrm{Li}_{12}$$

Maple supports isotopes and has a distinct set of properties for isotopes, including abundance, binding energy (**bindingenergy**), and mass excess (**massexcess**). For a complete list of isotope properties, refer to the **ScientificConstants/properties** help page.

Accessing an Element or Isotope Property Definition

The **GetElement** command in the **ScientificConstant**s package returns the complete definition of an element or isotope.

> *GetElement*('*Li*')

$$3, symbol = \mathrm{Li}, name = lithium, names = \{lithium\}, density = \left[value = 0.534, uncertainty = undefined, units = \frac{g}{cm^3}\right], electronaffinity = [value = 0.6180, uncertainty = 0.0005, units = eV], boilingpoint = [value = 1615., uncertainty = undefined, units = K], electronegativity = [value = 0.98, uncertainty = undefined, units = 1], ionizationenergy = [value = 5.3917, uncertainty = undefined, units = eV], atomicweight = [value = 6.941, uncertainty = 0.002, units = amu], meltingpoint = [value = 453.65, uncertainty = undefined, units = K]$$

> *GetElement*('*Li*[4]')

$$\mathrm{Li}_4, massexcess = [value = 25320.173, uncertainty = 212.132, units = keV], bindingenergy = [value = 4618.058, uncertainty = 212.132, units = keV], atomicmass = \left[value = 4.027182329 \times 10^6, uncertainty = 227.733, units = \mu amu\right]$$

Value, Units, and Uncertainty

To use constants or element properties, you must first construct a **ScientificConstants** object.

To construct a scientific constant, use the **Constant** command.

> *G* := *Constant*('*G*') :

To construct an element (or isotope) property, use the **Element** command.

> *LiAtomicWeight* := *Element*('Li', *atomicweight*)

$$LiAtomicWeight := Element(\mathrm{Li}, atomicweight)$$

Value

To obtain the value of a **ScientificConstants** object, use the **evalf** command.

> *evalf*(*G*)

$$1.069085060 \times 10^{-9}$$

> *evalf*(*LiAtomicWeight*)

$$2.541006042 \times 10^{-26}$$

Note: The value returned depends on the current system of units.

Units

To obtain the units for a **ScientificConstants** object, use the **GetUnit** command.

> *GetUnit*(*G*)

$$\frac{\mathrm{ft}^3}{\mathrm{lb}\ \mathrm{s}^2}$$

> *GetUnit*(*LiAtomicWeight*)

$$\mathrm{lb}$$

For information on changing the default system of units, for example, from SI to foot-pound-second, see *Changing the Current System of Units (page 97)*.

Value and Units

If you are performing computations with units, you can access the value and units for a **ScientificConstants** object by specifying the **units** option when constructing the object, and then evaluating the object.

> *evalf*(*Constant*('*G*', *units*))

$$1.069085060 \times 10^{-9}\ \frac{\mathrm{ft}^3}{\mathrm{lb}\ \mathrm{s}^2}$$

> *evalf*(*Element*('Li[5]', *atomicmass*, *units*))

$$1.835022162 \times 10^{-26}\ \mathrm{lb}$$

Uncertainty

The value of a constant is often determined by direct measurement or derived from measured values. Hence, it has an associated uncertainty. To obtain the uncertainty in the value of a **ScientificConstants** object, use the **GetError** command.

> *GetError*(*G*)

$$4.965723646 \times 10^{-14}$$

> *GetError*(*LiAtomicWeight*)

$$7.321728978 \times 10^{-30}$$

Performing Computations

You can use constant values in any computation. To use constant values with units, use a **Units** environment as described in *Performing Computations with Units (page 96)*. For information on computing with quantities that have an uncertainty, see the following section.

Modification and Extensibility

You can change the definition of a scientific constant or element (or isotope) property.

For more information, refer to the **ScientificConstants[ModifyConstant]** and **ScientificConstants[ModifyElement]** help pages.

You can extend the set of:

- Constants
- Elements (and isotopes)
- Element (or isotope) properties

For more information, refer to the **ScientificConstants[AddConstant]**, **ScientificConstants[AddElement]**, and **ScientificConstants[AddProperty]** help pages.

For more information about constants, refer to the **ScientificConstants** help page.

Uncertainty Propagation

Some computations involve uncertainties (or errors). Using the **ScientificErrorAnalysis** package, you can propagate the uncertainty in these values through the computation to indicate the possible error in the final result.

The **ScientificErrorAnalysis** package does *not* perform *interval arithmetic*. That is, the *error* of an object does *not* represent an *interval* in which possible values must be contained. (To perform interval arithmetic, use the **Tolerances** package. For more information, refer to the **Tolerances** help page..) The quantities represent unknown values with a central tendency. For more information on central tendency, refer to any text on error analysis for the physical sciences or engineering.

Quantities with Uncertainty

Creating: To construct quantities with uncertainty, use the **Quantity** command. You must specify the value and uncertainty. The uncertainty can be defined absolutely, relatively, or in units of the last digit. For more information on uncertainty specification, refer to the **ScientificErrorAnalysis[Quantity]** help page.

The output displays the value and uncertainty of the quantity.

> *restart* :

> *with*(*ScientificConstants*): *with*(*ScientificErrorAnalysis*):

> *Quantity*(105, 1.2)

$$Quantity(105, 1.2)$$

> *Quantity*(105, 0.03, '*relative*')

$$Quantity(105, 3.15) \quad (4.18)$$

To specify the error in units of the last digit, the value must be of floating-point type.

> *Quantity*(105.0, 12, '*uld*')

$$Quantity(105.0, 1.2)$$

To access the value and uncertainty of a quantity with uncertainty, use the **evalf** and **ScientificErrorAnalysis[GetError]** commands.

> *evalf*(**(4.18)**)

$$105.$$

> *GetError*(**(4.18)**)

$$3.15$$

Rounding: To round the error of a quantity with uncertainty, use the **ApplyRule** command. For a description of the predefined rounding rules, refer to the **ScientificErrorAnalysis/rules** help page.

> *GetError*(*ApplyRule*(**(4.18)**, 'round[2]'))

$$3.2$$

Units: Quantities with errors can have units. For example, the scientific constants and element (and isotope) properties in the **ScientificConstants** packages are quantities with errors and units.

To construct a new quantity with units and an uncertainty, include units in the **Quantity** calling sequence.

For an absolute error, you must specify the units in both the value and error.

> *with*(*Units*[*Standard*]) : *with*(*ScientificErrorAnalysis*) :

> *Quantity*(3.5 m, 0.1 m)

$$Quantity(3.5\ \mathrm{m}, 0.1\ \mathrm{m})$$

For a relative error, you can specify the units in only the value.

> *Quantity*(3.5m, 0.1, '*relative*')

$$Quantity(3.5\ \mathrm{m}, 0.35\ \mathrm{m})$$

For information on the correlation between, variance of, and covariance between quantities with uncertainty, refer to the **ScientificErrorAnalysis** help page.

Performing Computations with Quantities with Uncertainty

Many Maple commands support quantities with uncertainty.

> *q1* := *Quantity*(31., 2.):

> $q2 := Quantity(20., 1.)$:

Compute the value of the derivative of $q1 \cdot x^2 + sin(q2 \cdot x)$ at $x = sin(\pi/4)$.

> $d1 := diff(q1 \cdot x^2 + \sin(q2 \cdot x), x)$

$$d1 := 2\, x\, Quantity(31., 2.) + Quantity(20., 1.) \cos(x\, Quantity(20., 1.))$$

> $d2 := eval\left(d1, x = sin\left(\frac{\pi}{4}\right)\right)$:

To convert the solution to a single quantity with uncertainty, use the **combine/errors** command.

> $result := combine(d2, 'errors')$:

The value of the result is:

> $evalf(result)$

$$43.74124725$$

The uncertainty of the result is:

> $GetError(result)$

$$14.42690612$$

Additional Information

For information on topics including:

- Creating new rounding rules,
- Setting the default rounding rule, and
- Creating a new interface to quantities with uncertainty,

refer to the **ScientificErrorAnalysis** help page.

4.6 Restricting the Domain

By default, Maple computes in the complex number system. Most computations are performed without any restrictions or assumptions on the variables. Maple often returns results that are extraneous or unsimplified when computing in the field of complex numbers. Using restrictions, you can more easily and efficiently perform computations in a smaller domain.

Maple has facilities for performing computations in the real number system and for applying assumptions to variables.

Real Number Domain

To force Maple to perform computations in the field of real numbers, use the **RealDomain** package.

The **RealDomain** package contains a small subset of Maple commands related to basic precalculus and calculus mathematics, for example, **arccos**, **limit**, and **log**, and the symbolic manipulation of expressions and formulae, for example, **expand**, **eval**, and **solve**. For a complete list of commands, refer to the **RealDomain** help page.

After you load the **RealDomain** package, Maple assumes that all variables are real. Commands return simplified results appropriate to the field of real numbers.

Table 4.6: Restricting to Real Numbers

<table>
<tr><th>Without Loading RealDomain</th><th>After Loading RealDomain</th></tr>
<tr><td>In Maple's default environment, without loading RealDomain, the answers are correct for the entire complex field.

> $simplify\left(\sqrt{x^2}\right)$
$\mathrm{csgn}(x)\, x$ (4.19)

> $\ln(e^x)$
$\ln(e^x)$ (4.20)

> $(-32)^{\left(\frac{1}{5}\right)}$
$(-32)^{1/5}$ (4.21)</td><td>After loading RealDomain, the answers are simplified.

> $with(RealDomain)$:

> $simplify\left(\sqrt{x^2}\right)$
$|x|$ (4.22)

> $\ln(e^x)$
x (4.23)

> $(-32)^{\left(\frac{1}{5}\right)}$
-2 (4.24)</td></tr>
<tr><td>> $solve(x^2 = -1)$
$I, -I$

> $\arcsin(e^2)$
$\arcsin(e^2)$</td><td>Complex return values are excluded or replaced by undefined.

> $solve(x^2 = -1)$

> $\arcsin(e^2)$
$undefined$</td></tr>
</table>

After loading the **RealDomain** package, you can also use the context-sensitive items for simplify and solve to perform computations in the field of real numbers. For example, using the context panel item **Simplify > Simplify**,

$$\sqrt{x^2} \overset{\text{simplify}}{=} |x|$$

Assumptions on Variables

To simplify problem solving, it is recommended that you always apply any known assumptions to variables. You can impose assumptions using the **assume** command. To apply assumptions for a single computation, use the **assuming** command.

Note: The **assume** and **assuming** commands are not supported by the **RealDomain** package.

The assume Command

You can use the **assume** command to set variable properties, for example, **x::real**, and relationships between variables, for example, **x < 0** or **x < y**. For information on valid properties, refer to the **assume** help page. For information on the double colon (**::**) operator, refer to the **type** help page.

The **assume** command allows improved simplification of symbolic expressions, especially multiple-valued functions, for example, computing the square root.

To assume that **x** is a positive real number, use the following calling sequence. Then compute the square root of x^2.

> *assume*$(0 < x)$: $\sqrt{x^2}$

$$\sqrt{x\sim^2}$$

The trailing tilde (~) on the name **x** indicates that it carries assumptions.

When you use the **assume** command to place another assumption on **x**, all previous assumptions are removed.

> *assume*$(x < 0)$: $\sqrt{x^2}$

$$\sqrt{x\sim^2}$$

Displaying Assumptions: To view the assumptions on an expression, use the **about** command.

> *about*(x)

```
Originally x, renamed x~:
  is assumed to be: RealRange(-infinity,Open(0))
```

Imposing Multiple Assumptions: To simultaneously impose multiple conditions on an expression, specify multiple arguments in the **assume** calling sequence.

> *assume*$(0 < x, x < 2)$

To specify additional assumptions without replacing previous assumptions, use the **additionally** command. The syntax of the **additionally** calling sequence is the same as that of the **assume** command.

> *additionally*$(x :: integer)$: *about*(x)

```
Originally x, renamed x~:
  is assumed to be: 1
```

The only integer in the open interval **(0, 2)** is **1**.

Testing Properties: To test whether an expression always satisfies a condition, use the **is** command.

> *assume*$(15 < x, 7 < y)$: *is*$(100 < x\,y)$

$$true$$

The following test returns **false** because there are values of **x** and **y** (**x = 0**, **y = 10**) that satisfy the assumptions, but do not satisfy the relation in the **is** calling sequence.

> *assume*$(x :: nonnegint, 10 \leq y)$: *is*$(10 < x + y)$

$$false$$

To test whether an expression can satisfy a condition, use the **coulditbe** command.

> *coulditbe*$(10 < x + y)$

$$true$$

Removing Assumptions: To remove all assumptions on a variable, unassign its name.

> $x := 'x'$: $y := 'y'$:

For more information, see *Unassigning Names right single (page 67)*.

For more information on the **assume** command, refer to the **assume** help page.

The assuming Command

To perform a single evaluation under assumptions on the names in an expression, use the **assuming** command.

The syntax of the assuming command is <*expression*> **assuming** <*property or relation*>. Properties and relations are introduced in *The assume Command (page 104)*.

The **frac** command returns the fractional part of an expression.

> $\mathrm{frac}(x)$ assuming $x :: integer$

$$0$$

Using the **assuming** command is equivalent to imposing assumptions with the **assume** command, evaluating the expression, and then removing the assumptions.

> $about(x)$

```
x:
  nothing known about this object
```

If you do not specify the names to which to apply a property, it is applied to all names.

> $\sqrt{\left(\frac{a}{b}\right)^2}$ assuming *positive*

$$\sqrt{\frac{a^2}{b^2}}$$

Assumptions placed on names using the **assume** command are ignored by the **assuming** command, unless you include the **additionally** option.

> $assume(x < 1)$

> $is\left(1 - x^2 > 0\right)$ assuming $x > -1$

$$false$$

> $is\left(1 - x^2 > 0\right)$ assuming *additionally*, $x > -1$

$$true$$

The **assuming** command does not affect variables inside procedures. (For information on procedures, see *Procedures (page 305)*.) You must use the **assume** command.

```
> f := proc(x) sqrt(a^2) + x end proc;
```

$f := \textbf{proc}(x)\ \ Units{:}\text{-}Standard{:}\text{-}`+`(RealDomain{:}\text{-}\mathrm{sqrt}(RealDomain{:}\text{-}`\hat{}`(a, 2)), x)\ \ \textbf{end proc}$

> $f(1)$ assuming $a > 0$

$$\sqrt{a^2} + 1$$

> $assume(a > 0)$: $f(1)$

$$\sqrt{a\sim^2} + 1$$

For more information on the **assuming** command, refer to the **assuming** help page.

5 Mathematical Problem Solving

This chapter focuses on solving problems in specific mathematical disciplines. The areas described below are not all that Maple provides, but represent the most commonly used packages. Examples are provided to teach you how to use the different methods of calculation available in Maple, including tutors, assistants, commands, task templates, plotting, and context-sensitive operations.

The examples in this chapter assume knowledge of entering commands and mathematical symbols. For information, see *Entering Expressions (page 14)*. For information on basic computations, including integer operations and solving equations, see *Basic Computations (page 73)*.

5.1 In This Chapter

Section	Topics
Algebra (page 109) - Performing algebra computations	• Polynomial Algebra
Linear Algebra (page 116) - Performing linear algebra computations	• Creating Matrices and Vectors • Accessing Entries in Matrices and Vectors • Linear Algebra Computations • **Student LinearAlgebra** Package
Calculus (page 128) - Performing calculus computations	• Limits • Differentiation • Series • Integration • Differential Equations • Calculus Packages
Optimization (page 138) - Performing optimization computations using the **Optimization** package	• Point-and-Click Interface • Efficient Computation • MPS(X) File Support
Statistics (page 141) - Performing statistics computations using the **Statistics** package	• Probability Distributions and Random Variables • Statistical Computations • Plotting
Teaching and Learning with Maple (page 145) - Student and Instructor resources for using Maple in an academic setting	• **Student** Packages and Tutors • Study Guides • Step by Step Solutions • More Student and Instructor Resources
Clickable MathTM (page 157) - Solve math problems using some of the interactive methods available in Maple	• Step-by-Step examples

5.2 Algebra

Maple contains a variety of commands that perform integer operations, such as factoring and modular arithmetic, as described in *Integer Operations (page 76)*. The following section describes Maple's support of polynomial algebra.

For information on matrix and vector algebra, see *Linear Algebra (page 116)*.

For information directed to students of algebra or precalculus, see *Teaching and Learning with Maple (page 145)*.

Polynomial Algebra

A Maple polynomial is an expression in powers of an unknown. *Univariate* polynomials are polynomials in one unknown, for example, $x^3 - 2x + 13$. *Multivariate* polynomials are polynomials in multiple unknowns, such as $x^3 y - \frac{3}{2} x y^2 + 7 x$.

The coefficients can be integers, rational numbers, irrational numbers, floating-point numbers, complex numbers, variables, or a combination of these types.

> $a x^2 + 7x - \frac{b}{2}$

$$a x^2 + 7 x - \frac{1}{2} b$$

Arithmetic

The polynomial arithmetic operators are the standard Maple arithmetic operators excluding the division operator (/). (The division operator accepts polynomial arguments, but does not perform *polynomial division*.)

Polynomial division is an important operation. The **quo** and **rem** commands find the quotient and remainder of a polynomial division. See **Table 5.1**. (The **iquo** and **irem** commands find the quotient and remainder of an integer division. For more information, see *Integer Operations (page 76)*.)

Table 5.1: Polynomial Arithmetic Operators

Operation	Operator	Example
Addition	+	> $(x^2 + 1) + (3 x^3 - 5 x + 2)$ $3 x^3 + x^2 - 5 x + 3$
Subtraction	−	> $(x^2 + 1) - (3 x^3 - 5 x + 2)$ $-3 x^3 + x^2 + 5 x - 1$
Multiplication[1]	*	> $(x^2 + 1) \cdot (3 x^3 - 5 x + 2)$ $(x^2 + 1) (3 x^3 - 5 x + 2)$
Division: Quotient and Remainder	**quo** **rem**	> $quo(2 x^2 + x - 3, 3 x + 5, x)$ $\frac{2 x}{3} - \frac{7}{9}$ > $rem(2 x^2 + x - 3, 3 x + 5, x)$ $\frac{8}{9}$
Exponentiation[2]	^	> $(x^2 + 1)^3$ $(x^2 + 1)^3$

Operation	**Operator**	**Example**
[1]You can specify multiplication explicitly by entering *, which displays in 2-D Math as $\cdot$. In 2-D Math, you can also implicitly multiply by placing a space character between two expressions. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication. [2]In 2-D Math, exponents display as superscripts.		

To expand a polynomial, use the **expand** command.

> $expand\left(3\,x^2\cdot(3\,x+5)-\left(x^2-2\right)\right)$

$$9\,x^3+14\,x^2+2$$

If you need to determine whether one polynomial divides another, but do not need the quotient, use the **divide** command. The **divide** command tests for exact polynomial division.

> $divide\left(x^4y^2+x^3y^2-x^2\,y^2+13\,x^2+13\,x-13+y\cdot x^2+x\cdot y-y,\,x^2+x-1\right)$

$$true$$

Important: You must insert a space character or a multiplication operator ($\cdot$) between adjacent variables names. Otherwise, they are interpreted as a single variable.

For example, without a space, xy is considered a single variable, and x does not divide the single variable xy.

> $divide(xy,\,x)$

$$false$$

But, x divides the product of x and y.

> $divide(x\,y,\,x);\ divide(x\cdot y,\,x)$

$$true$$

$$true$$

For information on polynomial arithmetic over finite rings and fields, refer to the **mod** help page.

Sorting Terms

To sort the terms of a polynomial, use the **sort** command.

> $p1 := x^2+x^4+x^3-x:$

> $sort(p1)$

$$x^4+x^3+x^2-x$$

Note: The **sort** command returns the sorted polynomial, and updates the order of the terms in the polynomial.

The terms of **p1** are sorted.

> $p1$

$$x^4 + x^3 + x^2 - x$$

To specify the unknowns of the polynomial and their ordering, include a list of names.

> $sort\left(a^2x^3+x^2 + x\,a + a + b, [a]\right)$

$$x^3\,a^2 + x\,a + a + x^2 + b$$

> $sort\left(a^2x^3+x^2 + x\,a + a + b, [x, b]\right)$

$$a^2\,x^3 + x^2 + a\,x + b + a$$

By default, the **sort** command sorts a polynomial by decreasing *total degree* of the terms.

> $p2:=x^3+y^3+x^2y^2:$

> $sort(p2, [x, y])$

$$x^2\,y^2 + x^3 + y^3$$

The first term has total degree 4. The other two terms have total degree 3. The order of the final two terms is determined by the order of their names in the list.

To sort the terms by *pure lexicographic order*, that is, first by decreasing order of the first unknown in the list option, and then by decreasing order of the next unknown in the list option, specify the **'plex'** option.

> $sort(p2, [x, y], 'plex')$

$$x^3 + x^2\,y^2 + y^3$$

For information on enclosing keywords in right single quotes (**'**), see *Delaying Evaluation (page 291)*.

The first term contains x to the power 3; the second, x to the power 2; and the third, x to the power 0.

Using context-sensitive operations, you can perform operations, such as sorting, for polynomials and many other Maple objects.

To sort a polynomial:

1. Select the polynomial.
2. From the Context Panel, under the **Sorts** menu, select:

- **Single-variable**, and then the unknown, or,
- **Two-variable** (or **Three-variable**), **Pure Lexical** or **Total Degree**, and then the sort priority of the unknowns.

See **Figure 5.1**.

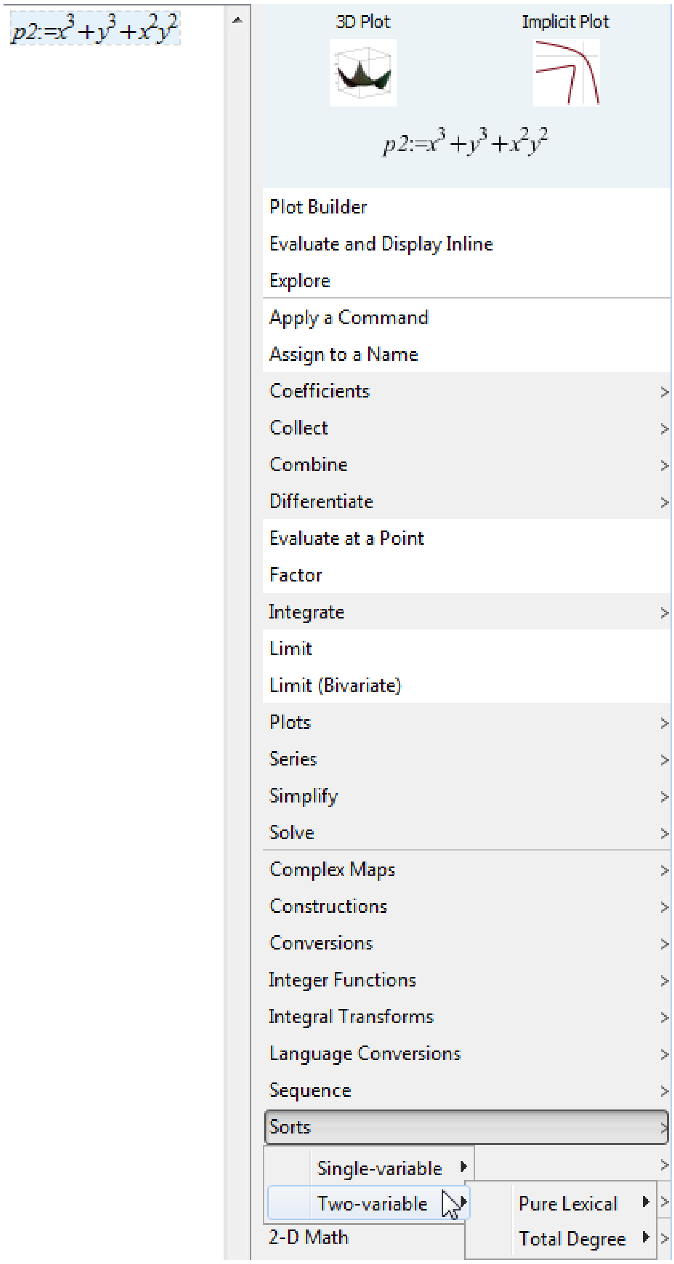


Figure 5.1: Sorting a Polynomial Using the Context Panel

Maple sorts the polynomial.

In Worksheet mode, Maple inserts the calling sequence that performs the sort followed by the sorted polynomial. For this example, choose **Sorts > Two-variable > Pure Lexical > y,x**.

> $x^3 + y^3 + x^2y^2$

$$x^3 + x^2\,y^2 + y^3 \tag{5.1}$$

> $sort(\ y\hat{}3 + y\hat{}2*x\hat{}2 + x\hat{}3,\ [y, x],\ 'plex'\ )$

$$y^3 + y^2\,x^2 + x^3 \tag{5.2}$$

You can use the Context Panel to perform context-sensitive operations on 2-D Math content including output. For more information, see *Computing with the Context Panel (page 48)* (for Document mode) or *The Context Panel (page 63)* (for Worksheet mode).

Collecting Terms

To collect the terms of polynomial, use the **collect** command.

$$> collect\left(2\,a\,x\,y + c\,x^2\,y - z\,y^2 + a\,z - 13\,b\,y + \frac{3\,y^2}{x}, y\right)$$

$$\left(-z + \frac{3}{x}\right)y^2 + \left(c\,x^2 + 2\,a\,x - 13\,b\right)y + a\,z$$

Coefficients and Degrees

Maple has several commands that return coefficient and degree values for a polynomial. See **Table 5.2**.

Table 5.2: Polynomial Coefficient and Degree Commands

Command	Description	Example
coeff	Coefficient of specified degree term	$> coeff\left(\frac{1}{2}x^3 - 2\,x + 5, x^3\right)$ $\frac{1}{2}$
lcoeff	Leading coefficient	$> lcoeff\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ $\frac{1}{2}$
tcoeff	Trailing coefficient	$> tcoeff\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ 5
coeffs	Sequence of all coefficients, in one-to-one correspondence with the terms **Note:** It does not return zero coefficients	$> coeffs\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ $5, -2, \frac{1}{2}$
degree	(Highest) degree	$> degree\left(\frac{1}{2}x^3 - 2\,x + 5\right)$ 3
ldegree	Lowest degree term with a non-zero coefficient	$> ldegree\left(\frac{1}{2}x^3 - 2\,x\right)$ 1

Factorization

To express a polynomial in fully factored form, use the **factor** command.

> $factor(x^4 - 1)$

$$(x-1)(x+1)(x^2+1)$$

The **factor** command factors the polynomial over the ring implied by the coefficients, for example, integers. You can specify an algebraic number field over which to factor the polynomial. For more information, refer to the **factor** help page. (The **ifactor** command factors an integer. For more information, see *Integer Operations (page 76)*.)

To solve for the roots of a polynomial, use the **solve** command. For information on the **solve** command, see *Solving Equations and Inequations (page 80)*. (The **isolve** command solves an equation for integer solutions. For more information, see *Integer Equations (page 92)*.)

Other Commands

Table 5.3 lists other commands available for polynomial operations.

Table 5.3: Select Other Polynomial Commands

Command	Description
content	Content (multivariate polynomial)
compoly	Decomposition
discrim	Discriminant
gcd	Greatest common divisor (of two polynomials)
gcdex	Extended Euclidean algorithm (for two polynomials)
CurveFitting[PolynomialInterpolation] See also the **CurveFitting Assistant** (**Tools** → **Tutors** → **Statistics** → **Curve Fitting**)	Interpolating polynomial (for list of points)
lcm	Least common multiple (of two polynomials)
norm	Norm
prem	Pseudo-remainder (of two multivariate polynomials)
primpart	Primitive part (multivariate polynomial)
randpoly	Random polynomial
PolynomialTools[IsSelfReciprocal]	Determine whether self-reciprocal
resultant	Resultant (of two polynomials)
roots	Exact roots (over algebraic number field)
sqrfree	Square-free factorization (multivariate polynomial)

Additional Information

Table 5.4: Additional Polynomial Help

Topic	Resource
General polynomial information	**polynom** help page
PolynomialTools package	**PolynomialTools** package overview help page
Algebraic manipulation of numeric polynomials	**SNAP** (Symbolic-Numeric Algorithms for Polynomials) package overview help page
Polynomial information and commands	Maple Help System Table of Contents: **Mathematics→Algebra→Polynomials** section

5.3 Linear Algebra

Linear algebra operations act on Matrix and Vector data structures.

You can perform many linear algebra operations using task templates. In the **Task Browser** (**Tools → Tasks → Browse**), expand the **Linear Algebra** folder.

Creating Matrices and Vectors

Creating Matrices

You can create a Matrix using

- The **Matrix** command
- The angle bracket shortcut notation
- The **Matrix palette** (see **Figure 5.2**).

When creating a Matrix using the Matrix command, there are several input formats available. For example, enter a list of lists. The dimensions of the matrix are inferred from the number of entries given.

$$> \mathit{Matrix}\left(\left[[1, \pi, 0], \left[e^2, \sin(t), \frac{87}{2}\right], [0, 0, 5e]\right]\right)$$

$$\begin{bmatrix} 1 & \pi & 0 \\ e^2 & \sin(t) & \frac{87}{2} \\ 0 & 0 & 5\,e \end{bmatrix}$$

Alternatively, use the angle bracket shortcut, <>. Separate items in a column with commas, and separate columns with vertical bars, |.

$$> \left\langle 1, \pi, 0 \middle| e^2, \sin(t), \frac{87}{2} \middle| 0, 0, 5e \right\rangle$$

$$\begin{bmatrix} 1 & e^2 & 0 \\ \pi & \sin(t) & 0 \\ 0 & \frac{87}{2} & 5\,e \end{bmatrix}$$

For information on the Matrix command options, see *Creating Matrices and Vectors with Specific Properties (page 120)*.

Use the Matrix palette to interactively create a matrix without commands:

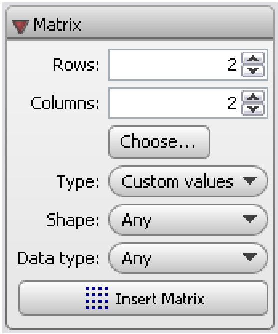


Figure 5.2: Matrix Palette

In the **Matrix** palette, you can specify the matrix size (see **Figure 5.3**) and properties. To insert a matrix, click the **Insert Matrix** button.

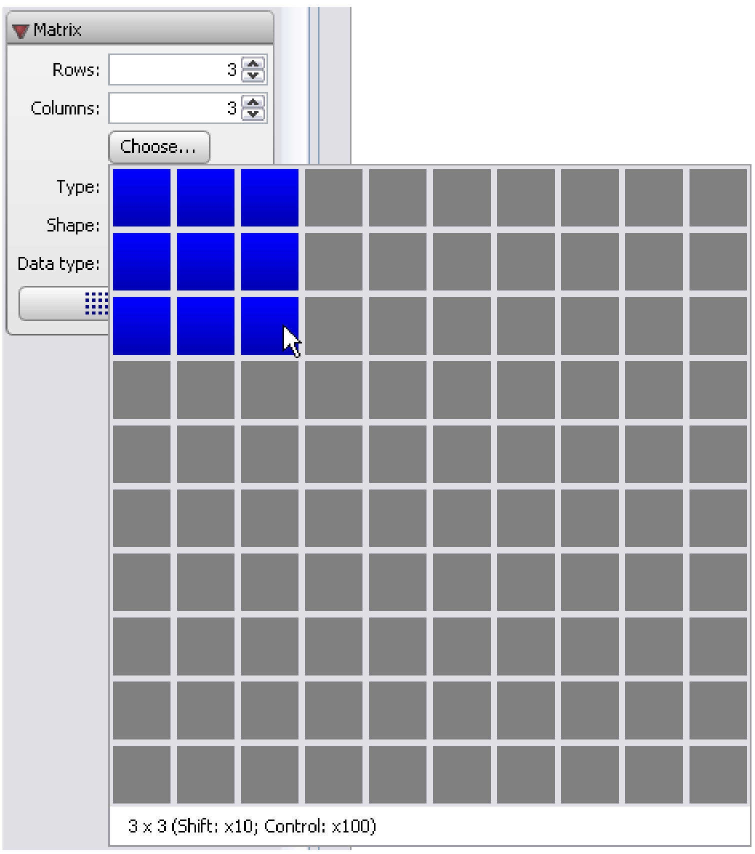


Figure 5.3: Matrix Palette: Choosing the Size

After inserting the matrix:

1. Enter the values of the entries. To move to the next entry placeholder, press **Tab**.
2. After specifying all entries, press **Enter**.

$$> \begin{bmatrix} 1 & e^2 & 0 \\ \pi & \sin(t) & 0 \\ 0 & \frac{87}{2} & 5e \end{bmatrix} :$$

Creating Vectors

You can create a Vector using angle brackets (< >).

To create a column vector, specify a comma-delimited sequence, **<a, b, c>**. The number of elements is inferred from the number of expressions.

$> \langle 1, 2, 3 \rangle$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

To create a row vector, specify a vertical-bar-delimited (|) sequence, **<a | b | c>**. The number of elements is inferred from the number of expressions.

$> \langle 1 \mid 2 \mid 3 \rangle$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

For information on the Vector command options, refer to the **Vector** help page.

You can also create vectors using the **Matrix** palette. If either the number of rows or number of columns specified is 1, then you have the option of inserting a matrix, or inserting a vector of the appropriate type. See **Figure 5.4**.

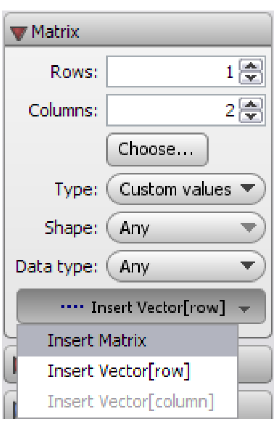


Figure 5.4: Insert Matrix or Insert Vector

Viewing Large Matrices and Vectors

Matrices 10×10 and smaller, and vectors with 10 or fewer elements, display in the document. For larger matrices or vectors, a portion is shown inline.

For example, insert a 15×15 matrix.

In the Matrix palette:

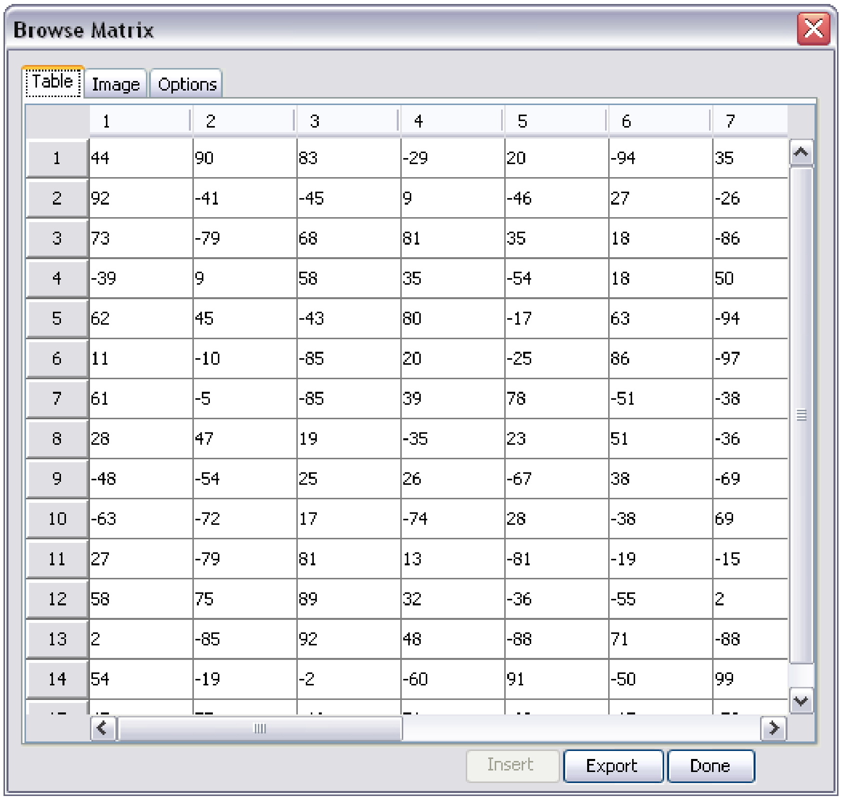
1. Specify the dimensions: 15 rows and 15 columns.
2. In the **Type** drop-down list, select a matrix type, for example, **Random**.
3. Click **Insert Matrix**. The command is inserted; execute it to see the result.

> *LinearAlgebra*:-*RandomMatrix*(15, 15)

$$\begin{bmatrix} 19 & 67 & 60 & -60 & -60 & -13 & -33 & 54 & -1 & 41 & \dots \\ 87 & -55 & 79 & -24 & 77 & -35 & 79 & 31 & 21 & 18 & \dots \\ 33 & 14 & -26 & 37 & -65 & 88 & 93 & 32 & -10 & -9 & \dots \\ 79 & 5 & 1 & 28 & -84 & -72 & 40 & -31 & -82 & -92 & \dots \\ -43 & -32 & -16 & -35 & 4 & -7 & 75 & -19 & 23 & -24 & \dots \\ 36 & 79 & -33 & 51 & 10 & -20 & 72 & -84 & 82 & -62 & \dots \\ 6 & 40 & -36 & 51 & -54 & -74 & 48 & -48 & 42 & -95 & \dots \\ 38 & 42 & -27 & -89 & 52 & 0 & -44 & -81 & -25 & 79 & \dots \\ -55 & -38 & -17 & -23 & -94 & -19 & -46 & 93 & 16 & 72 & \dots \\ 41 & 25 & 9 & 21 & 43 & -43 & 80 & -28 & 73 & -60 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

15 × 15 Matrix

To view the entire matrix or vector, double-click the summary placeholder. This launches the **Matrix Browser**. See **Figure 5.5**.



	1	2	3	4	5	6	7
1	44	90	83	-29	20	-94	35
2	92	-41	-45	9	-46	27	-26
3	73	-79	68	81	35	18	-86
4	-39	9	58	35	-54	18	50
5	62	45	-43	80	-17	63	-94
6	11	-10	-85	20	-25	86	-97
7	61	-5	-85	39	78	-51	-38
8	28	47	19	-35	23	51	-36
9	-48	-54	25	26	-67	38	-69
10	-63	-72	17	-74	28	-38	69
11	27	-79	81	13	-81	-19	-15
12	58	75	89	32	-36	-55	2
13	2	-85	92	48	-88	71	-88
14	54	-19	-2	-60	91	-50	99

Figure 5.5: Matrix Browser

To modify the entries using the Matrix Browser:

1. Select the **Table** tab.
2. Double-click an entry, and then edit its value. Press **Enter**.
3. Repeat for each entry to edit.
4. When you have finished updating entries, click **Done**.

You can view the matrix or vector as a table or as an image, which can be inserted into the document. For more information, refer to the MatrixBrowser help page.

To set the maximum dimension of matrices and vectors displayed inline:

- Use the **interface** command with the **rtablesize** option.

For example, **interface(rtablesize = 15)**.

For more information, refer to the **interface** help page.

Creating Matrices and Vectors with Specific Properties

By default, matrices and vectors can store any values. To increase the efficiency of linear algebra computations, create matrices and vectors with properties. You must specify the properties, for example, the matrix shape or data type, when defining the object.

The **Matrix** palette (**Figure 5.2**) supports several properties.

To specify the matrix type:

- Use the **Shape** and **Type** drop-down lists.

To specify the data type:

- Use the **Data type** drop-down list.

For example, define a diagonal matrix with small integer coefficients.

In the Matrix palette:

1. Specify the size of the matrix, for example, 3×3.
2. In the **Shapes** drop-down list, select **Diagonal**.
3. In the **Data type** drop-down list, select **integer[1]**.
4. Click the **Insert Matrix** button.
5. Enter the values in the diagonal entries.

$$> \begin{bmatrix} -23 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 32 \end{bmatrix}:$$

You cannot specify properties when defining vectors using the angle-bracket notation. You must use the **Vector** constructor.

To define a column vector using the Vector constructor, specify:

- The number of elements. If you explicitly specify all element values, this argument is not required.
- A list of expressions that define the element values.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the vector.

The following two calling sequences are equivalent.

$> Vector([0, 0, 0])$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$> Vector(3, 'shape' = 'zero')$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

To create a row vector using the **Vector** constructor, include **row** as an index.

$> Vector[row](3, 'fill' = 1)$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$> Vector[row]([127, 0, 34], 'datatype' = 'integer[1]')$

$$\begin{bmatrix} 127 & 0 & 34 \end{bmatrix}$$

The **Matrix** palette does not support some properties. To set all properties, use the **Matrix** constructor.

To define a matrix using the Matrix constructor, specify:

- The number of rows and columns. If you explicitly specify all element values, these arguments are not required.
- A list of lists that define the element values row-wise.
- Parameters such as **shape**, **datatype**, and **fill** that set properties of the matrix.

For example:

> *Matrix*([[1, 2, 3], [4, 5, 6]])

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

The **Matrix** palette cannot fill the matrix with an arbitrary value. Use the **fill** parameter.

> *Matrix*(3, 4, [[1, 2, 3], [4, 5, 6]], '*fill*' = 2 + *I*)

$$\begin{bmatrix} 1 & 2 & 3 & 2+\mathrm{I} \\ 4 & 5 & 6 & 2+\mathrm{I} \\ 2+\mathrm{I} & 2+\mathrm{I} & 2+\mathrm{I} & 2+\mathrm{I} \end{bmatrix}$$

For more information on the constructors, including other calling sequence syntaxes and parameters, refer to the **storage**, **Matrix**, and **Vector** help pages.

See also *Numeric Computations (page 128)*.

Accessing Entries in Matrices and Vectors

Matrices

To select an entry in a Matrix, enter the matrix name with a sequence of two non-zero integer indices, row first.

> $M := \langle -4.3, -6.7, 1.9 | 2.9, -1.2, 9.6 | 9.3, -8.0, -9.2 \rangle$

$$M := \begin{bmatrix} -4.3 & 2.9 & 9.3 \\ -6.7 & -1.2 & -8.0 \\ 1.9 & 9.6 & -9.2 \end{bmatrix}$$

> $M[1, 3]$

$$9.3$$

To select an entire row, enter a single index; to select an entire column, enter first the entire range of rows, $1 \,..\, -1$, then the column index.

> $M[2]$

$$\begin{bmatrix} -6.7 & -1.2 & -8.0 \end{bmatrix}$$

> $M[1..-1, 1]$

$$\begin{bmatrix} -4.3 \\ -6.7 \\ 1.9 \end{bmatrix}$$

Similarly, you can access submatrices. Enter the indices as a list or range.

> $M[2..3, 1..2]$

$$\begin{bmatrix} -6.7 & -1.2 \\ 1.9 & 9.6 \end{bmatrix}$$

Vectors

To select an entry in a vector, enter the vector name with a non-zero integer index.

> $a := <85.3, 47.1, 59.9, 38.1>$

$$a := \begin{bmatrix} 85.3 \\ 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

> $a[1]$

$$85.3$$

Negative integers select entries from the end of the vector.

> $a[-1]$

$$38.1$$

To create a Vector consisting of multiple entries, specify a **list** or **range** of integers in the index. For more information, refer to the **set** and **range** help pages.

> $a[[1, 2]]$

$$\begin{bmatrix} 85.3 \\ 47.1 \end{bmatrix}$$

> $a[2..4]$

$$\begin{bmatrix} 47.1 \\ 59.9 \\ 38.1 \end{bmatrix}$$

Linear Algebra Computations

Maple has extensive support for linear algebra. You can perform many matrix and vector computations using context-sensitive operations. Matrix operations such as multiplication and inverses can be done with the basic matrix arithmetic operators. The **LinearAlgebra** package provides the full range of Maple commands for linear algebra and vector space computations, queries, and linear system solving.

Matrix Arithmetic

The matrix and vector arithmetic operators are the standard Maple arithmetic operators up to the following two differences.

- The scalar multiplication operator is the asterisk (*****), which displays in 2-D Math as $\cdot$. The noncommutative matrix and vector multiplication operator is the period (**.**).
- There is no division operator (/) for matrix algebra. (You can construct the inverse of a matrix using the exponent -1.)

Table 5.5 lists the basic matrix operators.

> $A := \begin{bmatrix} 93 & 43 \\ 19 & 37 \end{bmatrix} : B := \begin{bmatrix} 48 & 20 \\ 19 & 37 \end{bmatrix} : C := \langle 23, 6 \rangle :$

Table 5.5: Matrix and Vector Arithmetic Operators

Operation	Operator	Example
Addition	$+$	> $A + B$ $\begin{bmatrix} 141 & 63 \\ 38 & 74 \end{bmatrix}$
Subtraction	$-$	> $A - B$ $\begin{bmatrix} 45 & 23 \\ 0 & 0 \end{bmatrix}$
Multiplication	$.$	> $A.C$ $\begin{bmatrix} 2397 \\ 659 \end{bmatrix}$
Scalar Multiplication[1]	*	> $12\,A$ $\begin{bmatrix} 1116 & 516 \\ 228 & 444 \end{bmatrix}$ > $4 \cdot C$ $\begin{bmatrix} 92 \\ 24 \end{bmatrix}$

Operation	Operator	Example
Exponentiation[2]	^	> A^3 $\begin{bmatrix} 986548 & 613868 \\ 271244 & 187092 \end{bmatrix}$ > B^{-1} $\begin{bmatrix} \frac{37}{1396} & -\frac{5}{349} \\ -\frac{19}{1396} & \frac{12}{349} \end{bmatrix}$

[1]You can specify scalar multiplication explicitly by entering *, which displays in 2-D Math as · . In 2-D Math, you can also implicitly multiply a scalar and a matrix or vector by placing a space character between them. In some cases, the space character is optional. For example, Maple interprets a number followed by a name as an implicit multiplication.

[2]In 2-D Math, exponents display as superscripts.

A few additional matrix and vector operators are listed in **Table 5.6**.

Define two column vectors.

> $d := <1, 2, 3> : e := <4, 5, 6> :$

Table 5.6: Select Matrix and Vector Operators

Operation	Operator	Example
Transpose	**^%T**[1]	> $d^{\%T}$ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
Hermitian Transpose	**^%H**[1]	> $\begin{bmatrix} I & -2\,I \\ 3+4\,I & 2-I \end{bmatrix}^{\%H}$ $\begin{bmatrix} -\mathrm{I} & 3-4\,\mathrm{I} \\ 2\,\mathrm{I} & 2+\mathrm{I} \end{bmatrix}$
Cross Product (3-D vectors only)	**&x**[2]	> *with*(*LinearAlgebra*): > *d* &x *e* $\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$

[1]Exponential operators display in 2-D Math as superscripts.

[2]After loading the **LinearAlgebra** package, the cross product operator is available as the infix operator **&x** . Otherwise, it is available as the **LinearAlgebra[CrossProduct]** command.

For information on matrix arithmetic over finite rings and fields, refer to the **mod** help page.

Point-and-Click Interaction

Using the Context Panel, you can perform many matrix and vector operations.

Matrix operations available in the Context Panel include the following.

- Perform standard operations: determinant, inverse, norm (1, Euclidean, infinity, or Frobenius), transpose, and trace
- Compute eigenvalues, eigenvectors, and singular values
- Compute the dimension or rank
- Convert to the Jordan form, or other forms
- Perform Cholesky decomposition and other decompositions

For example, compute the infinity norm of a matrix. See **Figure 5.6**.

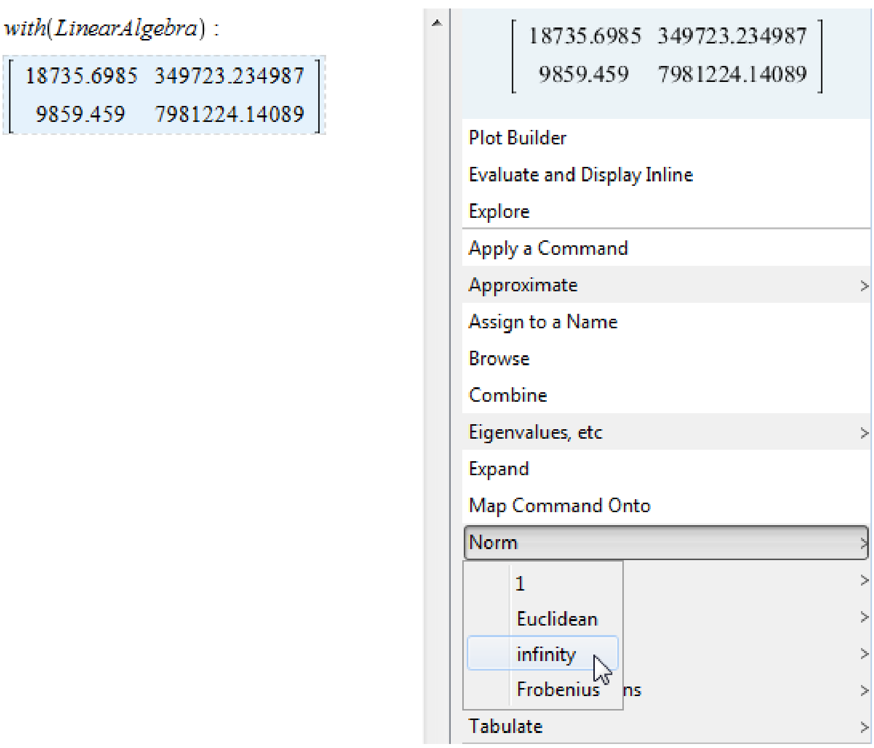


Figure 5.6: Computing the Infinity Norm of a Matrix

In Document mode, Maple inserts a right arrow and the name of the computation performed, followed by the norm.

$$\begin{bmatrix} 18735.6985 & 349723.234987 \\ 9859.459 & 798124.14089 \end{bmatrix} \xrightarrow{\text{infinity-norm}} 807983.5999$$

Vector operations available in the Context Panel include the following.

- Compute the dimension
- Compute the norm (1, Euclidean, and infinity)
- Compute the transpose
- Select an element

For more information on context-sensitive operations, see *Computing with the Context Panel (page 48)* (for Document mode) or *The Context Panel (page 63)* (for Worksheet mode).

LinearAlgebra Package Commands

The **LinearAlgebra** package contains commands that construct and manipulate matrices and vectors, compute standard operations, perform queries, and solve linear algebra problems.

Table 5.7 lists some **LinearAlgebra** package commands. For a complete list, refer to the **LinearAlgebra/Details** help page.

Table 5.7: Select LinearAlgebra Package Commands

Command	Description
Basis	Return a basis for a vector space
CrossProduct	Compute the cross product of two vectors
DeleteRow	Delete a row or rows of a matrix
Dimension	Determine the dimension of a matrix or a vector
Eigenvalues	Compute the eigenvalues of a matrix
Eigenvectors	Compute the eigenvectors of a matrix
FrobeniusForm	Reduce a matrix to Frobenius form
GaussianElimination	Perform Gaussian elimination on a matrix
HessenbergForm	Reduce a square matrix to Hessenberg form
HilbertMatrix	Construct a generalized Hilbert matrix
IsOrthogonal	Test if a matrix is orthogonal
LeastSquares	Compute the least-squares approximation to **A . x = b**
LinearSolve	Solve the linear system **A . x = b**
MatrixInverse	Compute the inverse of a square matrix or pseudo-inverse of a non-square matrix
QRDecomposition	Compute the QR factorization of a matrix
RandomMatrix	Construct a random matrix
SylvesterMatrix	Construct the Sylvester matrix of two polynomials

For information on arithmetic operations, see *Matrix Arithmetic (page 124)*.

For information on selecting entries, subvectors, and submatrices, see *Accessing Entries in Matrices and Vectors (page 122)*.

Example: Determine a basis for the space spanned by the set of vectors **{(2, 13, -15), (7, -2, 13), (5, -4, 9)}**. Express the vector **(25, -4, 9)** with respect to this basis.

```
> with(LinearAlgebra):
```

```
> v1 := <2, 13, −15>: v2 := <7, −2, 13>: v3 := <5, −4, 9>:
```

Find a basis for the vector space spanned by these vectors, and then construct a matrix from the basis vectors.

```
> basis := Matrix(Basis([v1, v2, v3]));
```

$$basis := \begin{bmatrix} 2 & 7 & 5 \\ 13 & -2 & -4 \\ -15 & 13 & 9 \end{bmatrix}$$

To express **(25, -4, 9)** in this basis, use the **LinearSolve** command**.**

```
> LinearSolve(basis, <25, -4, 9>)
```

$$\begin{bmatrix} \frac{170}{91} \\ -\frac{285}{91} \\ \frac{786}{91} \end{bmatrix}$$

Numeric Computations

You can very efficiently perform computations on large matrices and vectors that contain floating-point data using the built-in library of numeric linear algebra routines. Some of these routines are provided by the Numerical Algorithms Group (NAG®). Maple also contains portions of the CLAPACK and optimized ATLAS libraries.

For information on performing efficient numeric computations using the **LinearAlgebra** package, refer to the **EfficientLinearAlgebra** help page.

See also *Creating Matrices and Vectors with Specific Properties (page 120)* and *Reading from Files (page 329)*.

Student LinearAlgebra Package

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands.

In the **Student[LinearAlgebra]** subpackage, the environment differs from that of the **LinearAlgebra** package in that floating-point computations are generally performed using software precision, instead of hardware precision, and symbols are generally assumed to represent real, rather than complex, quantities. These defaults, and others, can be controlled using the **SetDefault** command. For more information, refer to the **Student[SetDefault]** help page.

For information on using Maple as a teaching and learning tool, see *Teaching and Learning with Maple (page 145)*.

5.4 Calculus

This section describes the key Maple calculus commands. Many of these commands are also available in the Context Panel for an expression.

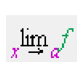
For a complete list of calculus commands, refer to the **Mathematics** (including **Calculus**, **Differential Equations**, **Power Series**, and **Vector Calculus** subfolders) and **Education >Student Package** sections of the Maple Help System Table of Contents.

The **Task Browser** (**Tools→Tasks→Browse**) contains numerous calculus task templates.

For more about resources for teaching or learning calculus, see *Teaching and Learning with Maple (page 145)*.

Limits

To compute the limit of an expression as the independent variable approaches a value:

1. In the **Calculus** palette, click the limit item .
2. Specify the independent variable, limit point, and expression, and then evaluate it. Press **Tab** to move to the next placeholder.

For example:

$$> \lim_{x \to 0} \left(\frac{x}{\sin(x)} \right)$$

$$1$$

The limit Command

By default, Maple searches for the real bidirectional limit (unless the limit point is ∞ or -∞). To specify a direction, include one of the options **left**, **right**, **real**, or **complex** in a call to the **limit** command. See **Table 5.8**.

Table 5.8: Limits

Limit	Command Syntax	Output
$\lim_{x \to 0} \left(\frac{1}{x} \right)$	$> limit\left(\frac{1}{x}, x = 0 \right)$	*undefined*
$\lim_{x \to 0^+} \left(\frac{1}{x} \right)$	$> limit\left(\frac{1}{x}, x = 0, 'right' \right)$	∞
$\lim_{x \to 0^-} \left(\frac{1}{x} \right)$	$> limit\left(\frac{1}{x}, x = 0, 'left' \right)$	$-\infty$

Using the **limit** command, you can also compute multidimensional limits.

$$> limit\left(\frac{x^2}{y}, \{x = 1, y = \infty\} \right)$$

$$0$$

For more information on multidimensional limits, refer to the **limit/multi** help page.

Numerically Computing a Limit

To numerically compute a limit:

- Use the **evalf(Limit(arguments))** calling sequence.

Important: Use the inert **Limit** command, not the **limit** command. For more information, refer to the **limit** help page.

The **Limit** command accepts the same arguments as the **limit** command.

For example:

$$> evalf\left(Limit\left(\frac{\sin(x)}{\cos(x) + \tan(x)}, x = 1.225 \right) \right)$$

$$0.3020605357$$

For information on the **evalf** command, see *Numerical Approximation (page 287)*.

The **Limit** command does not compute the limit. It returns an unevaluated limit.

$$> Limit\left(\frac{\sin(x)}{\cos(x) + \tan(x)}, x = 1.225 \right)$$

$$\lim_{x \to 1.225} \frac{\sin(x)}{\cos(x) + \tan(x)}$$

For more information on the **Limit** command, refer to the **Limit** help page.

Differentiation

Maple can perform symbolic and numeric differentiation.

To differentiate an expression:

1. In the **Calculus** palette, click the differentiation item $\frac{d}{dx} f$ or the partial differentiation item $\frac{\partial}{\partial x} f$.
2. Specify the expression and independent variable, and then evaluate it.

For example, to differentiate $x \sin(a\,x)$ with respect to x:

> *unassign*('a') :

> $\frac{d}{d\,x}\,(x \sin(a\,x))$

$$\sin(a\,x) + x\,a \cos(a\,x)$$

You can also differentiate using the Context Panel. For more information, see *The Context Panel (page 27)*.

To calculate a higher order or partial derivative, edit the derivative symbol inserted. For example, to calculate the second derivative of $x \sin(a\,x) + x^2$ with respect to x:

> $\frac{d^2}{d\,x^2}\left(x \sin(a\,x) + x^2\right)$

$$2\,a \cos(a\,x) - x\,a^2 \sin(a\,x) + 2$$

To calculate the mixed partial derivative of $x \sin(3\,y) + y\,x^5$:

> $\frac{\partial^2}{\partial y\,\partial x}\left(x \sin(3\,y) + y\,x^5\right)$

$$3 \cos(3\,y) + 5\,x^4$$

Note: To enter another ∂ symbol, you can copy and paste the existing symbol, or enter d and use symbol completion.

The diff Command

Maple computes derivatives using the **diff** command. To directly use the **diff** command, specify the expression to differentiate and the variable.

> $x \sin(a\,x) + x^2$

$$x \sin(a\,x) + x^2 \tag{5.3}$$

> *diff*(**(5.3)**, *x*)

$$\sin(a\,x) + x\,a \cos(a\,x) + 2\,x \tag{5.4}$$

For information on equation labels such as **(5.3)**, see *Equation Labels (page 68)*.

You can calculate a higher order derivative by specifying a sequence of differentiation variables. Maple recursively calls the **diff** command.

> $diff(\mathbf{(5.3)}, x, x)$

$$2\,a\cos(a\,x) - x\,a^2\sin(a\,x) + 2 \tag{5.5}$$

To calculate a partial derivative, use the same syntax. Maple assumes that the derivatives commute.

> $diff\left(x\sin(3\,y) + y\sqrt{x}, x, y\right)$

$$3\cos(3\,y) + \frac{1}{2\sqrt{x}}$$

To enter higher order derivatives, it is convenient to use the syntax **diff(f, x$n)**. This syntax can also be used to compute the symbolic n^{th} order derivative.

For example:

> $diff(\cos(t), t\$n)$

$$\cos\left(t + \frac{n\,\pi}{2}\right)$$

Differentiating an Operator

You can also specify a mathematical function as a *functional operator* (a mapping). For a comparison of operators and other expressions, see *Distinction between Functional Operators and Other Expressions (page 275)*.

To find the derivative of a functional operator:

- Use the **D** operator.

The **D** operator returns a functional operator.

For example, find the derivative of an operator that represents the mathematical function $F : x \to x\cos(x)$.

First, define the operator *F*.

1. In the **Expression** palette, click the single-variable function definition item $f := a \to y$.
2. Enter placeholder values.

- To move to the next placeholder, press the **Tab** key. **Note:** If pressing the **Tab** key inserts a tab, click the Tab icon  in the toolbar.

> $F := x \to x\cos(x):$

Now, define the operator, *G*, that maps x to the derivative of $x\cos(x)$.

> $G := \mathrm{D}(F)$

$$G := x \mapsto \cos(x) - x\sin(x)$$

F and *G* evaluated at $\frac{\pi}{2}$ return the expected values.

$> F\left(\frac{\pi}{2}\right); G\left(\frac{\pi}{2}\right)$

$$0$$

$$-\frac{\pi}{2}$$

For more information on the **D** operator, refer to the **D** help page. For a comparison of the **diff** command and **D** operator, refer to the **diffVersusD** help page.

Directional Derivative

To compute and plot a directional derivative, use the **Directional Derivative Tutor**. The tutor computes a floating-point value for the directional derivative.

To launch the tutor:

- From the **Tools** menu, select **Tutors**, **Calculus - Multivariate**, and then **Directional Derivatives**. Maple launches the **Directional Derivative Tutor**. See **Figure 5.7**.

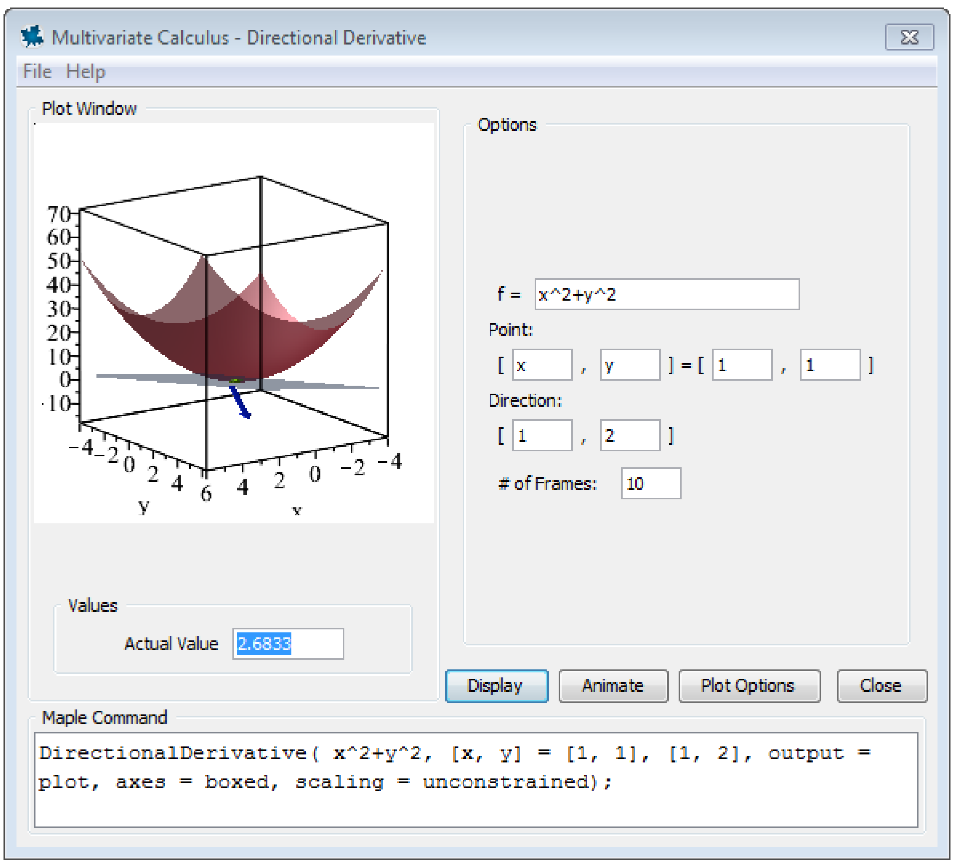


Figure 5.7: Directional Derivative Tutor

To compute a symbolic value for the directional derivative, use the **Student[MultivariateCalculus][DirectionalDerivative]** command. The first list of numbers specifies the point at which to compute the derivative. The second list of numbers specifies the direction in which to compute the derivative.

For example, at the point $[1, 2]$, the gradient of $x^2 + y^2$ points in the direction $[2, 4]$, which is the direction of greatest increase. The directional derivative in the orthogonal direction $[-2, 1]$ is zero.

> *with*(*Student*[*MultivariateCalculus*]):

> $DirectionalDerivative\left(x^2+y^2, [x, y] = [1, 2], [1, 2]\right);$

$$2\sqrt{5}$$

> $DirectionalDerivative\left(x^2+y^2, [x, y] = [1, 2], [-2, 1]\right);$

$$0$$

Series

To generate the **Taylor series** expansion of a function about a point, use the **taylor** command.

> $taylor(\sin(4\,x)\cos(x),\, x=0)$

$$4\,x - \frac{38}{3}\,x^3 + \frac{421}{30}\,x^5 + \mathrm{O}\left(x^7\right)$$

Note: If a Taylor series does not exist, use the **series** command to find a general series expansion.

For example, the **cosine integral function** does not have a taylor series expansion about 0. For more information, refer to the **Ci** help page.

> $taylor(Ci(x),\, x = 0)$

```
Error, does not have a taylor expansion, try series()
```

To generate a truncated series expansion of a function about a point, use the **series** command.

> $series(Ci(x),\, x = 0)$

$$\gamma + \ln(x) - \frac{1}{4}\,x^2 + \frac{1}{96}\,x^4 + \mathrm{O}\left(x^6\right)$$

By default, Maple performs series calculations up to order 6. To use a different order, specify a non-negative integer third argument.

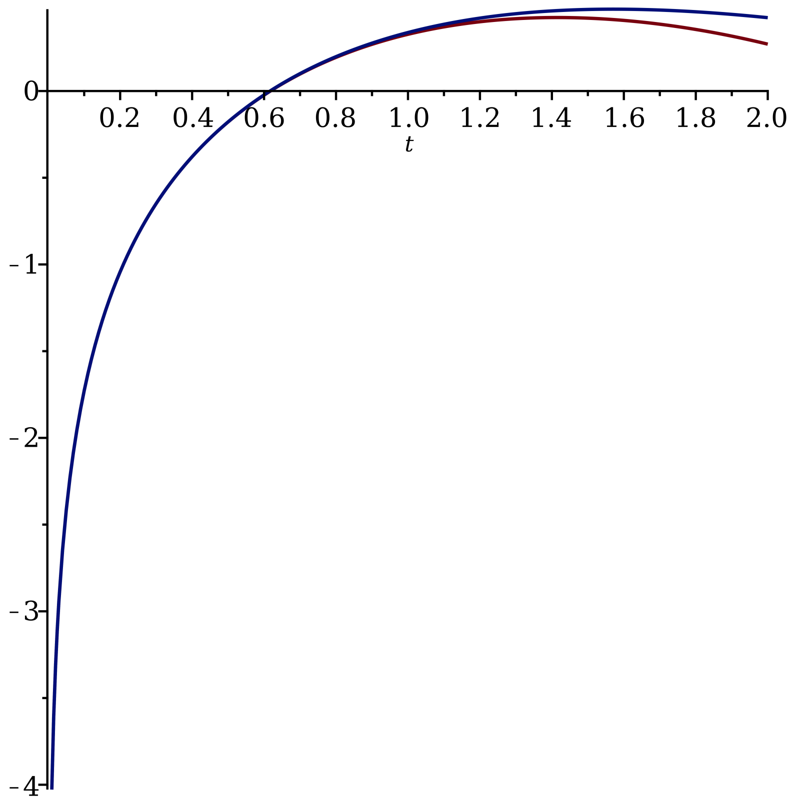
> $expansion := series(Ci(t)\ ,\, t = 0,\, 4)$

$$expansion := \gamma + \ln(t) - \frac{1}{4}\,t^2 + \mathrm{O}\left(t^4\right)$$

To set the order for all computations, use the **Order** environment variable. For information about the **Order** variable and the $\mathrm{O}\left(t^4\right)$ term, refer to the **Order** help page.

The expansion is of type **series**. Some commands do not accept arguments of type **series**. To use the expansion, you must convert it to a polynomial using the **convert/polynom** command.

> $plot\left(\{Ci(t), convert(expansion, polynom)\}, t = \frac{1}{100}..2\right)$



For information on Maple types and type conversions, see *Maple Expressions (page 269)*.

For information on plotting, see *Plots and Animations (page 183)*.

Integration

Maple can perform symbolic and numeric integration.

To compute the indefinite integral of an expression:

1. In the **Calculus** palette, click the indefinite integration item $\int f \,\mathrm{d}x$.
2. Specify the integrand and variable of integration, and then evaluate it.

For example, to integrate $x \sin(a\, x)$ with respect to *x*:

> $\int x \sin(a\, x) \,\mathrm{d}x$

$$\frac{\sin(a\, x) - x\, a \cos(a\, x)}{a^2}$$

Recall that you can also enter symbols, including $\int$ and d, using symbol completion.

- Enter the symbol name (or part of the name), for example, **int** or **d**, and then press the completion shortcut key.

For more information, see *Symbol Names (page 21)*.

You can also compute an indefinite integral using the Context Panel. For more information, see *The Context Panel (page 27)*.

To compute the definite integral of an expression:

1. In the **Calculus** palette, click the definite integration item $\int_a^b f\,dx$.
2. Specify the endpoints of the interval of integration, integrand expression, and variable of integration, and then evaluate it.

For example, to integrate $e^{-a\,t}\ln(t)$ over the interval (0, ∞):

$$> \int_0^{\infty} e^{-a\,t}\ln(t)\,dt$$

$$\lim_{t\to\infty}\left(-\frac{e^{-a\,t}\ln(t)+\mathrm{Ei}_1(a\,t)+\gamma+\ln(a)}{a}\right)$$

Maple treats the parameter **a** as a complex number. As described in *Assumptions on Variables (page 104)*, you can compute under the assumption that **a** is a positive, real number using the **assuming** command.

$$> \int_0^{\infty} e^{-a\,t}\ln(t)\,dt \text{ assuming } a>0$$

$$-\frac{\gamma+\ln(a)}{a}$$

To compute iterated integrals, line integrals, and surface integrals, use the task templates (**Tools** → **Tasks** → **Browse**) in the **Multivariate** and **Vector Calculus** folders.

The int Command

$\int f\,dx$ and $\int_a^b f\,dx$ use the **int** command. To use the **int** command directly, specify the following arguments.

- Expression to integrate
- Variable of integration

$$> x\sin(a\,x)$$

$$x\sin(a\,x) \tag{5.6}$$

$$> int(\mathbf{(5.6)}, x)$$

$$\frac{\sin(a\,x)-x\,a\cos(a\,x)}{a^2} \tag{5.7}$$

For a definite integration, set the variable of integration equal to the interval of integration.

> $int\left(\mathbf{(5.6)}, x = 0 .. \frac{\pi}{a}\right)$

$$\frac{\pi}{a^2} \tag{5.8}$$

Numeric Integration

To perform numeric integration:

- Use the **evalf(Int(arguments))** calling sequence.

Important: Use the inert **Int** command, not the **int** command. For more information, refer to the **int** help page.

In addition to the arguments accepted by the **int** command, you can include optional arguments such as **method**, which specifies the numeric integration method.

> $evalf\left(Int\left(\frac{1}{\Gamma(x)}, x = 0 ..2, 'method' = \_Dexp\right)\right)$

$$1.626378399$$

For information on the **evalf** command, see *Numerical Approximation (page 287)*.

For information on numeric integration, including iterated integration and controlling the algorithm, refer to the **evalf/Int** help page.

Calculus Packages

In addition to top-level calculus commands, Maple contains calculus packages.

VectorCalculus Package

The **VectorCalculus** package contains commands that perform multivariate and vector calculus operations on **VectorCalculus vectors** (vectors with an additional coordinate system attribute) and **vector fields** (vectors with additional coordinate system and **vectorfield** attributes), for example, **Curl**, **Flux**, and **Torsion**.

> $with(VectorCalculus):$

> $BasisFormat(false):$

> $SetCoordinates('cartesian[x, y, z]'):$

> $VectorField1 := VectorField(\langle -y, x, z\rangle)$

$$VectorField1 := \begin{bmatrix} -y \\ x \\ z \end{bmatrix}$$

Note: For information on changing the display format in the VectorCalculus package, see the **VectorCalculus[BasisFormat]** help page.

Find the curl of **VectorField1**.

> *Curl*(*VectorField1*);

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Find the flux of **VectorField1** through a sphere of radius **r** at the origin.

> *Flux*(*VectorField1*, *Sphere*(⟨0, 0, 0⟩, *r*))

$$\frac{4\, r^3\, \pi}{3}$$

Compute the torsion of a space curve. The curve must be a vector with parametric function components.

> *simplify*(*Torsion*(⟨*t*, t^2, t^3⟩, *t*)) *assuming t*::*real*

$$\frac{3}{9\, t^4 + 9\, t^2 + 1}$$

For information on the **assuming** command, see *The assuming Command (page 106)*.

For more information on the **VectorCalculus** package, including a complete list of commands, refer to the **VectorCalculus** help page.

To find other calculus packages, such as **VariationalCalculus**, refer to the **index/package** help page.

Student Calculus Packages

The **Student** package contains subpackages that help instructors teach concepts and allow students to visualize and explore ideas. These subpackages also contain computational commands. The **Student** calculus subpackages include **Calculus1**, **MultivariateCalculus**, and **VectorCalculus**. The **Student[VectorCalculus]** package provides a simple interface to a subset of the functionality available in the **VectorCalculus** package.

For information on using Maple as a teaching and learning tool, including the comprehensive interactive Study Guides, see *Teaching and Learning with Maple (page 145)*.

Differential Equations

Maple has a powerful set of solvers for ordinary differential equations (ODEs) and partial differential equations (PDEs), and systems of ODEs and PDEs.

For information on solving ODEs and PDEs, see *Other Specialized Solvers (page 87)*.

Student ODEs Package

The Student subpackage for ODEs has commands for computation, visualization, and step-by-step solutions, designed to support a first year course in ordinary differential equations.

5.5 Optimization

Using the **Optimization** package, you can numerically solve optimization problems. The package uses fast Numerical Algorithms Group (NAG) algorithms to *minimize* or *maximize* an objective function.

The **Optimization** package solves constrained and unconstrained problems.

- **Linear programs**
- **Quadratic programs**
- **Nonlinear programs**
- Linear and nonlinear **least-squares** problems

The **Optimization** package contains local solvers. In addition, for univariate finitely-bounded nonlinear programs with no other constraints, you can compute global solutions using the **NLPSolve** command. To find global solutions generally, purchase the **Global Optimization Toolbox**. For more information, visit **http://www.maplesoft.com/products/toolboxes**.

Point-and-Click Interface

The primary method for solving optimization problems is the **Optimization Assistant**.

To launch the Optimization Assistant:

- From the **Tools** menu, select **Tutors**, **Optimization**, and then **Optimization**.

Maple launches the **Optimization Assistant**. See **Figure 5.8**.

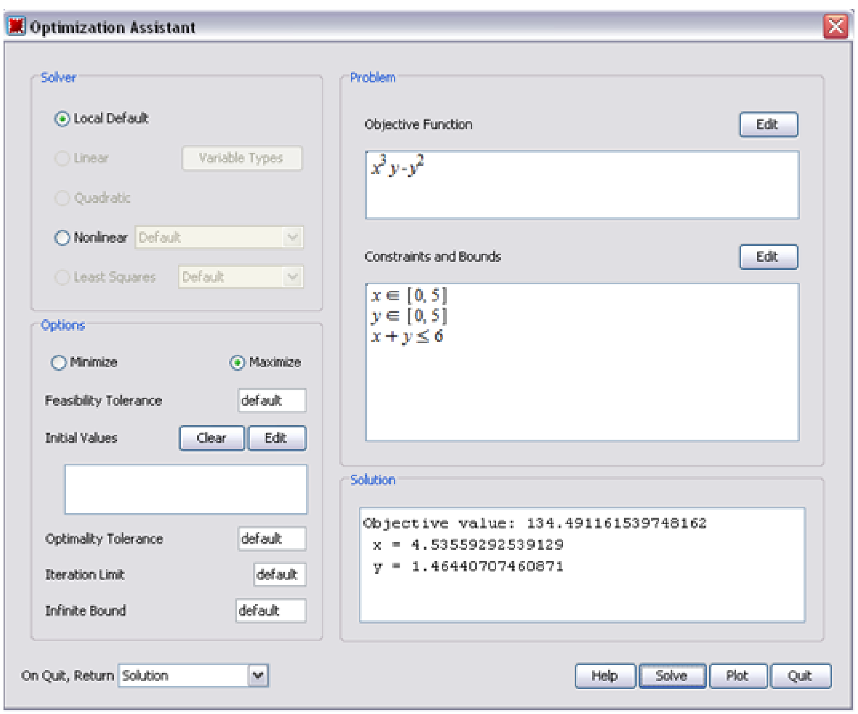


Figure 5.8: Optimization Assistant

To solve a problem:

1. Enter the objective function, constraints, and bounds.
2. Select the **Minimize** or **Maximize** radio button.

3. Click the **Solve** button. The solution is displayed in the **Solution** text box.

You can also enter the problem (objective function, constraints, and bounds) in the calling sequence of the **Optimization[Interactive]** command.

For example, find the maximum value of $x^3y - y^2$ subject to the constraints $x + y \leq 6$, $x \in [0,5]$, $y \in [0,5]$.

> $Optimization[Interactive]\left(x^3\, y - y^2, \{x + y \leq 6, x = 0\,..5, y = 0\,..5\}\right)$

$$[134.491161539748163, [x = 4.53559292539129, y = 1.46440707460871]]$$

- When the Optimization Assistant opens, select **Maximize**, then **Solve**.

After finding a solution, you can plot it. To plot a solution:

- In the **Optimization Assistant** window, click the **Plot** button. The **Optimization Plotter** window is displayed. See **Figure 5.9**.

Note: When you close the **Optimization Assistant**, you can choose to return the solution, problem, command used, plot, or nothing, using the drop-down in the bottom right corner of the assistant window.

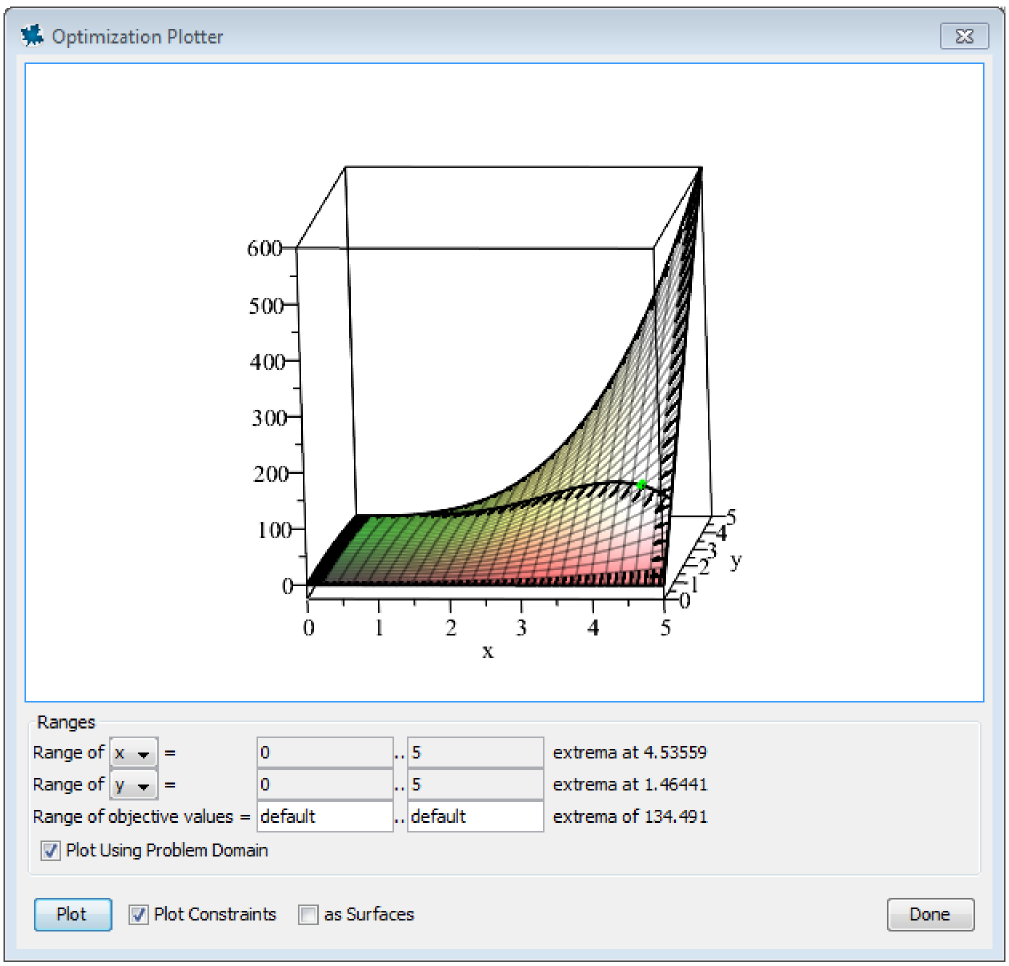


Figure 5.9: Optimization Assistant Plotter Window

For information on the algorithms used to solve optimization problems, refer to the **Optimization/Methods** help page.

Large Optimization Problems

The **Optimization Assistant** accepts input in an algebraic form. You can specify input in other forms, described in the **Optimization/InputForms** help page, in command calling sequences.

The Matrix form, described in the **Optimization/MatrixForm** help page, is more complex but offers greater flexibility and efficiency.

For example, solve the linear program:

Maximize $c^T x$ subject to $Ax \leqslant b$, where x is the vector of problem variables.

1. Define the column vector, **c**, of the linear objective function.

```
> restart;
> with(LinearAlgebra) :
> c := RandomVector[column](20, outputoptions = ['datatype'='float']) :
```

2. Define the matrix **A**, the coefficient matrix for the linear inequality constraints.

```
> A := RandomMatrix(19, 20, outputoptions = ['datatype' = 'float']):
```

3. Define the column vector **b**, the linear inequality constraints.

```
> b := RandomVector[column](19, outputoptions = [ 'datatype' = 'float']):
```

4. The **QPSolve** command solves quadratic programs.

```
> Optimization[LPSolve](c, [A, b], maximize, assume = nonnegative)
```

$$\left[43.2673034492019, \begin{bmatrix} 0.113733122260125 \\ 0. \\ 0. \\ 0.418532445477082 \\ 0.477295668318831 \\ 0. \\ 0.412320813472658 \\ 0. \\ 0. \\ 0. \\ \vdots \end{bmatrix}_{\text{20 element Vector[column]}} \right]$$

This example uses a random data set to demonstrate the problem. You could also read data from an external file as Matrices, and use that data. For details and an example, see *Reading from Files (page 329)*.

Note: For information on creating matrices and vectors (including how to use the **Matrix** palette to easily create matrices), see *Linear Algebra (page 116)*.

For additional information on performing efficient computations, refer to the **Optimization/Computation** help page.

MPS(X) File Support

To import linear programs from a standard MPS(X) data file, use the **ImportMPS** command.

Optimization Package Commands

Each **Optimization** package command solves the problem using a different optimization method. These are described in **Table 5.9**, along with the general input form for each command.

Table 5.9: Optimization Package Commands

Command	Description
LPSolve	Solve a linear program (LP), which involves computing the minimum (or maximum) of a linear objective function subject to linear constraints; input is in equation or Matrix form
LSSolve	Solve a least-squares (LS) problem, which involves computing the minimum of a real-valued objective function having the form $\frac{1}{2}\left(f_1(x)^2 + f_2(x)^2 + \ldots + f_q(x)^2\right)$, where x is a vector of problem variables, possibly subject to constraints; input is in equation or Matrix form
Maximize	Compute a local maximum of an objective function, possibly subject to constraints
Minimize	Compute a local minimum of an objective function, possibly subject to constraints
NLPSolve	Solve a non-linear program (NLP), which involves computing the minimum (or maximum) of a real-valued objective function, possibly subject to constraints; input is in equation or Matrix form
QPSolve	Solve a quadratic program (QP), which involves computing the minimum (or maximum) of a quadratic objective function, possibly subject to linear constraints; input is in equation or Matrix form

For a complete list of commands and other **Optimization** package information, refer to the **Optimization** help page.

5.6 Statistics

The **Statistics** package provides tools for mathematical statistics and data analysis. The package supports a wide range of common statistical tasks including quantitative and graphical data analysis, simulation, and curve fitting.

In addition to standard data analysis tools, the **Statistics** package provides a wide range of symbolic and numeric tools for computing with random variables. The package supports over 35 major probability distributions and can be extended to include new distributions.

Probability Distributions and Random Variables

The **Statistics** package supports:

- Continuous distributions, which are defined along the real line by **probability density functions**. Maple supports many continuous distributions, including the normal, Student-t, Laplace, and logistic distributions.
- Discrete distributions, which have nonzero probability only at discrete points. A discrete distribution is defined by a **probability function**. Maple supports many discrete distributions, including the Bernoulli, geometric, and Poisson distributions.

For a complete list of distributions, refer to the **Statistics/Distributions** help page.

You can define random variables by specifying a distribution in a call to the **RandomVariable** command.

> $with(Statistics)$:

> $X := RandomVariable(Poisson(\lambda))$:

Find the probability distribution function for **X**. (For information on statistics computations, see *Statistical Computations (page 142)*.)

> $PDF(X, t)$

$$\sum_{k=0}^{\infty} \frac{\lambda^k \mathrm{e}^{-\lambda} \mathrm{Dirac}(t-k)}{k!}$$

Adding Custom Distributions

To add a new distribution, specify a probability distribution in a call to the **Distribution** command.

> $U := Distribution\left(PDF = \left(t \rightarrow \begin{cases} 0 & t < 0 \\ \frac{1}{3} & t < 3 \\ 0 & otherwise \end{cases}\right)\right):$

To construct a piecewise-continuous function in 1-D Math, use the **piecewise** command, for example, `t -> piecewise(t < 0, 0, t < 3, 1/3, 0)`.

Define a new random variable with this distribution.

> $Z := RandomVariable(U): PDF(Z, t)$

$$\begin{cases} 0 & t < 0 \\ \frac{1}{3} & t < 3 \\ 0 & otherwise \end{cases}$$

Calculate the mean value of the random variable.

> $Mean(Z)$

$$\frac{3}{2}$$

Statistical Computations

In addition to basic functions, like mean, median, standard deviation, and percentile, the **Statistics** package contains commands that compute, for example, the interquartile range and hazard rate.

Example 1 - Interquartile Range

Compute the average absolute range from the **interquartile** of the **Rayleigh** distribution with scale parameter 3.

> $InterquartileRange(Rayleigh(3))$

$$\sqrt{36}\,\sqrt{\ln(2)} - \sqrt{-18 \ln\left(\frac{3}{4}\right)}$$

To compute the result numerically:

- Specify the **'numeric'** option.

> *InterquartileRange*(*Rayleigh*(3), '*numeric*')

$$2.71974481762339$$

Example 2 - Hazard Rate

Compute the **hazard rate** of the Cauchy distribution with location and scale parameters **a** and **b** at an arbitrary point **t**.

> *unassign*('*b*') :

> *HazardRate*(*Cauchy*(*a*, *b*), *t*)

$$\frac{1}{\pi b\left(1+\frac{(t-a)^2}{b^2}\right)\left(\frac{1}{2}-\frac{\arctan\left(\frac{t-a}{b}\right)}{\pi}\right)}$$

You can specify a value for the point **t**.

> $HazardRate\left(Cauchy(a, b), \frac{1}{2}\right)$

$$\frac{1}{\pi b\left(1+\frac{\left(\frac{1}{2}-a\right)^2}{b^2}\right)\left(\frac{1}{2}-\frac{\arctan\left(\frac{\frac{1}{2}-a}{b}\right)}{\pi}\right)}$$

You can also specify that Maple compute the result numerically.

> $HazardRate\left(Cauchy(10, 1), \frac{1}{2}, 'numeric'\right)$

$$0.003608801461$$

For more information, refer to the **Statistics/DescriptiveStatistics** help page.

Plotting

You can generate statistical plots using the visualization commands in the **Statistics** package. Available plots include:

- Bar chart
- Frequency plot
- Histogram
- Pie chart
- Scatter plot

For example, create a scatter plot for a distribution of points that vary from $\sin\left(\frac{2\pi x}{200}\right)$ by a small value determined by a normally distributed sample.

> *restart*;

> *with*(*Statistics*) :

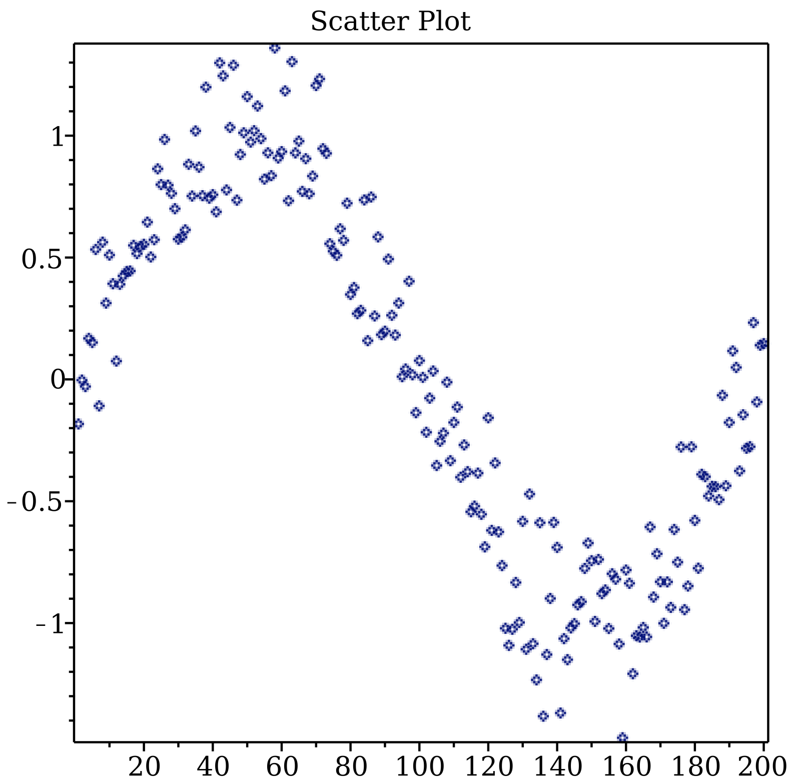
> $N := 200$:

> $U := \mathit{Sample}(\mathit{Normal}(0, 1), N)$:

> $X := \langle \mathit{seq}(x, x = 1 .. N) \rangle$:

> $Y := < \mathit{seq}\left(\sin\left(\frac{2\,\pi\,x}{N}\right) + \frac{U[x]}{5}, x = 1 ..N\right) >$:

> $\mathit{ScatterPlot}(X, Y, '\mathit{title}' = \text{"Scatter Plot"})$;



To fit a curve to the data points, include the optional **fit** equation parameter.

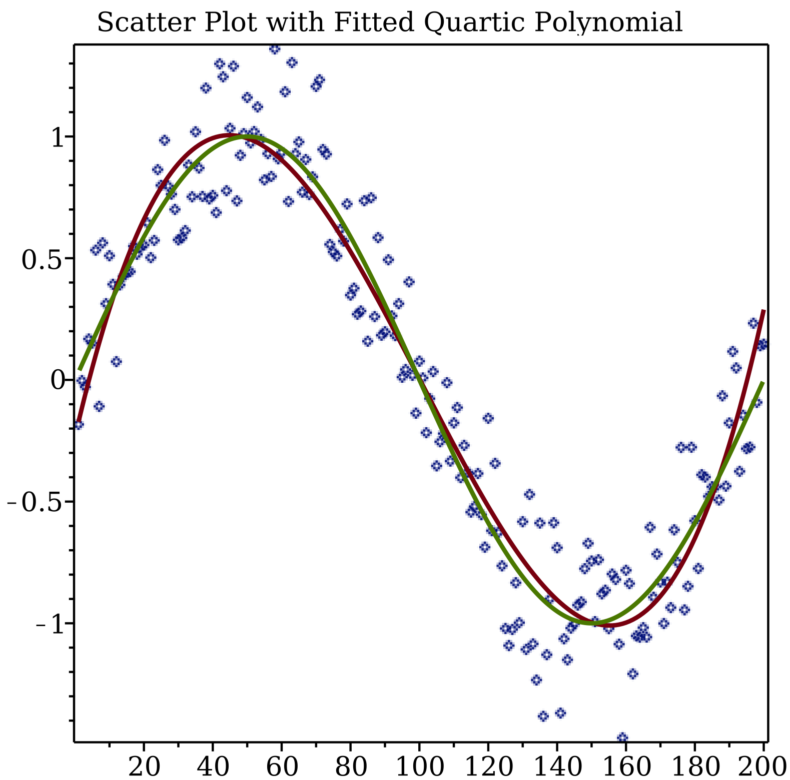
Using the **plots[display]** command, create a plot that contains:

- a scatter plot of the data points
- a quartic polynomial fitted to the data points: $f(x) = a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e$
- the function $\sin\left(\frac{2\,\pi\,x}{N}\right)$

> $P := \mathit{ScatterPlot}\left(X, Y, \mathit{fit} = \left[a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e, x\right], \mathit{thickness} = 2\right)$:

> $Q := \mathit{plot}\left(\mathit{sin}\left(\frac{2\,\pi\,x}{N}\right), x = 1 ..N, \mathit{thickness} = 2, \mathit{color} = \text{"Niagara 3"}\right)$:

> *plots*[*display*](*P*, *Q*, '*title*' = "Scatter Plot with Fitted Quartic Polynomial")



For more information on statistical plots, refer to the **Statistics/Visualization** help page.

For an overview of plotting, see *Plots and Animations (page 183)*.

Student Statistics Package

The **Student[Statistics]** package helps instructors teach concepts and allows students to understand and explore concepts in an introductory statistics course. The **Student[Statistics]** package provides a simple interface to a limited subset of the functionality available in the **Statistics** package, as well as some additional resources. You can create and explore random distributions, perform hypothesis testing, and more.

The **Student Statistics Example** worksheet provides some starting examples.

Additional Information

For more information on the **Statistics** package, including regression analysis, estimation, data manipulation, and data smoothing, refer to the **Statistics** help page.

5.7 Teaching and Learning with Maple

Table 5.10 lists the available resources for instructors and students. For additional resources, see *Available Resources (page 40)*.

Table 5.10: Student and Instructor Resources

<table>
<tr><th>Resource</th><th>Description</th></tr>
<tr><td>Student Packages and Tutors</td><td>The Student package contains computational and visualization (plotting and animation) functionality, and point-and-click interfaces for explaining and exploring concepts (Tools→Tutors). For more information, refer to the Student help page.</td></tr>
<tr><td>Study Guides</td><td>Study guides - Complete lessons with interactive examples for academic courses. Maple includes these study guides:
• Precalculus Study Guide contains worked problems, each solved as in a standard textbook, using Maple commands and custom Maplet graphical interfaces.
• Calculus Study Guide contains worked problems, each solved as in a standard textbook, using Maple commands and custom Maplet graphical interfaces.
• Multivariate Calculus Study Guide contains worked problems, each solved as in a standard textbook, using Maple commands and custom Maplet graphical interfaces.</td></tr>
<tr><td>Teacher Resource Center</td><td>The Maple Teacher Resource Center contains resources and tips for teachers using Maplesoft products to help in the classroom. Available resources include:
• Classroom content for subjects including Precalculus, Calculus, and Engineering
• Training videos
• E-books
(<u>http://www.maplesoft.com/TeacherResource</u>)</td></tr>
<tr><td>Maple Portal</td><td>The Maple Portal includes material designed for all Maple users as well as a specific portal for students. The Maple Portal includes:
• How Do I... topics that give quick answers to essential questions
• Tutorials that provide an overview of topics from getting started to plotting and working with matrices
• Navigation to portals with specialized information for students and engineers
Access the portal the table of contents in the Maple Help System, under Getting Started.</td></tr>
<tr><td>Mathematics and Engineering Dictionary</td><td>The Maple Help System has an integrated dictionary of over 5000 mathematics and engineering terms. You can search the dictionary by entering a term in the Help System search field.</td></tr>
<tr><td>Maple Application Center</td><td>The Maple Application Center contains tutorials and applications that help instructors begin using Maple and use Maple in the classroom. Browse the many resources by category or search for a topic.
(<u>http://www.maplesoft.com/applications</u>)</td></tr>
<tr><td>Student Help Center</td><td>The Maple Student Help Center contains tutorials and applications that help students learn how to use Maple, explore mathematical concepts, and solve problems. Available resources include:
• Free course lessons for many subjects including precalculus to vector calculus; high school, abstract, and linear algebra; engineering; physics; differential equations; cryptography; and classical mechanics.
• Applications for students, written by students, providing examples in many subject areas.
• Student FAQs with answers from experts.
(<u>https://www.maplesoft.com/studentcenter/index.aspx</u>)</td></tr>
</table>

Student Packages and Tutors

The **Student** package is a collection of subpackages for teaching and learning mathematics and related subjects. The **Student** package contains packages for a variety of subjects, including precalculus, calculus, and linear algebra.

Instructors can:

- Teach concepts without being distracted by the mechanics of the computations.
- Create examples and quickly update them during a lesson to demonstrate different cases or show the effect of the variation of a parameter.
- Create plots and animations to visually explain concepts, for example, the geometric relationship between a mathematical function and its derivatives (**Tools→Tutors→Calculus - Single Variable→Derivatives**). See **Figure 5.10**.

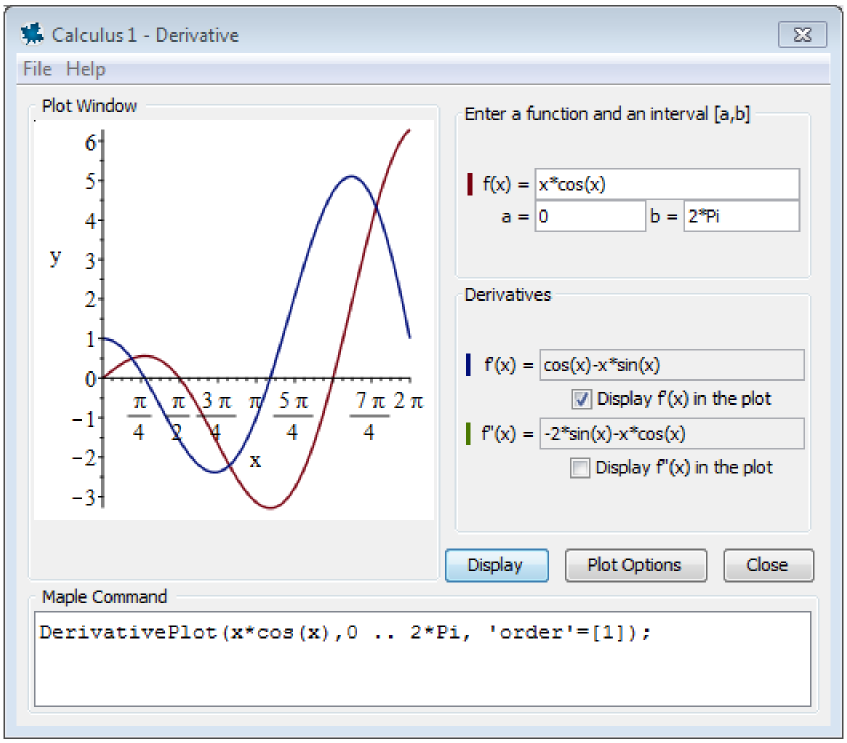


Figure 5.10: Calculus 1 Derivatives Tutor

Students can:

- Perform step-by-step computations, for example, compute a derivative by applying differentiation rules using commands or a tutor (**Tools→Tutors→Calculus - Single Variable→Differentiation Methods**). See **Figure 5.11**.
- Perform computations.
- Visually explore concepts.

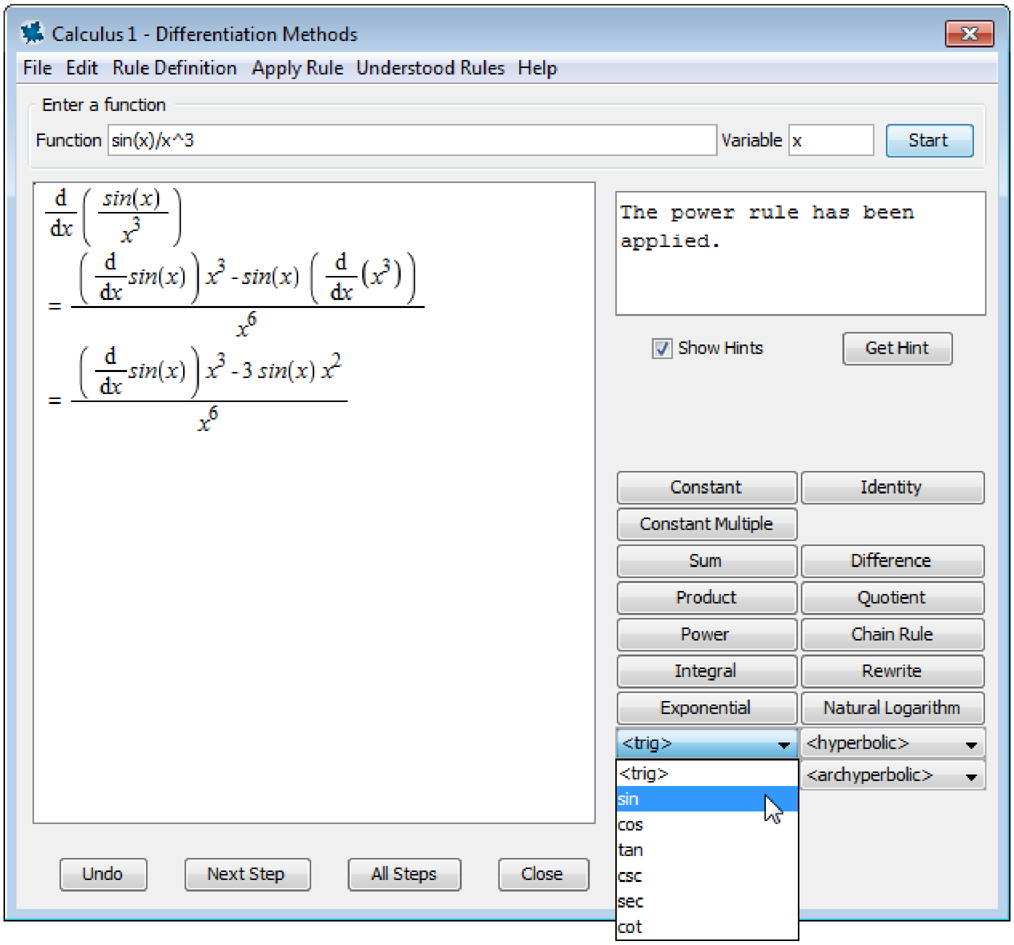


Figure 5.11: Calculus 1 Differentiation Methods Tutor

Tutors provide point-and-click interfaces to the **Student** package functionality.

To launch a tutor:

1. From the **Tools** menu, select **Tutors**.
2. Select a subject, for example, **Calculus - Multivariate**.
3. Select a tutor, for example, **Gradients**.

Maple inserts the *Student*[*MultivariateCalculus*][*GradientTutor*]() calling sequence (in Worksheet mode), and launches the **Multivariate Calculus Gradient Tutor**.

By rotating the three-dimensional plot, you can show that the gradient points in the direction of greatest increase of the surface (see **Figure 5.12**) and show the direction of the gradient vector in the x-y plane by rotating the plot (see **Figure 5.13**).

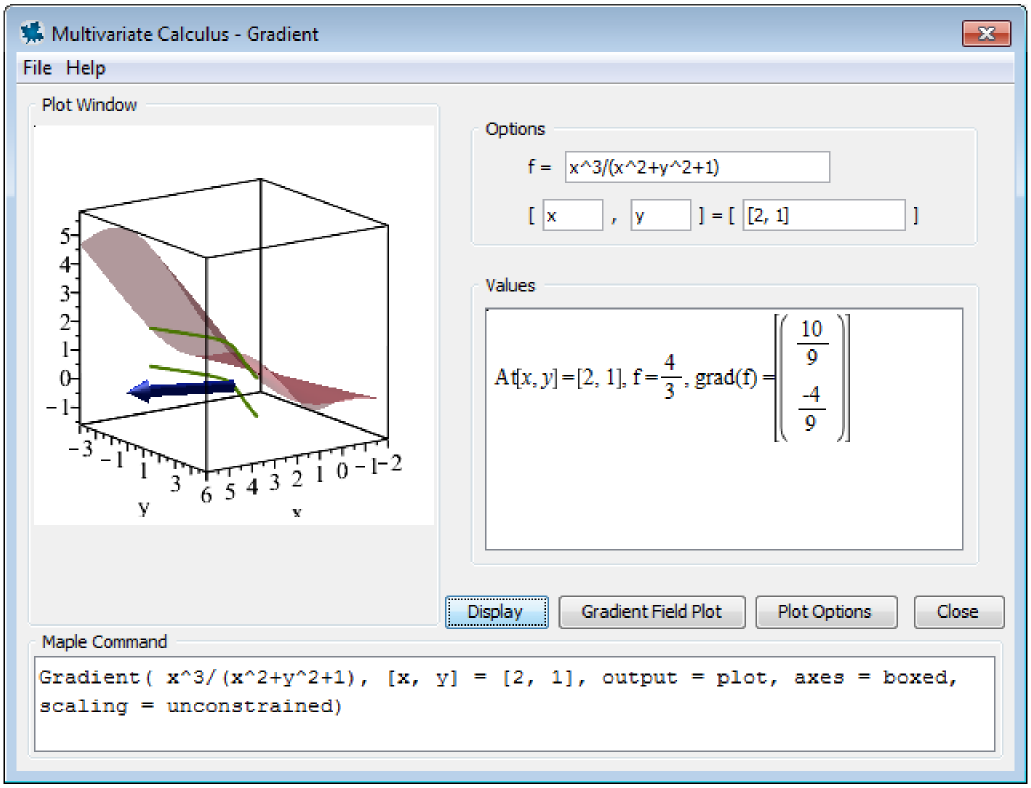


Figure 5.12: Multivariate Calculus Gradient Tutor

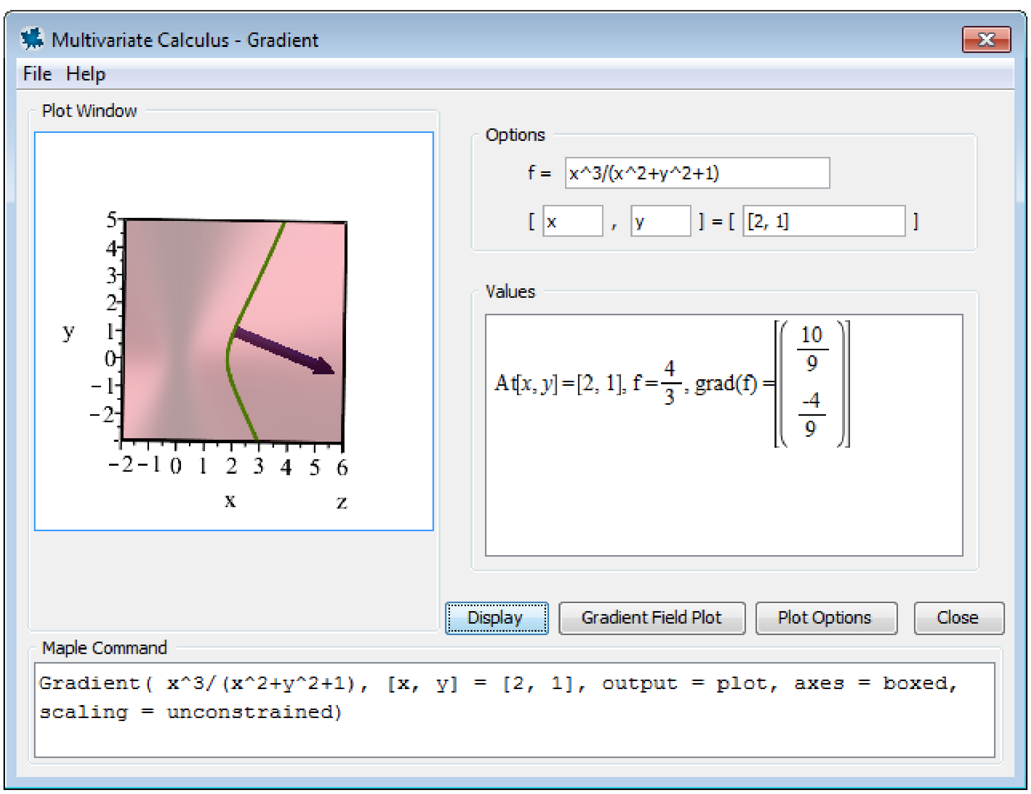


Figure 5.13: Multivariate Calculus Gradient Tutor Showing x-y Plane

When you close the tutor, Maple inserts the 3-D plot.

> *Student*[*MultivariateCalculus*][*GradientTutor*]( );

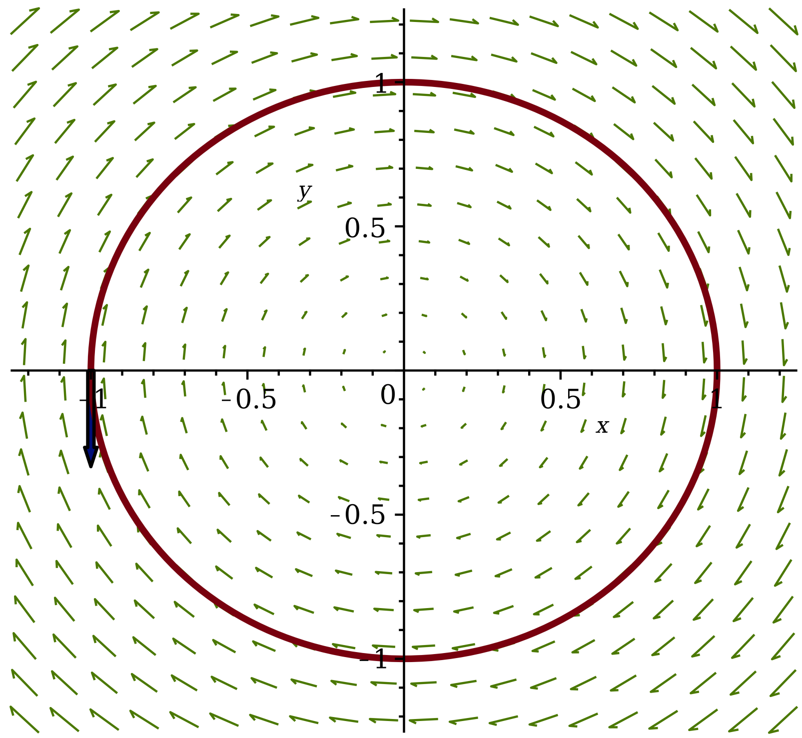
Many **Student** package commands can return a value, mathematical expression, plot, or animation. This allows you to compute the final answer, see the general formula applied to a specific problem, or visualize the underlying concepts.

For example, the **Student[VectorCalculus][LineInt]** (line integral) command can return the following.

- Plot that visually indicates the vector field, path of integration, and tangent vectors to the path
- Unevaluated line integral
- Numeric value of the line integral

> *with*(*Student*[*VectorCalculus*]):

> *LineInt*(*VectorField*(< *y*, −*x*>), *Circle*(< 0, 0>, 1), '*output*' = '*plot*')



The path of integration, vector(s) tangent to the path, and

> *LineInt*(*VectorField*(< *y*, −*x*>), *Circle*(< 0, 0> , 1), '*output*' = '*integral*')

$$\int_0^{2\pi} \left(-\sin(t)^2 - \cos(t)^2\right) \mathrm{d}t \tag{5.9}$$

To evaluate the integral returned by the **output** = **integral** calling sequence, use the **value** command.

> *value*(**(5.9)**)

$$-2\pi \tag{5.10}$$

By default, the **LineInt** command returns the value of the integral.

> *LineInt*(*VectorField*(< *y* − *x*, −*x* −*y*>), *Circle*(< 0, 0>, *r*))

$$-2\pi r^2$$

For more information on the **Student** package, refer to the **Student** help page.

Calculus Problem Solving Examples

Maple is a powerful application with many resources to guide you. The following examples provide you with scenarios to learn about using Maple resources and the Maple program.

When using Maple to solve a problem, consider the following process.

1. Formulate your problem.
2. Obtain Maple resources that allow you to solve it.

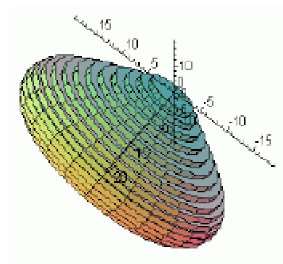
Problem

Scenario A:

Your company is designing a bottle for its new spring water product. The bottle must contain 18 ounces of water and the height is fixed. The design includes an undulating curved surface. You know the amplitude and equation of the curve, but you must find the radius. You require the **Volume of Revolution**.

Scenario B:

You want to teach your students the concept of a **Volume of Revolution**. Specifically, you want to plot and compute the volume of a solid generated by rotating $f(x)$, $a \leq x \leq b$, about an axis or a line parallel to an axis.



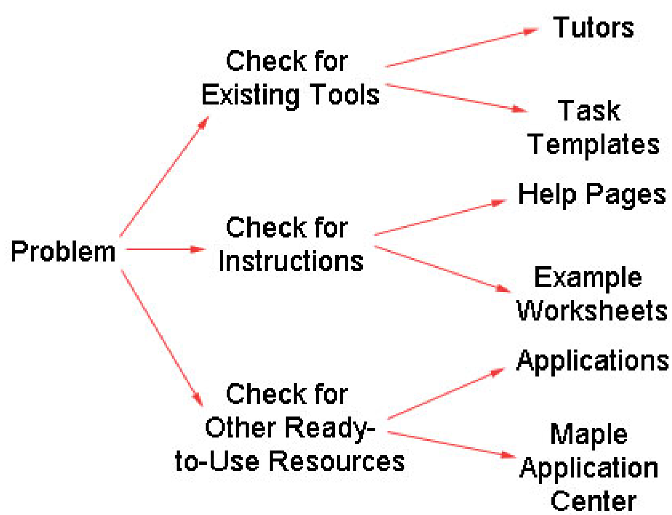


Figure 5.14: Flowchart of solving a problem

Check for Existing Tools: Tutor

Begin by examining the **Tools** menu for a **Tutor** to a Volume of Revolution problem.

To access a Tutor for the Volume of Revolution:

1. From the **Tools** menu, select **Tutors**, and then **Calculus-Single Variable**. Notice that a Volume of Revolution tutor exists.
2. Click the **Volume of Revolution** menu item. The following Maple command is entered in your document.

```
> Student[Calculus1][VolumeOfRevolutionTutor]( );
```

The **Volume of Revolution Tutor** is displayed. See **Figure 5.15**. Use this tutor to enter a function and an interval, view and manipulate the corresponding plot, and view the full Maple command associated with your entries and selections.

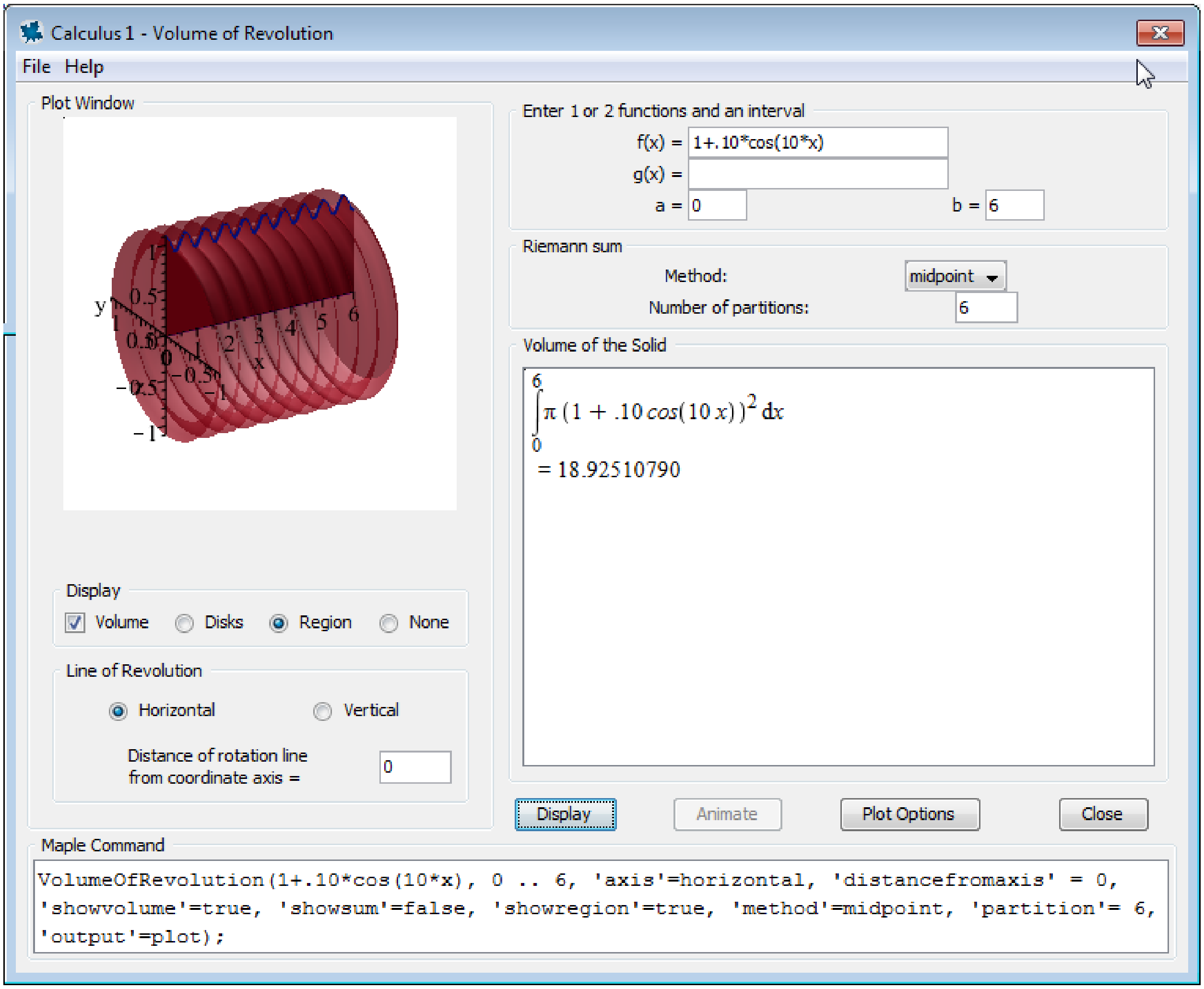


Figure 5.15: Volume of Revolution Tutor

After you **Close** the tutor, the plot is inserted into your worksheet.

Check for Existing Tools: Task Template

1. From the **Tools** menu, select **Tasks**, and then **Browse**. The **Browse Tasks** dialog opens, displaying a list of tasks in the left pane. The tasks are sorted by subject to help you quickly find the desired task.
2. Expand the **Calculus - Integral**→ **Applications** → **Solids of Revolution** folder.
3. From the displayed list, select **Volume**. The **Volume of Revolution** task is displayed in the right pane of the **Browse Tasks** dialog.
4. Select the **Insert into New Worksheet** check box.

Click Insert Default Content. Before inserting a task, Maple checks whether the task variables have assigned values in your worksheet. If any task variable is assigned, the **Task Variables** dialog opens allowing you to modify the names. Maple uses the edited variable names for all variable instances in the inserted task. The content is inserted into your document. See **Figure 5.16**.

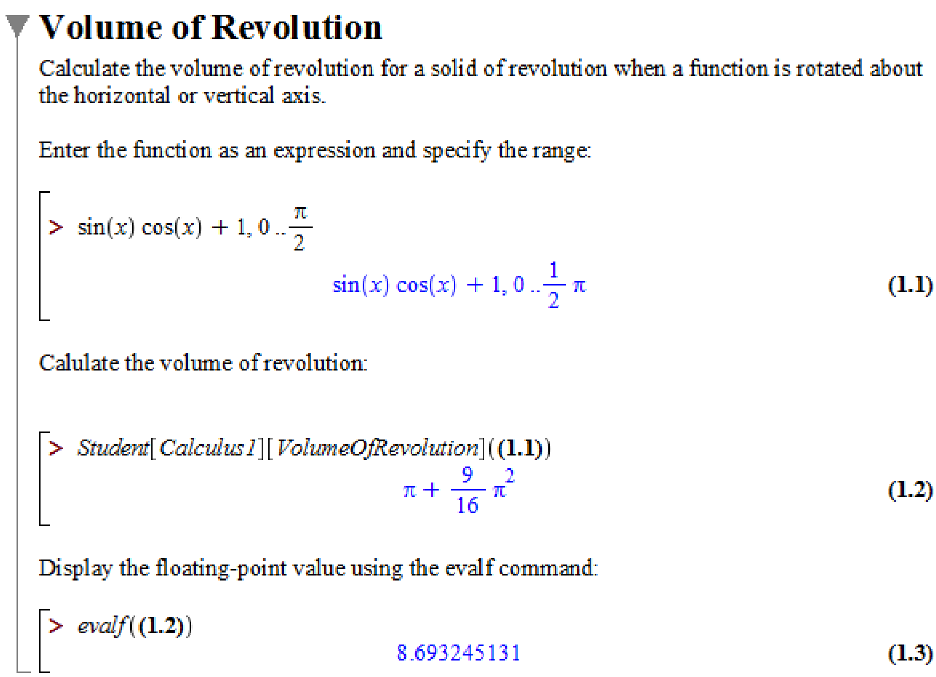


Figure 5.16: Inserted Task Template

6. When a Task Template is inserted, parameters are marked as placeholders, denoted by purple font. To navigate between placeholders, press the **Tab** key. After updating any parameters, execute the command by pressing **Enter**.

Check for Instructions: Help Page and Example Worksheet

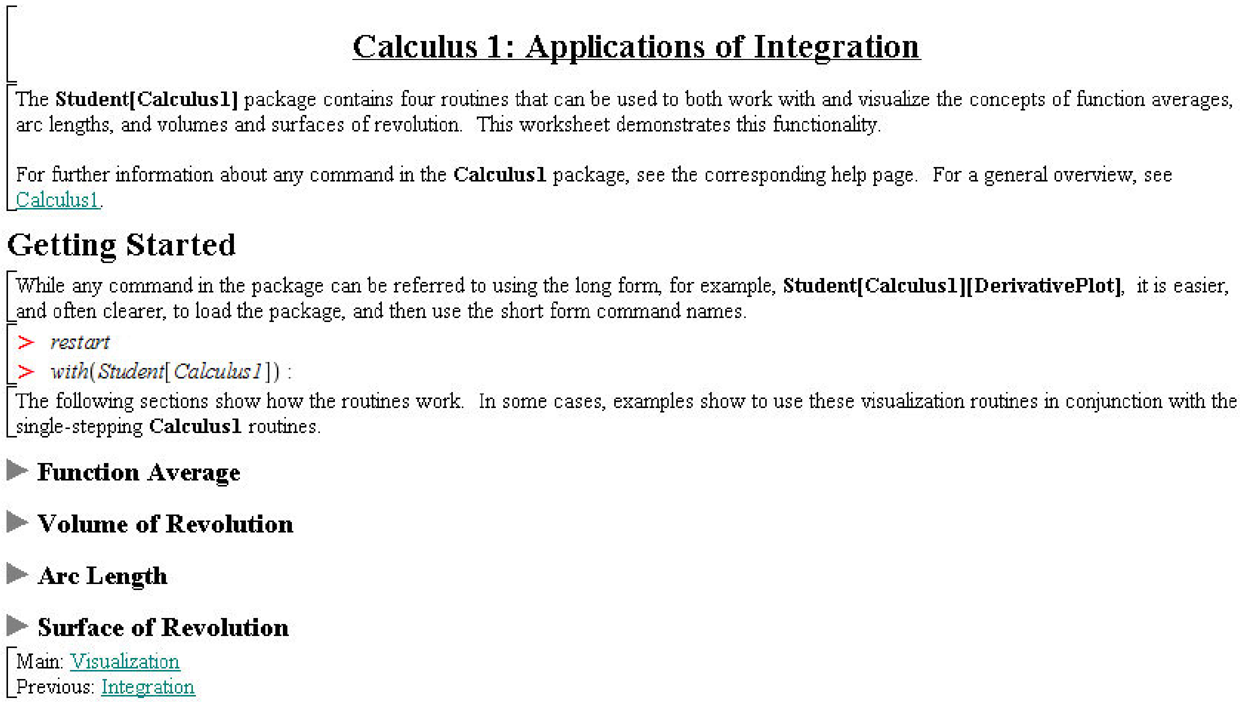
The help system provides command syntax information.

To access a help page:

1. From the **Help** menu, select **Maple Help**.
2. In the search field, enter **volume of revolution** and click **Search**. The search results include the command help page, the dictionary definition, and the associated tutor help page.
3. Review the calling sequence, parameters, and description in the **Student[Calculus1][VolumeOfRevolution]** help page.
4. Copy the examples into your worksheet: from the help system **Edit** menu, select **Copy Examples**.
5. Close the Help Navigator.
6. In your document, from the **Edit** menu, select **Paste**. The examples are pasted into your document.
7. Execute the examples and examine the results.

To access an example worksheet:

1. In the worksheet, enter **examples/index**. The **Examples and Applications Index** opens.
2. Expand the **Calculus** topic.
3. Click the **examples/Calculus1IntApps** link. The **Calculus1: Applications of Integration** worksheet opens. See **Figure 5.17**.
4. Expand the **Volume of Revolution** topic.
5. Examine and execute the examples.



Calculus 1: Applications of Integration

The **Student[Calculus1]** package contains four routines that can be used to both work with and visualize the concepts of function averages, arc lengths, and volumes and surfaces of revolution. This worksheet demonstrates this functionality.

For further information about any command in the **Calculus1** package, see the corresponding help page. For a general overview, see Calculus1.

Getting Started

While any command in the package can be referred to using the long form, for example, **Student[Calculus1][DerivativePlot]**, it is easier, and often clearer, to load the package, and then use the short form command names.

```
> restart
> with(Student[Calculus1]) :
```

The following sections show how the routines work. In some cases, examples show to use these visualization routines in conjunction with the single-stepping **Calculus1** routines.

▶ **Function Average**

▶ **Volume of Revolution**

▶ **Arc Length**

▶ **Surface of Revolution**

Main: Visualization
Previous: Integration

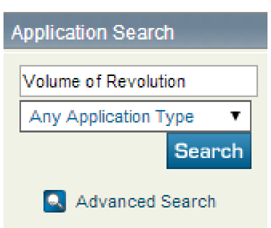
Figure 5.17: Example Worksheet

Check for Other Ready-To-Use Resources: Application Center

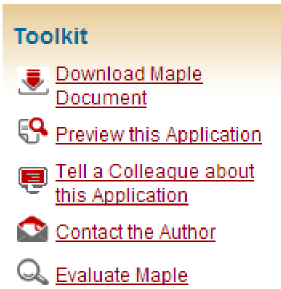
The Maple Application Center contains free user-contributed applications related to mathematics, education, science, engineering, computer science, statistics and data analysis, finance, communications, graphics, and more.

To access a free application for volume of revolution:

1. Go to the Maplesoft website, **http://www.maplesoft.com**.
2. In the menu of the main webpage, click **Support & Resources**, and then **Maple Application Center**.
3. In the **Application Search** section, enter **Volume of Revolution** in the **Keyword or phrase** field.



4. Click **Search**.
5. From the search results page, under **Displaying applications**, click the **Click here** link.
6. From the list of archived applications, select any of the Maple document you want to view.
7. Click on the **Download Maple Document** link to download the **.mw** file.



8. Execute the worksheet and examine the results.

5.8 Clickable Math™

For years, Maple has led the way in making math software easy to use. With its collection of Clickable Math™ tools, including palettes, interactive assistants, context-sensitive operations, tutors, and more, Maple has set the standard for making it easy to learn, teach, and do mathematics.

The Study Guides are a core feature of our Clickable Math resources. These are described in *Teaching and Learning with Maple (page 145)*.

In addition, two key features of the Clickable Math tool collection are Smart Popups and Drag-to-Solve™.

Smart Popups

Smart Popups are interactive popup options that are invoked when you select certain types of equations, expressions or subexpressions.

With Smart Popups you can:

- Select operations to apply to just one part of your equation or mathematical expression, leaving the rest unchanged.
- Preview the result of the operation before going ahead.
- Explore your expression to deepen your understanding of the problem.
- Easily determine if your subexpression can be factored, what its plot looks like, what mathematical identities could be applied, and more.

Smart popups, if any, are shown at the top of the Context Panel. For more information on Smart Popups, as well as examples, see the Clickable Math: Smart Popups help page.

Drag-to-Solve

The Drag-to-Solve feature enables you to solve your equations step-by-step by dragging terms to where you want them to be.

With Drag-to-Solve you can:

- Easily take complete control over each individual step of your calculation.
- Let Maple apply the appropriate addition, subtraction, division, or multiplication operation to both sides of your equation, to avoid mechanical errors.
- Keep the full record of steps produced by Maple to document your work.

For more information on Drag-to-Solve, as well as examples, see the Clickable Math: Drag-to-Solve help page.

Examples

This chapter is designed to show several ways to solve the same problem in Maple. Throughout these examples, you will need to insert new document block regions. This is done using the Create document block icon ( ) on the toolbar or through the **Edit** menu, by selecting **Document Blocks>Create Document Block**. Also, these examples only give the keyboard keys needed for a Windows operating system. Refer to *Shortcut Keys by Platform (page xiii)* for the keys needed for your operating system.

Example 1 - Graph a Function and its Derivatives

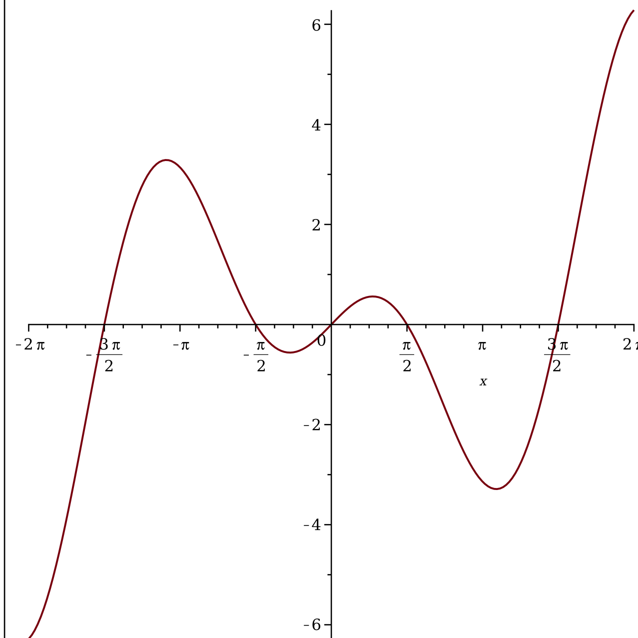
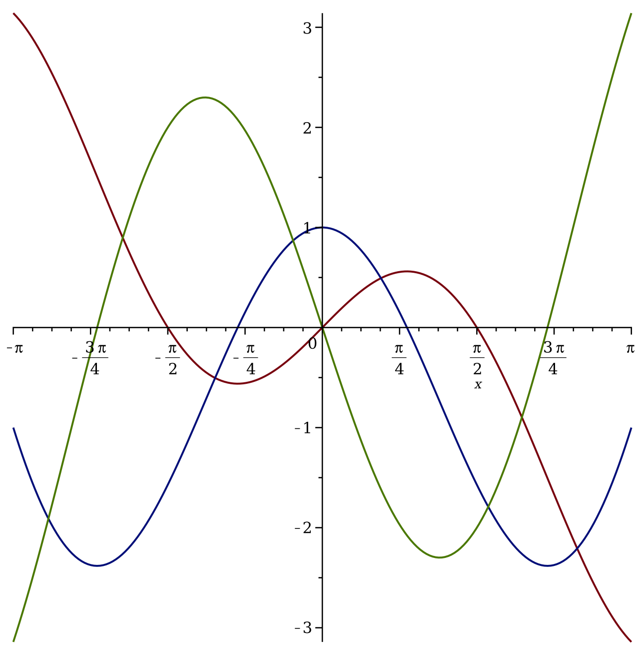
On the interval $[-\pi, \pi]$, graph f, f', and f'' for $f(x) = x\cos(x)$.

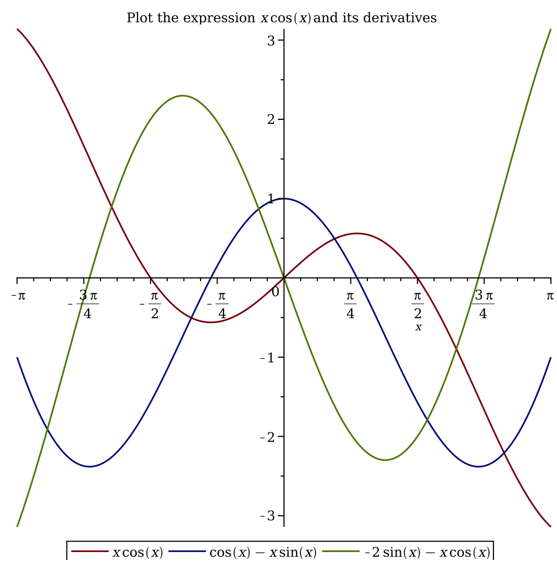
We solve this problem using the following methods:

- *Solution by Context-Sensitive Operations (page 158)*
- *Solution by Tutor (page 160)*
- *Access the Tutor from a Task Template (page 162)*

Solution by Context-Sensitive Operations

Action	Result in Document
1. Enter the expression $x\cos(x)$.	$x\cos(x)$
Make a copy of the expression and calculate the derivative: 2. Insert a new document block region by selecting from the **Format** menu **Create Document Block**. 3. Highlight the original expression. **Ctrl** + drag the expression to the new document block. 4. Select the expression and, from the Context Panel select **Differentiate → With Respect To → x**.	$x\cos(x) \xrightarrow{\text{differentiate w.r.t. x}} \cos(x) - x\sin(x)$
Make a copy of the derivative and calculate the second derivative: 5. Insert a new document block, and **Ctrl** + drag the derivative to the document block. 6. From the Context Panel for the derivative, select **Differentiate → With Respect To → x.**	$\cos(x) - x\sin(x) \xrightarrow{\text{differentiate w.r.t. x}}$ $-2\sin(x) - x\cos(x)$

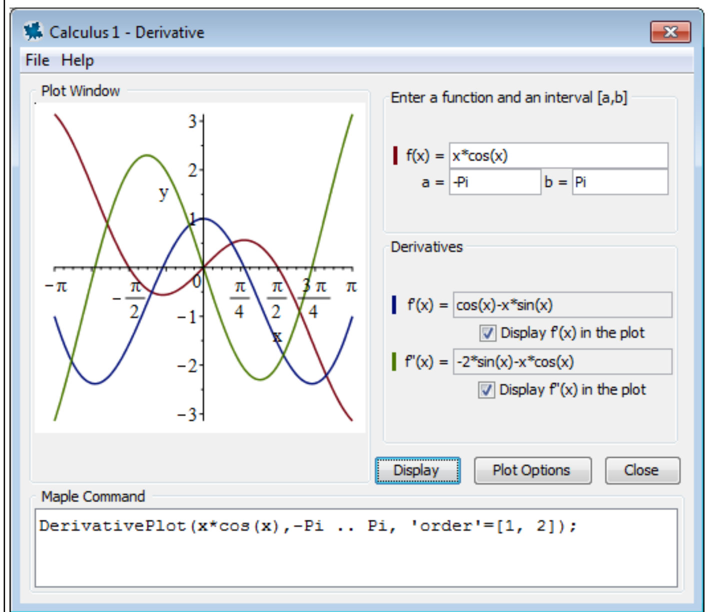
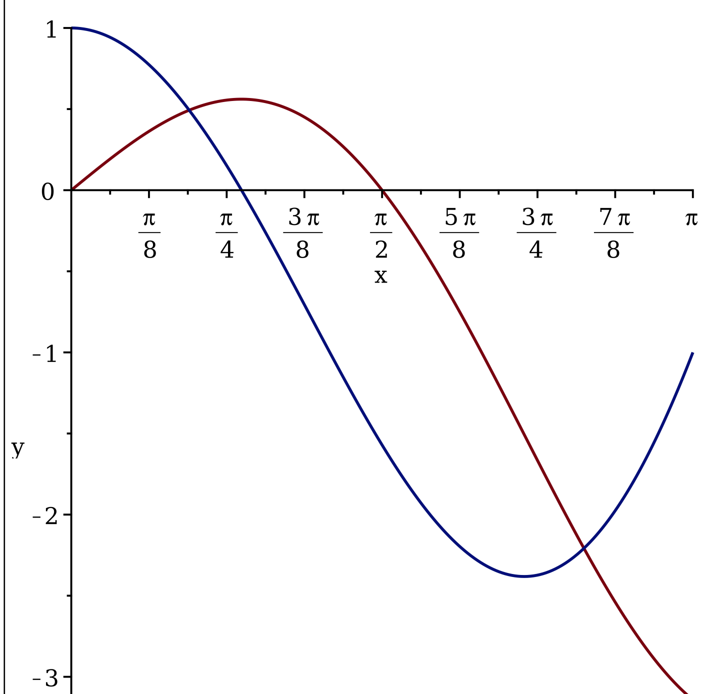
Action	Result in Document
Plot the original expression: 7. Insert a new document block, and **Ctrl** + drag the original expression to the new block. 8. From the Context Panel for the expression, select **Plots** → **2-D Plot**. 9. Modify the plot through the Context Panel: select **Axes** → **Properties**. 10. In the **Axis Properties** dialog, de-select **Use data extents** and change the range to **-3.14** to **3.14**. Click **OK**.	$x\cos(x)$ → 
Add the first and second derivatives to the plot: 11. Select and then **Ctrl** + drag the derivative of the expression onto the plot region. Do the same for the second derivative.	$x\cos(x)$ → 

Action	Result in Document
Enhance the plot by adding a legend: 12. From the Context Panel for the plot region, select **Legend** → **Show Legend**. 13. In the legend, double-click **Curve 1**. Notice that the **Text** icon is selected in the toolbar, Text. Delete the text and select the **Math** icon in the toolbar, Math. This allows you to enter 2-D Math in a text region. Enter the original expression, $x\cos(x)$. 14. Repeat for Curve 2 and Curve 3.	
Add a title: 15. From the Context Panel, select **Title** → **Add Title**. Replace the text **New title** with the text "**Plot the expression** ". 16. Click the **Math** icon, and enter the expression $x\cos(x)$. Click the **Text** icon once again and enter " **and its derivatives**".	$x\cos(x)$ → 

Solution by Tutor

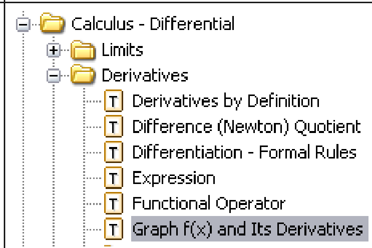
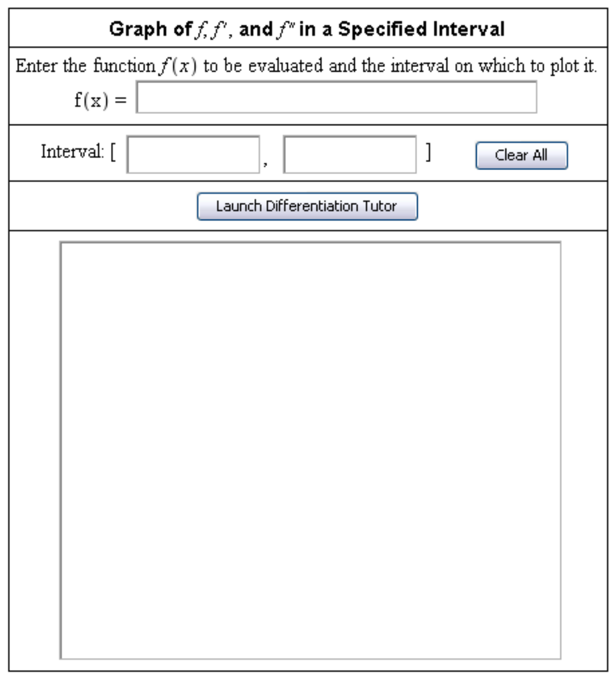
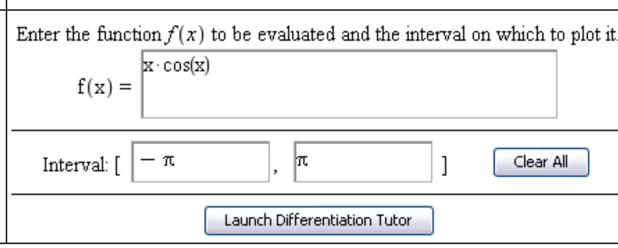
The **Student Calculus 1** package contains a tutor called **Derivatives**, which displays a plot of the expression along with its derivatives. In this example, we solve the same problem as previously, using this tutor

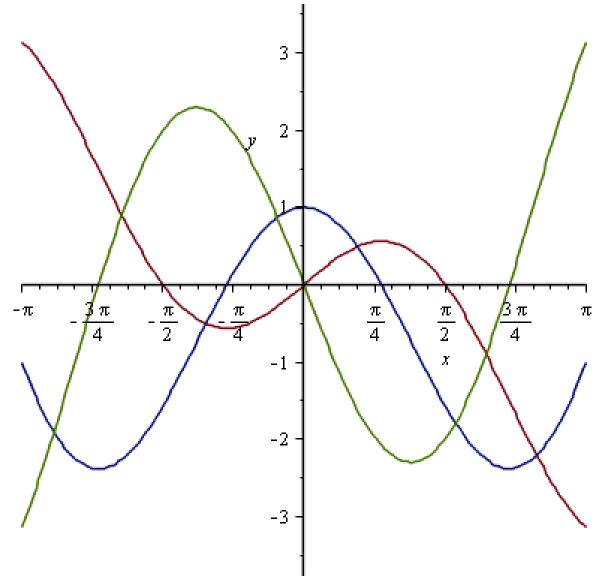
Action	Result in Document
1. Load the **Student Calculus 1** package. From the **Tools** menu, select **Load Package** → **Student Calculus 1**. 2. **Ctrl** + drag the expression $x\cos(x)$ to a blank document block region.	Loading **Student:-Calculus1**

Action	**Result in Document**
3. From the Context Panel for the expression, select **Student Calculus 1 → Tutors → Derivatives**. **Note:** The **Student Calculus 1** menu is now available in the Context Panel because we loaded the package in step 1. In the **Derivative Tutor**, the color swatch shown beside the original expression is the color used for the curve in the plot region. Similarly for $f'(x)$ and $f''(x)$. 4. Change the lower endpoint to -Pi. Select the check box to display $f''(x)$ in the plot. Click **Display** to make these changes take effect.	
5. You can change the expression and modify plot options from within this tutor. For each change made, click **Display** to view the altered plot. When complete, click **Close** to display the resulting plot in the document.	$x\cos(x) \xrightarrow{\text{derivative tutor}}$ 

Access the Tutor from a Task Template

Maple also comes with a Task Template to solve this problem without using any commands.

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Launch the Task Template Browser by selecting Tools → Tasks → Browse.
2. In the table of contents of the Task Browser dialog, select Calculus -Differential→ Derivatives → Graph $f(x)$ and its Derivatives.</td><td>
</td></tr>
<tr><td>3. Click Insert Minimal Content at the top of the dialog to insert the task template into the current document.</td><td>
</td></tr>
<tr><td>4. Enter the new expression $x*\cos(x)$ in the f(x) region.
5. Enter the interval $[-\pi, \pi]$. To insert the symbol for pi, you can use command completion or select π from the Common Symbols palette.</td><td>
</td></tr>
</table>

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>6. Click Launch Differentiation Tutor to launch the same tutor as in the previous solution.
7. When complete, click Close. A plot of the expression and its derivatives displays in the plot region of the inserted task template.</td><td>
</td></tr>
</table>

Example 2 - Solve for x in a Quadratic Equation

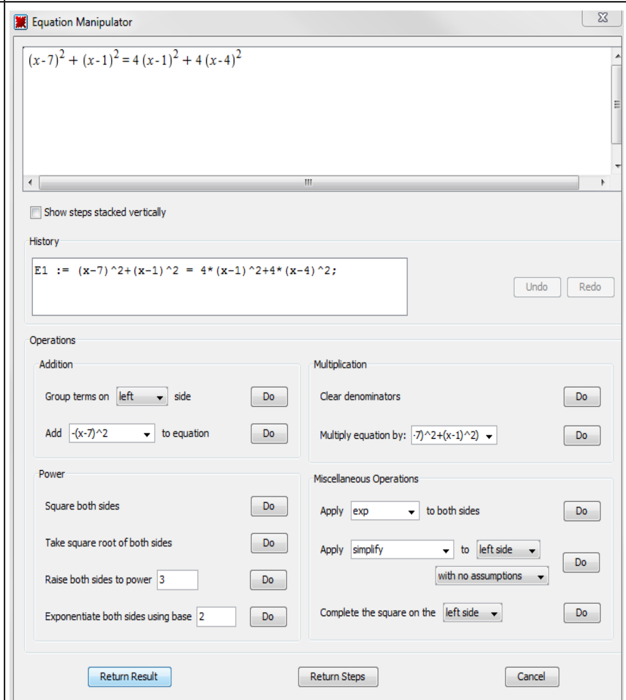
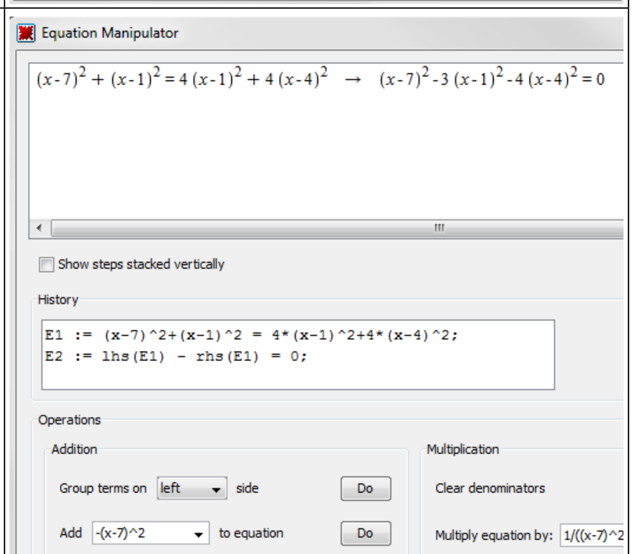
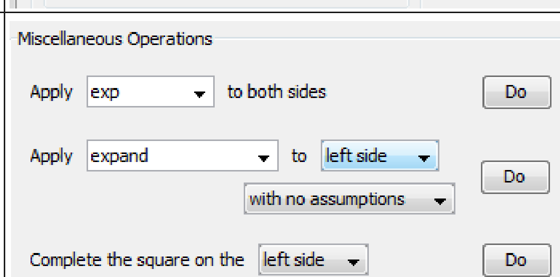
Solve for x in the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$.

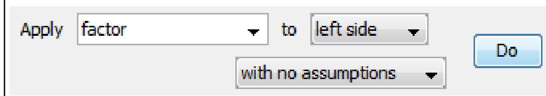
We solve this problem using the following methods:

- *Solution through Equation Manipulator (page 163)*
- *Instant Solution (page 165)*
- *Step-by-step Interactive Solution (page 165)*
- *Graphical Solution (page 166)*
- *Graphical Solution Using Smart Popups (page 168)*

Solution through Equation Manipulator

Maple provides a dialog that allows you to single-step through the process of manipulating an expression. This manipulator is available from the Context Panel.

<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td>1. Enter the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ in a new document block region.</td><td>$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$</td></tr>
<tr><td>2. From the Context Panel for this equation, select Manipulate Equation. The Manipulate Equation dialog displays.</td><td>
</td></tr>
<tr><td>Group all of the terms to the left:
3. In the Addition region, the Group terms row allows you to group terms on a specified side. With the left side already selected, click Do.</td><td>
</td></tr>
<tr><td>Expand the left side of the equation:
4. In the Miscellaneous Operations region, we can manipulate the equation by applying a command from the drop-down menus. Since we want to expand the left side of the equation only, click the first drop-down menu in the second row and select expand. Click Do. Note: This example is carried out with no assumptions. You can assume the solution to be real, positive, nonnegative or integer from the drop-down menu.</td><td>
</td></tr>
</table>

Action	Result in Document
Factor the equation: 5. From the same drop-down menu, select **factor** and click **Do**.	
6. Click **Return Steps** to close the dialog and return all of the steps to the Maple document.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{manipulate equation}}$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$ $-6\,x^2+24\,x-18=0$ $-6\,(x-1)\,(x-3)=0$
7. **Ctrl** + drag the factored form of the original equation to a new document block region. 8. From the Context Panel, select **Solve** → **Obtain Solutions for** → **x**. Or, Alternatively, click on the output from step 6 and select **Solve** → **Obtain Solutions for** → **x** in the Context Panel.	$-6\,(x-1)\,(x-3)=0 \xrightarrow{\text{solutions for x}} 1, 3$

Instant Solution

To apply an instant solution to this problem, use the Context Panel.

Action	Result in Document
1. **Ctrl** + drag the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a new document block region.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$
2. From the Context Panel for the expression, select **Solve** → **Obtain Solutions for** → **x**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{solutions for x}} 1, 3$

Step-by-step Interactive Solution

This equation can also be solved interactively in the document, by applying context-sensitive operations or commands one step at a time.

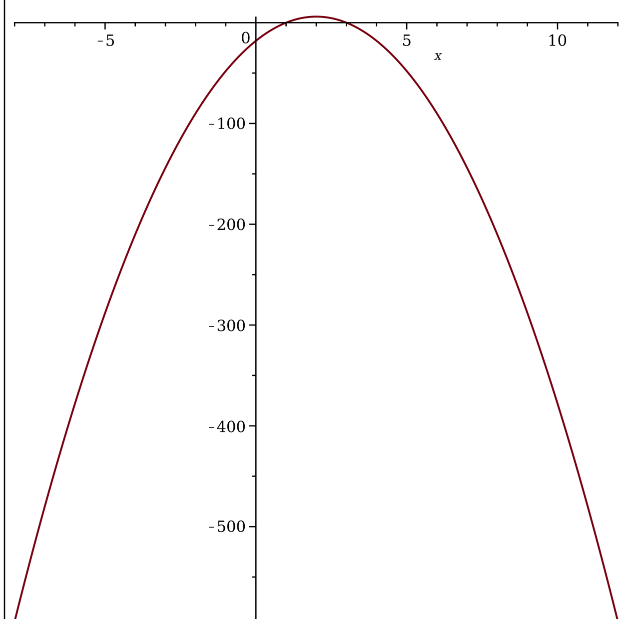
Action	Result in Document
1. **Ctrl** + drag the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a blank document block region.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ (5.11)
Group all terms on the right: 2. From the Context Panel for this equation, select **Move to Right**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $\xrightarrow{\text{move to right}} 0=3\,(x-1)^2+4\,(x-4)^2-(x-7)^2$

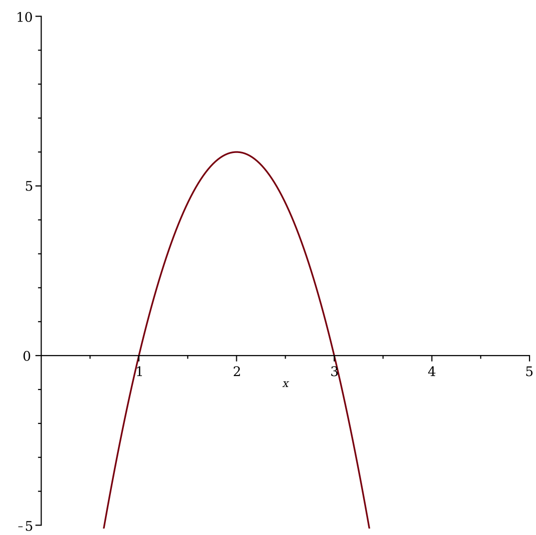
Action	Result in Document
Expand the expression on the right-hand side: 3. From the Context Panel for this output (or **Ctrl** + drag the equation to a blank document block region), select **Right-hand side**. 4. From the Context Panel for this result, select **Expand**.	$0 = 3(x-1)^2 + 4(x-4)^2 - (x-7)$ $\xrightarrow{\text{right hand side}}$ $3(x-1)^2 + 4(x-4)^2 - (x-7)^2 \overset{\text{expand}}{=}$ $6x^2 - 24x + 18$
Use Maple's factor command on the resulting right-hand side: 5. From the Context Panel for the result, select **Right-hand Side**. 6. From the Context Panel for the result, select **Factor**.	$0 = 6x^2 - 24x + 18 \xrightarrow{\text{right hand side}}$ $6x^2 - 24x + 18 \overset{\text{factor}}{=} 6(x-1)(x-3)$
Solve for x: 7. From the Context Panel for the result, select **Solve** → **Obtain Solutions for** → **x**.	$6(x-1)(x-3) \xrightarrow{\text{solutions for x}} 1, 3$

Graphical Solution

Now that we have seen several methods to solve this problem, we can check the answer by plotting the expression.

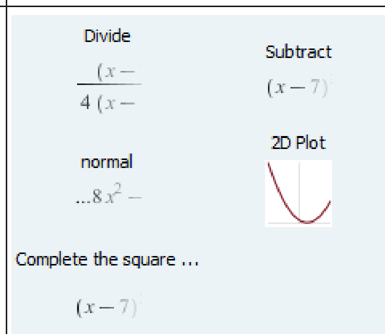
Action	Result in Document
1. **Ctrl** + drag the equation $(x-7)^2 + (x-1)^2 = 4\left((x-1)^2 + (x-4)^2\right)$ to a new document block region and press **Enter**.	$(x-7)^2 + (x-1)^2 = 4\left((x-1)^2 + (x-4)^2\right)$ $(x-7)^2 + (x-1)^2 = 4(x-1)^2 + 4(x-4)^2$
First, manipulate the equation to become an expression: 2. Select the output and from the Context Panel select **Move to Left**. Note the difference in the alignment when using context-sensitive operations on output rather than input. The result is centered in the document with the self-documenting arrow positioned at the left.	$(x-7)^2 + (x-1)^2 = 4(x-1)^2 + 4(x-4)^2$ $\xrightarrow{\text{move to left}} (x-7)^2 - 3(x-1)^2 - 4(x-4)^2 = 0$

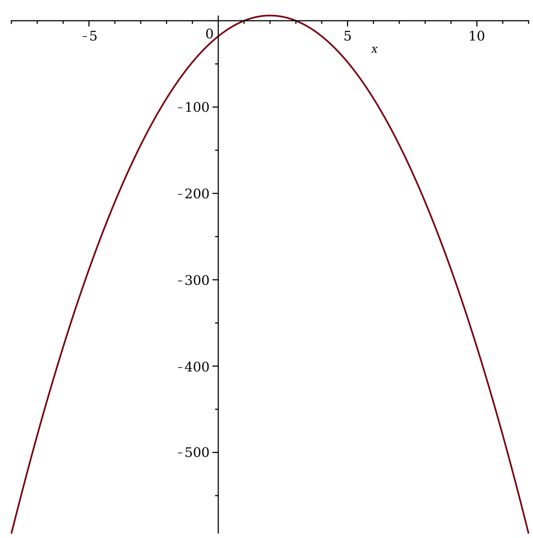
Action	Result in Document
3. Select the output and from the Context Panel select **Left-hand Side**.	$(x-7)^2 - 3\,(x-1)^2 - 4\,(x-4)^2 = 0$ $\xrightarrow{\text{left hand side}}$ $(x-7)^2 - 3\,(x-1)^2 - 4\,(x-4)^2$
4. Select the output and from the Context Panel select **Expand**.	$(x-7)^2 - 3\,(x-1)^2 - 4\,(x-4)^2$ $\overset{\text{expand}}{=}$ $-6\,x^2 + 24\,x - 18$
Now that the equation is in its simplest form, plot the result: 5. **Ctrl** + drag the output to a new document block. 6. From the Context Panel for thee expression select **Plots** → **2-D Plot**. Or, Right-click the output from step 4 and select **Plots** → **2-D Plot**.	$-6\,x^2 + 24\,x - 18 \;\rightarrow$ 

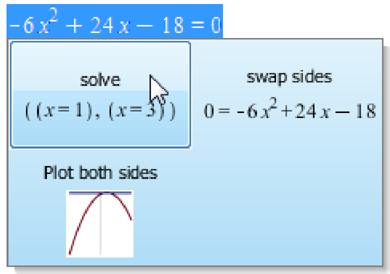
Action	**Result in Document**
Change the x- and y- axis ranges using the Context Panel: 7. By default, the plot uses a range chosen to fit the key features of the graph. To better see the x-intercepts, change the range: from the Context Panel, select **Axes → Properties**. In the **Horizontal** tab of the **Axes Properties** dialog, clear the **Use data extents** check box and change the **Range min** and **Range max** to **0** and **5**, respectively. Click the **Vertical** tab and clear the **Use data extents** check box. Change the **Range min** and **Range max** to **-5** and **10**, respectively. 8. Click **OK** to apply the changes and return to the plot. The interception points of this graph with the x-axis are 1 and 3, the same solutions that we found previously.	

Graphical Solution Using Smart Popups

Use Smart Popup menu options to find a graphical solution.

Copy the equation $(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ to a new document block region and press **Enter**.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ (5.12)
Highlight the right-hand side of the output expression. Smart popups are shown in the top of the Context Panel.	$(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$
Select the **Subtract** menu option.	
The results of the calculation are displayed in the document.	$(x-7)^2+(x-1)^2=4\left((x-1)^2+(x-4)^2\right)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ (5.13) subtract 4*(x-1)^2+4*(x-4)^2 from both sides → $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$ (5.14)

Select the left side of the equation. A smart popup window is displayed.	$(x-7)^2-3\,(x-1)^2-4\,(x-4)^2$
Select **normal**. The results of the calculation are displayed in the document.	$(x-7)^2+(x-1)^2=4\big((x-1)^2+(x-4)^2\big)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ (5.15) subtract 4*(x-1)^2+4*(x-4)^2 from both sides → $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$ (5.16) normal (x-7)^2-3*(x-1)^2-4*(x-4)^2 → $-6\,x^2+24\,x-18=0$ (5.17)
Once again, select the left side of the equation. A smart popup window is displayed. Select **2D plot**.	$-6\,x^2+24\,x-18$
The resulting plot is displayed in the document.	$(x-7)^2+(x-1)^2=4\big((x-1)^2+(x-4)^2\big)$ $(x-7)^2+(x-1)^2=4\,(x-1)^2+4\,(x-4)^2$ (5.18) subtract 4*(x-1)^2+4*(x-4)^2 from both sides → $(x-7)^2-3\,(x-1)^2-4\,(x-4)^2=0$ (5.19) normal (x-7)^2-3*(x-1)^2-4*(x-4)^2 → $-6\,x^2+24\,x-18=0$ (5.20) 2D Plot -6*x^2+24*x-18 → 

You can also solve the equation using smart popups. Highlight the simplified equation and from the Smart Popup menu, select **solve**.	
The solution to the equation is displayed.	$[[x=1],[x=3]]$ (5.21)

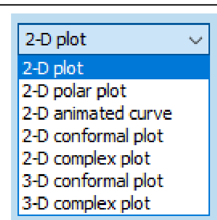
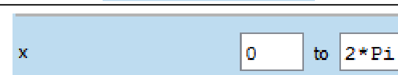
Example 3 - Solve a Quadratic Trig Equation

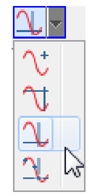
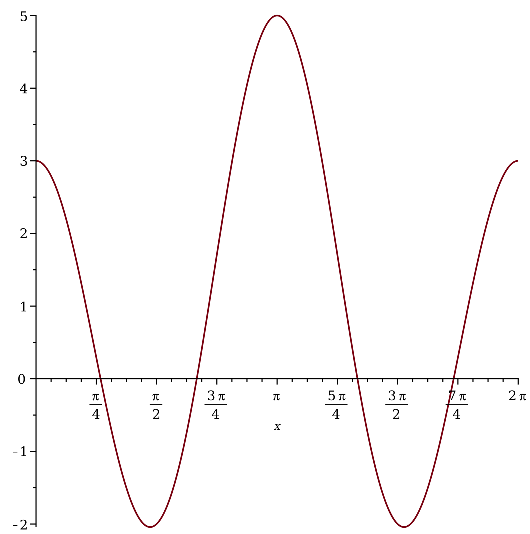
Find all of the solutions to the equation $6\cos^2(x)-\cos(x)-2=0$ in the interval $[0, 2\pi]$.

We solve this problem using the following methods:

- *Graphical Solution (page 170)*
- *Solution by Task Template (page 172)*
- *Analytic Solution (page 172)*

Graphical Solution

Action	Result in Document
1. **Ctrl** + drag the equation $6\cos^2(x)-\cos(x)-2=0$ to a blank document block. 2. From the Context Panel for the equation, select **Left-hand Side**.	$6\cos(x)^2-\cos(x)-2=0 \xrightarrow{\text{left hand side}}$ $6\cos(x)^2-\cos(x)-2$
3. Click the output and select **Plot Builder** in the Context Panel. A plot appears. In the Plot Builder panel, in the plot type list, the initally selected plot type is **2-D plot**.	
4. Modify the plot range to $x=0$ to $2*\text{Pi}$.	

Action	**Result in Document**
5. Click **Plot** to display the plot in the document. 6. From the graph, we can see all of the solutions within the interval $[0, 2\pi]$. To approximate the values, click the plot, select the type of coordinates that you want to view from the selection menu ( ) in the toolbar, and then use the point probe tool to view the coordinates of the mouse pointer.	 

Solution by Task Template

Action	Result in Document
1. From the **Tools** menu, select **Tasks** → **Browse**. Expand the **Algebra** folder and select **Solve Analytically in a Specified Interval**.	Algebra Complete the Square Complex Arithmetic Conic - Analysis and Graph ⋮ Solve a Set of Equations Symbolically Solve an Equation Numerically Solve an Equation Symbolically Solve an Inequality Solve Analytically in Specified Interval
2. Click **Insert Minimal Content**.	**Solve Analytically in a Specified Interval** Enter an expression: > $12\sin^2(x)-5\sin(x)-3$ $12\sin(x)^2-5\sin(x)-3$ **(15)** Find the roots in a specified interval: > $Student[Calculus1][Roots]((15), 0..2\pi)$ $\left[\arcsin\left(\frac{3}{4}\right), -\arcsin\left(\frac{3}{4}\right)+\pi, \arcsin\left(\frac{1}{3}\right)+\pi, -\arcsin\left(\frac{1}{3}\right)+2\pi\right]$ **(16)** Express the roots in floating-point form: > $evalf((16))$ [0.8480620790, 2.293530575, 3.481429564, 5.943348398] **(17)**
3. Replace the current equation with the one from this example, $6\cos^2(x)-\cos(x)-2=0$, and then execute the commands. Notice that equation labels are used to reference the results.	**Solve Analytically in a Specified Interval** Enter an expression: > $6\cos^2(x)-\cos(x)-2=0$ $6\cos(x)^2-\cos(x)-2=0$ **(15)** Find the roots in a specified interval: > $Student[Calculus1][Roots]((15), 0..2\pi)$ $\left[\arccos\left(\frac{2}{3}\right), \frac{2}{3}\pi, \frac{4}{3}\pi, -\arccos\left(\frac{2}{3}\right)+2\pi\right]$ **(16)** Express the roots in floating-point form: > $evalf((16))$ [0.8410686706, 2.094395103, 4.188790204, 5.442116637] **(17)**

Analytic Solution

Action	Result in Document
1. **Ctrl** + drag the equation $6\cos^2(x)-\cos(x)-2=0$ to a blank document block region.	$6\cos^2(x)-\cos(x)-2=0$
2. From the Context Panel, select **Left-hand Side**.	$6\cos^2(x)-\cos(x)-2=0 \xrightarrow{\text{left hand side}}$ $6\cos(x)^2-\cos(x)-2$
3. From the Context Panel, select **Factor**. 4. Click the new factored output and select **Solve** → **Solve** in the Context Panel. Alternatively, you can select each factor, **Ctrl** + drag the expressions to separate document block regions and for each one select **Solve** → **Solve**.	$6\cos(x)^2-\cos(x)-2=6\cos(x)^2-\cos(x)-2 \overset{\text{factor}}{=}$ $(2\cos(x)+1)(3\cos(x)-2) \xrightarrow{\text{solve}}$ $\left\{x=\frac{2\pi}{3}\right\}, \left\{x=\arccos\left(\frac{2}{3}\right)\right\}$

Example 4 - Find the Inverse Function

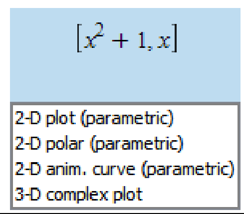
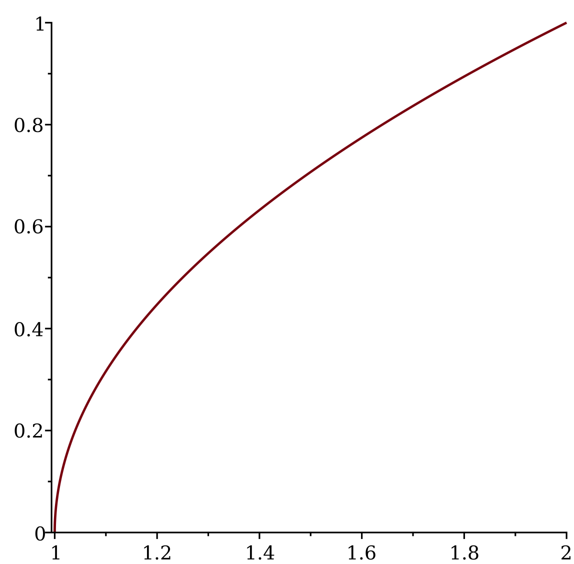
If $f(x) = x^2 + 1, x \geq 0$, find and graph the rule for $f^{-1}(x)$, its functional inverse.

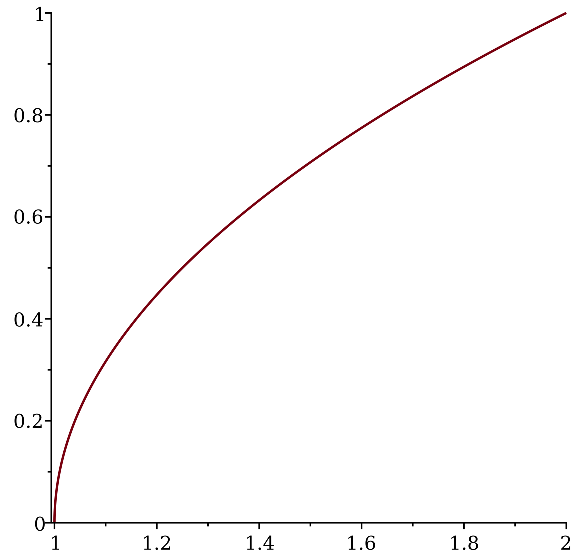
We solve this problem using the following methods:

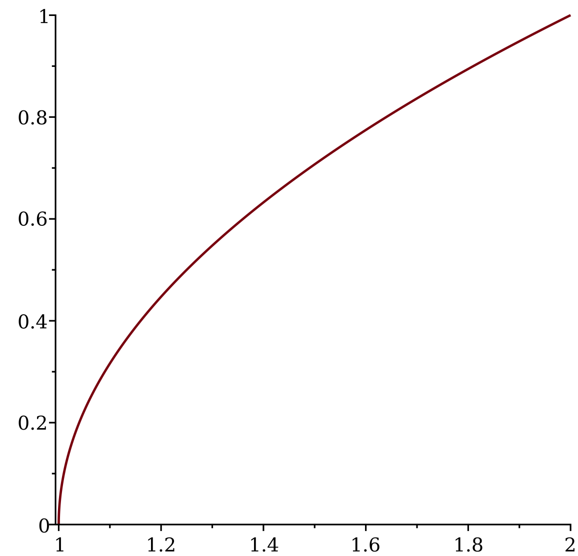
- *Implement the Definition Graphically (page 173)*
- *Solution by Tutor (page 176)*

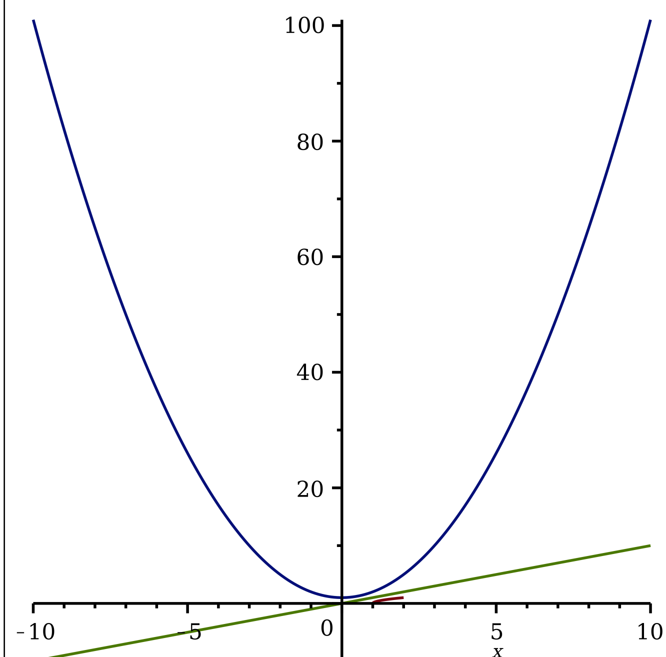
Implement the Definition Graphically

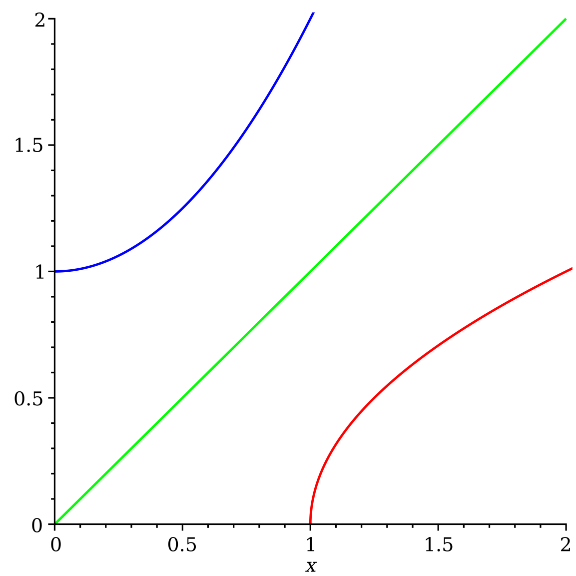
The graph of the inverse function is the set of ordered pairs formed by interchanging the ordinates and abscissas.

Action	Result in Document
1. In a blank document block, enter $[x^2 + 1, x]$ and press **Enter**.	$[x^2 + 1, x]$ $[x^2 + 1, x]$
2. Click the output and select **Plot Builder** in the Context Panel. In the Plot Builder panel select **2-D plot (parametric)** for the plot type. The plot appears.	
3. Adjust the domain for x to the interval .	→ 

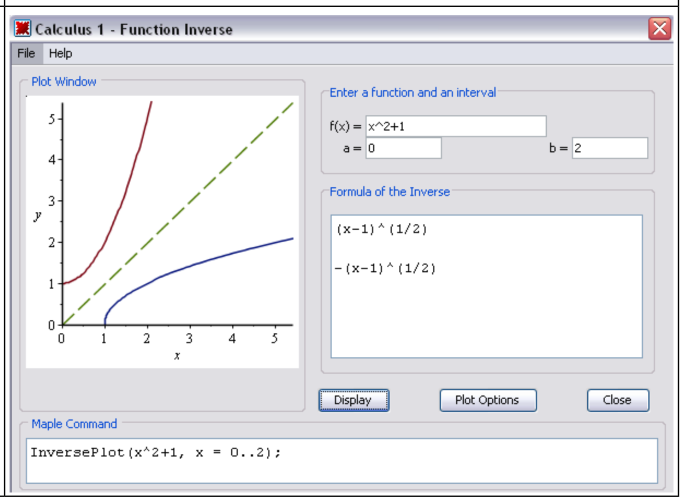
Action	**Result in Document**
4. Copy and paste the expression $x^2 + 1$ onto this graph. Notice that the axis ranges alter.	
5. Copy and paste the expression x onto this graph. The resulting graph shows $f(x)$, $f'(x)$, and the line $y = x$.	

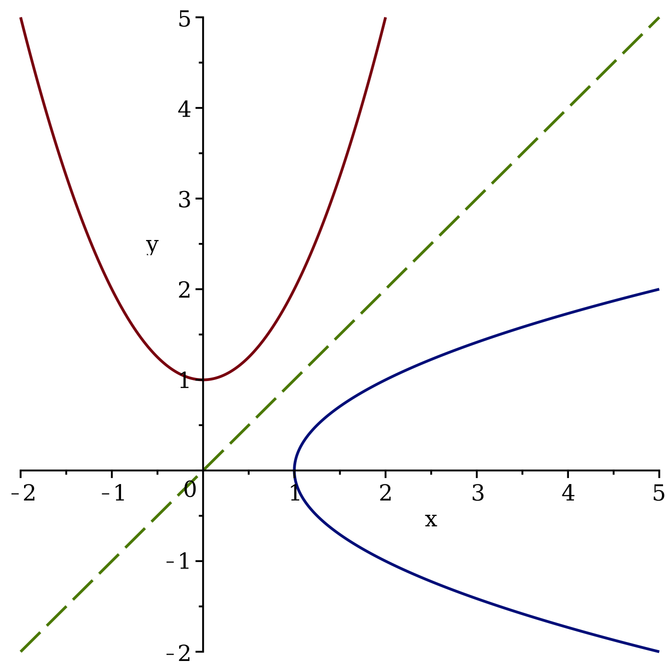
<table>
<tr><th>Action</th><th>Result in Document</th></tr>
<tr><td></td><td>


</td></tr>
<tr><td>Adjust the x and y axis ranges:
6. Select the plot and from the Plot menu, select Axes → Properties.
7. In the Axis Properties dialog, de-select Use data extents and change the range to 0 to 2.
8. Click the Vertical tab and repeat step 9. Click OK to apply these settings and close the dialog.</td><td></td></tr>
</table>

Action	Result in Document
	→ 

Solution by Tutor

Action	Result in Document
1. Load the **Student Calculus 1** package. From the **Tools** menu, select **Load Package** → **Student Calculus 1**.	Loading **Student:-Calculus1**
2. Enter the expression $x^2 + 1$ in a blank document block.	$x^2 + 1$
3. From the Context Panel, select **Student Calculus 1** → **Tutors** → **Function Inverse**. The **Function Inverse Tutor** displays. 4. Adjust the domain to .	

Action	Result in Document
5. When you are finished, click **Close**. The plot of the function, its inverse, and the line $y = x$ is returned to the document.	$x^2 + 1 \xrightarrow{\text{inverse tutor}}$ 

Example 5 - Methods of Integration - Trig Substitution

Evaluate the integral $\int \frac{1}{\sqrt{4 - x^2}}\, dx$ by making the substitution $x = 2 \sin(u)$.

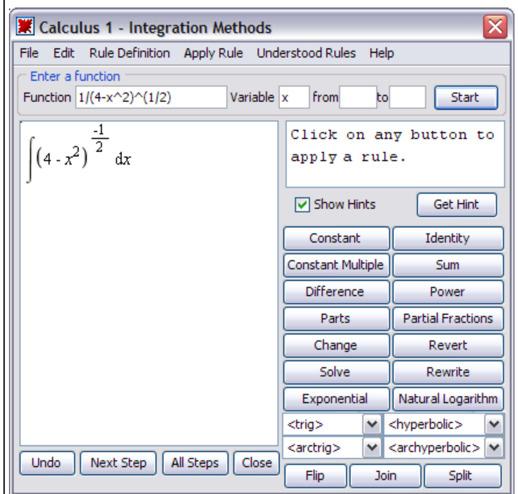
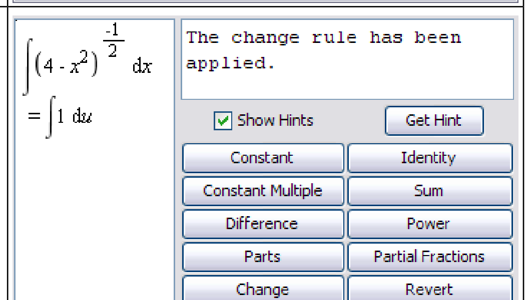
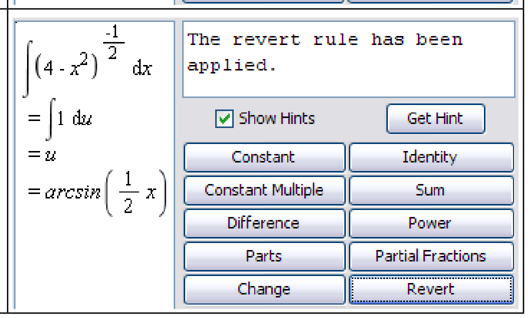
We solve this problem using the following methods:

- *Immediate Evaluation of the Integral (page 177)*
- *Solution by Integration Methods Tutor (page 178)*
- *Solution by First Principles (page 179)*

Immediate Evaluation of the Integral

Action	Result in Document
1. Enter the integral $\int \frac{1}{\sqrt{4 - x^2}}\, dx$ in a blank document block region.	$\int \frac{1}{\sqrt{4 - x^2}}\, dx$
2. From the Context Panel for the expression and select **Evaluate and Display Inline**.	$\int \frac{1}{\sqrt{4 - x^2}}\, dx = \arcsin\left(\frac{x}{2}\right)$

Solution by Integration Methods Tutor

Action	Result in Document
1. Load the **Student Calculus 1** package. From the **Tools** menu, select **Load Package** → **Student Calculus 1**.	Loading **Student:-Calculus1**
2. **Ctrl** + drag the integrand $\frac{1}{\sqrt{4-x^2}}$ to a blank document block region.	$\frac{1}{\sqrt{4-x^2}}$
3. From the Context Panel, select **Student Calculus 1** → **Tutors** → **Integration Methods**. The **Integration Methods Tutor** displays.	
4. Perform a change of variables by selecting **Change** and entering **x = 2*sin(u)**.	
5. Apply the constant rule by clicking **Constant**. 6. To revert back to the original variable, click **Revert**.	

Action	Result in Document
7. Now that the integral has been evaluated, click **Close** to close the tutor and return the evaluated integral to the document.	$\frac{1}{\sqrt{4-x^2}}$ $\xrightarrow{\text{integration methods tutor}}$ $\int \frac{1}{\sqrt{-x^2+4}}\,dx$ $= \int 1\,du$ [*change*, $x = 2\sin(u)$, u] $= u$ [*constant*] $= \arcsin\left(\frac{x}{2}\right)$ [*revert*] $\int \frac{1}{\sqrt{-x^2+4}}\,dx = \arcsin\left(\frac{x}{2}\right)$ (5.22)

Solution by First Principles

Action	Result in Document
1. **Ctrl** + drag the integrand $\frac{1}{\sqrt{4-x^2}}$ to a blank document block region and press **Enter**.	$\frac{1}{\sqrt{4-x^2}}$ $\frac{1}{\sqrt{-x^2+4}}$
Perform trig substitution: 2. From the Context Panel for the output, select **Evaluate at a point**. In the dialog that displays, enter **2*sin(u)**.	$\xrightarrow{\text{evaluate at point}}$ $\frac{1}{\sqrt{-4\sin(u)^2+4}}$
3. From the Context Panel for the output, select **Simplify** → **Symbolic**.	$\xrightarrow{\text{simplify symbolic}}$ $\frac{1}{2\cos(u)}$ (5.23)
Calculate $\frac{du}{dx}$: 4. In a blank document block, enter the substitution equation: $x = 2\sin(u)$ and press **Enter**. 5. From the Context Panel for the output, select **Differentiate** → **Implicitly**. In the dialog that displays, change the **Dependent Variable** to **x** and change **Differentiate with respect to** to **u**.	$x = 2\sin(u)$ $x = 2\sin(u)$ $\xrightarrow{\text{implicit differentiation}}$ $2\cos(u)$ (5.24)
Calculate the integral in terms of u: 6. Referencing the results by their equation labels, multiply the original simplified expression by this derivative.	(5.23)·(5.24) 1 (5.25)

Action	**Result in Document**
7. Integrate the resulting expression.	$\int$ **(5.25)** du u (5.26)
Revert the substitution: 8. Place the equation $x = 2\sin(u)$ in a blank document block. Delete u and insert the equation label for the previous result, the value of the integral in terms of u. Press **Enter**. 9. From the Context Panel for the output, select **Solve** → **Solve for Variable** → **u**. The solution is $\arcsin\left(\frac{1}{2}x\right)$.	$x = 2\sin($**(5.26)**$)$ $x = 2\sin(u)$ $\xrightarrow{\text{solve for u}}$ $\left[\left[u = \arcsin\left(\frac{x}{2}\right)\right]\right]$

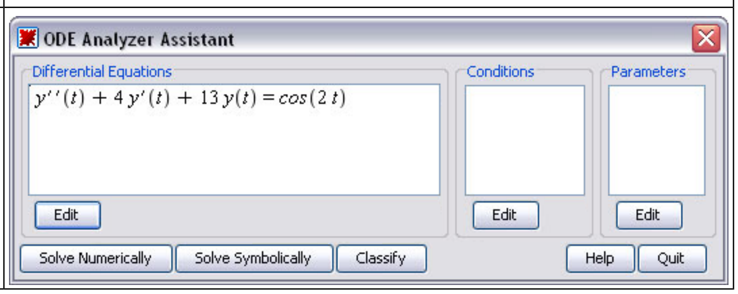
Example 6 - Initial Value Problem

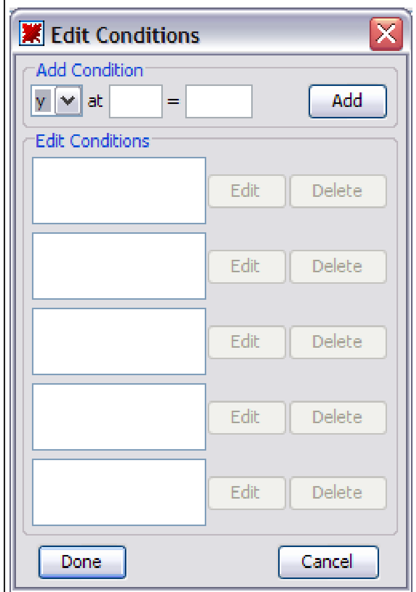
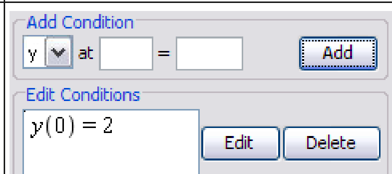
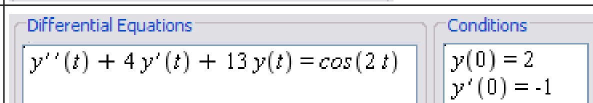
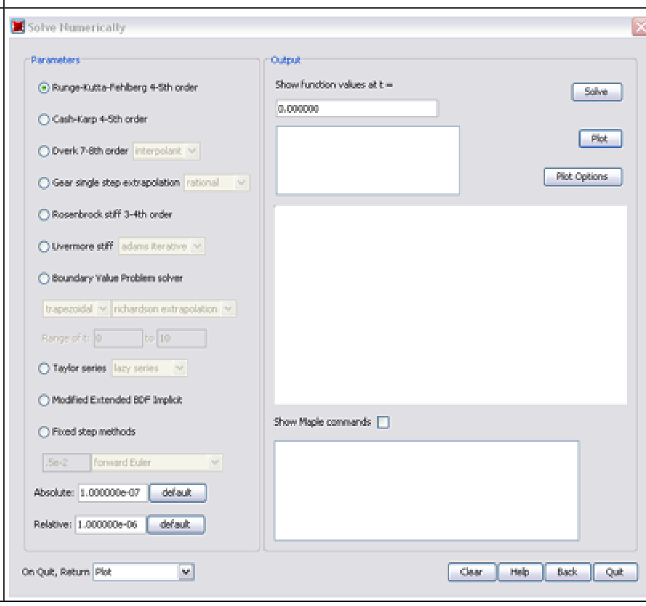
Solve and plot the solution of the initial value problem

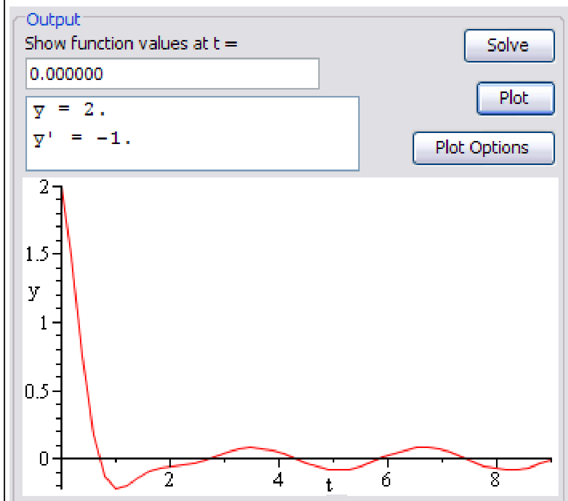
$$y''(t) + 4\,y'(t) + 13\,y(t) = \cos(2\,t)$$
$$y(0) = 2$$
$$y'(0) = -1$$

Solution by ODE Analyzer Assistant

The ODE Analyzer Assistant lets you solve ODEs numerically or symbolically and displays a plot of the solution.

Action	**Result in Document**
1. Enter the ODE in a blank document block region.	$y''(t) + 4\,y'(t) + 13\,y(t) = \cos(2\,t)$
2. Select the equation and from the Context Panel, select **Solve DE Interactively**. The **ODE Analyzer Assistant** displays with the ODE automatically inserted.	

Action	Result in Document
To insert the initial conditions: 3. In the **Conditions** region, click **Edit**. The **Edit Conditions** dialog opens.	
4. In the **Add Condition** region, with **y** selected in the drop-down menu, enter **0** in the first text field to the right and **2** in the second text field. Click **Add**. Your entry should match the one shown to the right.	
5. To enter the initial condition for y', select **y'** from the drop-down menu. In the text fields, enter **0** and **-1**. Click **Add**. Click **Done** to close this dialog and return to the main dialog. Notice that the initial conditions are in the **Conditions** section.	
6. Click **Solve Numerically**. A new dialog appears.	

Action	Result in Document
7. Click **Solve** to solve the initial value problem. 8. Click **Plot** to plot the solution of the DE.	
9. Click the **Plot Options** button to modify the default graph, if desired. 10. Click **Quit** to close the ODE Analyzer and return a plot of the solution to the document.	$y''(t) + 4\,y'(t) + 13\,y(t) = \cos(2\,t) \xrightarrow{\text{solve DE interactively}}$ $y(t) = e^{-2\,t}\sin(3\,t)\,\_C2 + e^{-2\,t}\cos(3\,t)\,\_C1 + \frac{9\cos(2\,t)}{145} + \frac{8\sin(2\,t)}{145}$

6 Plots and Animations

Maple can generate many forms of plots, allowing you to visualize a problem and further understand concepts.

- Maple accepts explicit, implicit, and parametric forms to display 2-D and 3-D plots and animations.
- Maple recognizes many coordinate systems.
- All plot regions in Maple are active; therefore, you can drag expressions to and from a plot region.
- Maple offers numerous plot options, such as axis styles, title, colors, shading options, surface styles, and axis ranges, which give you complete control to customize your plots.

For a reference to the types of plots available in Maple, see the **Plotting Guide**.

6.1 In This Chapter

Section	Topics
Creating Plots (page 183) - Interactive and command-driven methods to display 2-D and 3-D plots	• **Interactive Plot Builder** • Context Panel • The **plot** and **plot3d** Commands • The **plots** Package • Multiple Plots in the Same Plot Region
Customizing Plots (page 206) - Methods for applying plot options before and after a plot displays	• **Interactive Plot Builder** Options • Context Panel Options • The **plot** and **plot3d** Command Options
Analyzing Plots (page 213) - Plot analyzing tools	• Point Probe • Rotate • Pan • Zoom
Representing Data (page 214) - Templates for visual representation of your data	• The **Live Data Plots** Palette
Creating Animations (page 215) - Interactive and command-driven methods to display animations	• **Interactive Plot Builder** • The **plots[animate]** Command • The **plot3d[viewpoint]** Command
Playing Animations (page 219) - Tools to run animations	• Animation Context Bar
Customizing Animations (page 221) - Methods for applying plot options before and after an animation displays	• **Interactive Plot Builder** Animation Options • Context Menu Options • The **animate** Command Options
Exporting (page 226) - Methods for exporting plots	• Saving Plots to File Formats
Code for Color Plates (page 226) - Information on color plates	• Accessing Code for the Color Plates

6.2 Creating Plots

Maple offers several methods to easily plot an expression. These methods include:

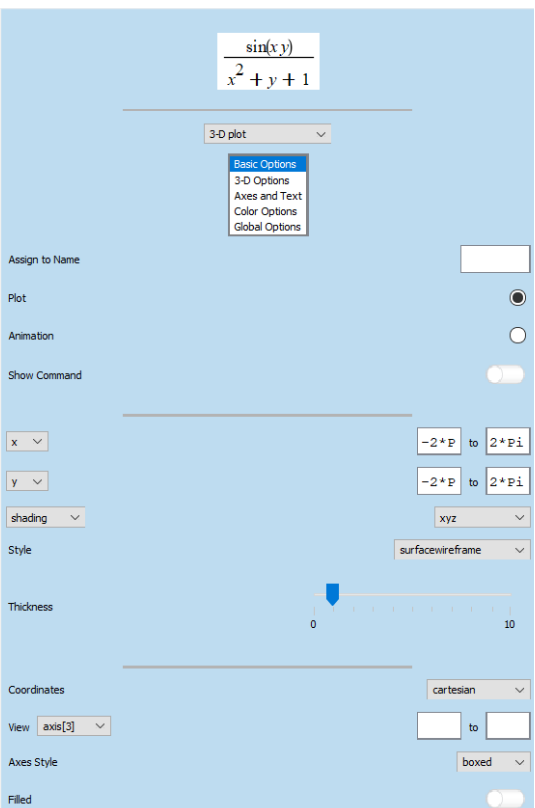
- The **Interactive Plot Builder**
- The context panel
- Commands

Each method offers a unique set of advantages. The method you use depends on the type of plot to display, as well as your personal preferences.

Interactive Plot Builder

The Interactive Plot Builder is a point-and-click interface to Maple plotting functionality . The interface displays plot types based on the expression you specify. Depending on the plot type you select, you can create a:

- 2-D / 3-D plot
- 2-D polar plot
- 2-D / 3-D contour plot
- 2-D / 3-D conformal plot of a complex-valued function
- 2-D / 3-D complex plot
- 2-D / 3-D parametric plot
- 2-D density plot
- 2-D / 3-D implicit plot
- 2-D inequality plot
- 3-D spacecurve
- 2-D animated curve



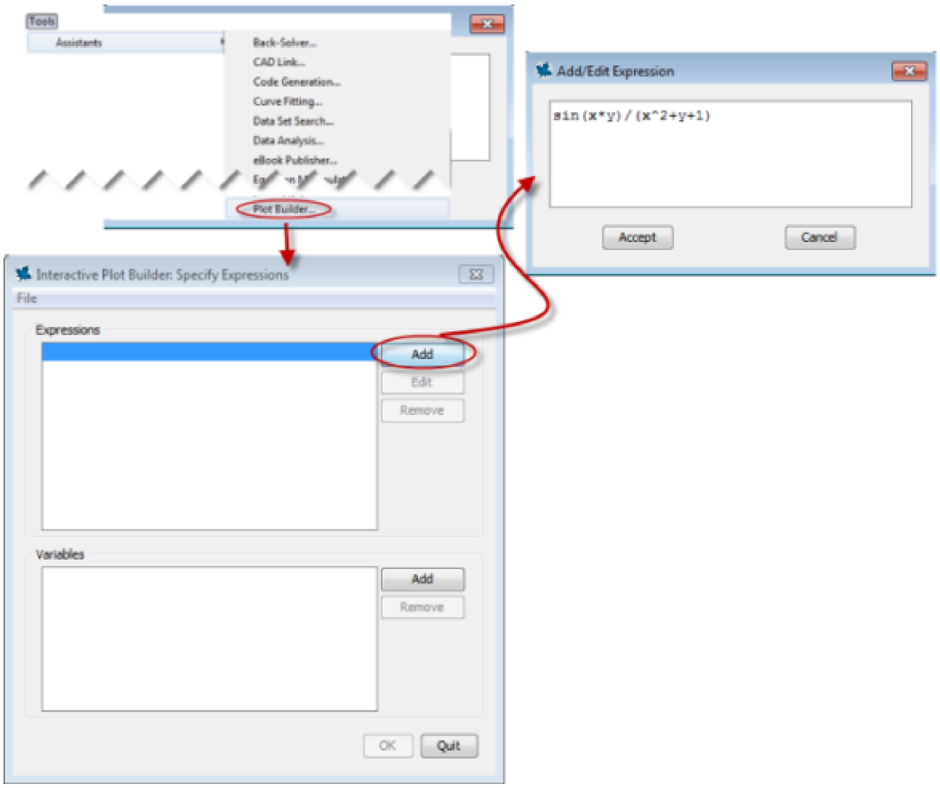
Launching the PlotBuilder

Assistants Access

The Plot Builder can be accessed from the **Standard Menu Bar**, under Tools -> Assistants.

When launched in this way, an interactive dialog is displayed where you can enter the expression to be plotted. This is equivalent to calling the **PlotBuilder** command with no arguments.

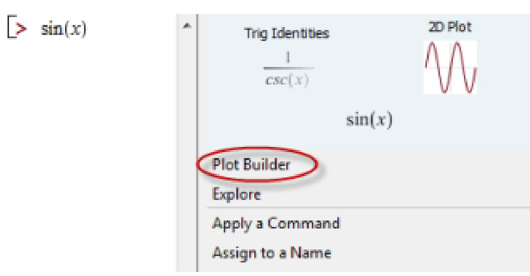
Note: The Tools menu also offers tutors to easily generate plots in several academic subjects. For more information, see *Teaching and Learning with Maple (page 145)*.



Context Panel Access

The Plot Builder can also be launched using the **context panel** for an expression.

Click on the expression you'd like to plot and from the context panel select **Plot Builder**. In this case, the **Plot Builder Panel** opens.

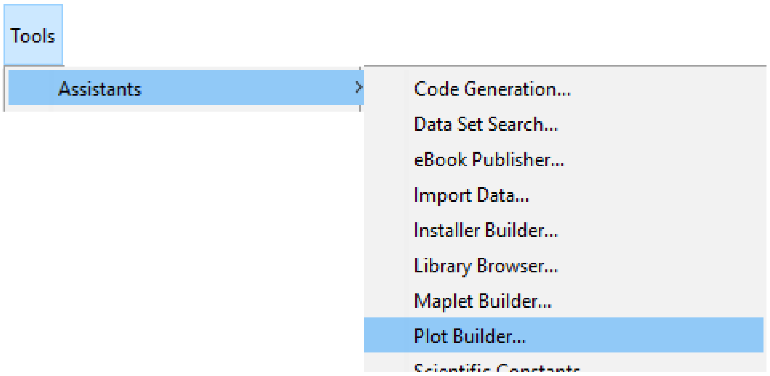


Example 1 - Display a plot of a single variable expression

Maple can display two-dimensional graphs and offers numerous plot options such as color, title, and axis styles to customize the plot.

Launch the Interactive Plot Builder:

1. Make sure that the cursor is in a Maple input region.
2. From the **Tools** menu, select **Assistants**, and then **Plot Builder**.

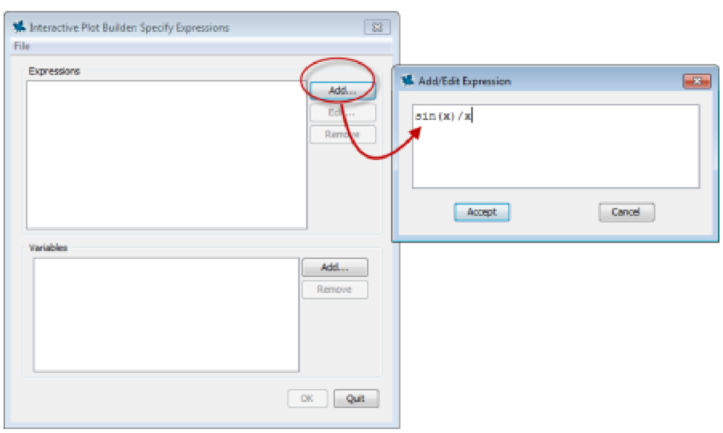


Notes: In worksheet mode, Maple inserts **PlotBuilder()** in the Maple document and opens the **Specify Expressions** window.

Entering this command at the Maple prompt also opens the Plot Builder.

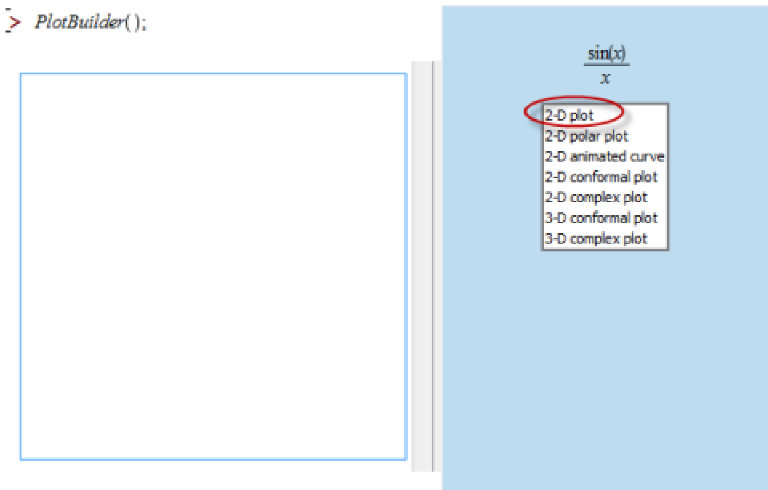
Enter an expression:

3. In the **Interactive Plot Builder:Specify Expressions** window:
   a. Add the expression, **sin(x)/x**.
   b. Click **Accept** in the Add/Edit Expression window.
   c. Click **OK** in **Interactive Plot Builder:Specify Expressions** window.

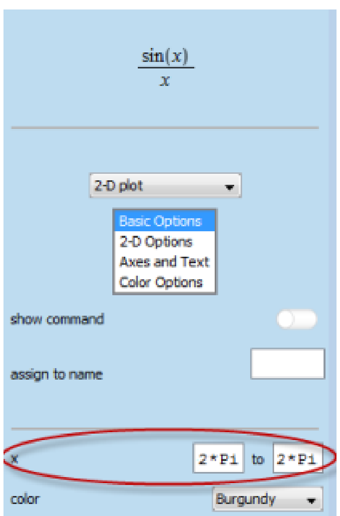


Display the PlotBuilder panel and plot the expression:

4. The PlotBuilder inserts plot component, with a plot of the expression, into the worksheet, just under the PlotBuilder() command. Click on this to make the PlotBuilder panel slide into view.
5. Select 2-D plot from the list of available plot types for this expression. Notice the rest of the panel is now populated with the available plotting options for a 2-D plot.



6. Ensure that the **x**-axis range is from $-2\ \mathrm{Pi}..2\ \mathrm{Pi}$.



Note: You can toggle the **show command** option to view the Maple syntax used to generate this plot.

Example 2 - Display a plot of multiple expressions in 1 variable

Maple can display multiple expressions in the same plot region to compare and contrast. The **Interactive Plot Builder** accepts multiple expressions.

Launch the Interactive Plot Builder and enter the expressions:

If the PlotBuilder is passed a list, set, or sequence containing multiple expressions then the context panel becomes automatically populated with a thumbnail view. The common plot type for expressions in a single unknown is a 2-D curve, and for expressions in two unknowns it is a 3-D surface plot.

As an example, use the PlotBuilder to plot, $\sin(x^2)$, $\frac{d}{dx}\sin(x^2)$ and $\int \sin(x^2)\,dx$.

> $PlotBuilder\left(\sin(x^2), \frac{d}{dx}\sin(x^2), \int \sin(x^2)\,dx\right)$

Notice in the PlotBuilder window, from the thumbnail view you can change the common plot type for all expressions using the ListBox menu. You can select any individual thumbnail plot to change its associated properties or its own plot type in the PlotBuilder's usual individual plot view.

By default, Maple displays each plot in a plot region using a different color. You can also apply a line style such as solid, dashed, or dotted for each expression in the graph. For more information, refer to the **plot/options** help page. To see the Maple syntax used to generate this plot, see *Maple commands from Creating Plots: Interactive Plot Builder (page 192)*

Example 3 - Display a plot of a multi-variate expression

Maple can display three-dimensional plots and offers numerous plot options such as light models, surface styles, and shadings to allow you to customize the plot.

Enter an expression, then launch the Interactive Plot Builder from the context panel:

1. Enter the expression **(1+sin(x*y))/(x^2+y^2)**.
2. Launch the Plot Builder from the context panel.

In the Select Plot Type list:

2. Notice the available plot types for an expression with 2 variables, as well as the plot objects for each type. For this example, select **3-D plot**.
3. Select **Basic Options** from the list of option types.

Restricting how much of the plot axes to display:

4. For the view option, select **axis[3]** to adjust how much of the z-axis to display. Enter the range values in the two boxes. You can repeat this for the y-axis (axis[2]) and x-axis (axis[1]) as well.

Style and color changes:

5. From the **Style** combo box, select **surface**.
6. From the **shading and color** list box select **shading**, then select **z (grayscale)** from the color list box.
7. Select the **Axes and Text** menu.
8. In the text box next to **label**, enter **z**.
9. Select the **3-D Options** menu
10. In the text box beside **grid size**, enter **40**,**40**.

Note: You can toggle the **show command** option to view the Maple syntax used to generate this plot.

Example 4 - Display a conformal plot

Maple can display a conformal plot of a complex expression mapped onto a two-dimensional grid or plotted on the Riemann sphere in 3-D.

Enter an expression, then launch the Interactive Plot Builder from the context panel:

1. Enter the expression **z^3**.
2. Launch the Plot Builder from the context panel.
3. Select **2-D conformal plot** as the plot type.
4. Change the range of the **z** parameter to **0** .. **2+2*I.**
5. From the **axes style** box, ensure **normal** is selected.
6. Select 2-D Options from the plot options list box, then enter [30,30] as the **grid size.**

Example 5 - Display a plot in polar coordinates

Cartesian (ordinary) coordinates is the Maple default. Maple also supports numerous other coordinate systems, including hyperbolic, inverse elliptic, logarithmic, parabolic, polar, and rose in two-dimensions, and bipolar cylindrical, bispherical, cylindrical, inverse elliptical cylindrical, logarithmic cosh cylindrical, Maxwell cylindrical, tangent sphere, and toroidal in three-dimensional plots. For a complete list of supported coordinate systems, refer to the **coords** help page.

Enter an expression, then launch the Interactive Plot Builder from the context panel:

1. Enter the expression **1+4*cos(4*theta)**.
2. Launch the Plot Builder from the context panel.

Select the plot type and change the x-axis range:

2. Select 2-D polar plot as the plot type.
3. Change the angle of **theta** to **0 .. 8*Pi**.

Change the plot color:

3. From the **color** group box, select **magenta**.

Note: You can toggle the **show command** option to view the Maple syntax used to generate this plot.

Context Panel

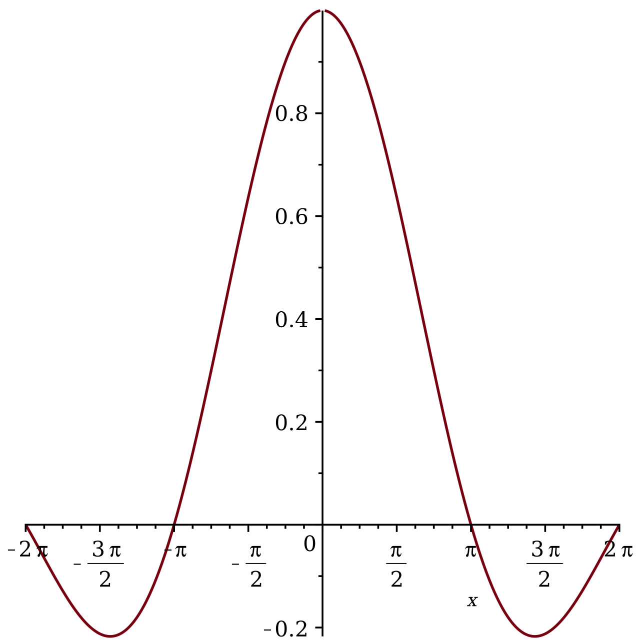
The context panel is a collection of tools and operations that are appropriate for a particular expression. The plotting options in the context panel change according to the expression, table, or region that you click on.

When you invoke the **Interactive Plot Builder** through the context panel, the expression automatically passes to the builder, and Maple does not display the **Specify Expression** window.

One advantage of using the context panel is the simplicity of creating an expression using the tools and operations in the panel. By using this method, you do not need any knowledge of plot command syntax.

1. Enter and evaluate an expression, for example,

$$\frac{d}{dx}\sin(x^2)$$



2. Click the expression.
3. From the context panel, select **Plots → 3-D Plot → x,y**.

> $\int \sin(x^2)\,dx,$

$$plot\left(\left[\sin(x^2), \frac{d}{dx}\sin(x^2), \int \sin(x^2)\,dx\right], x = -\pi..\pi\right) \qquad (6.1)$$

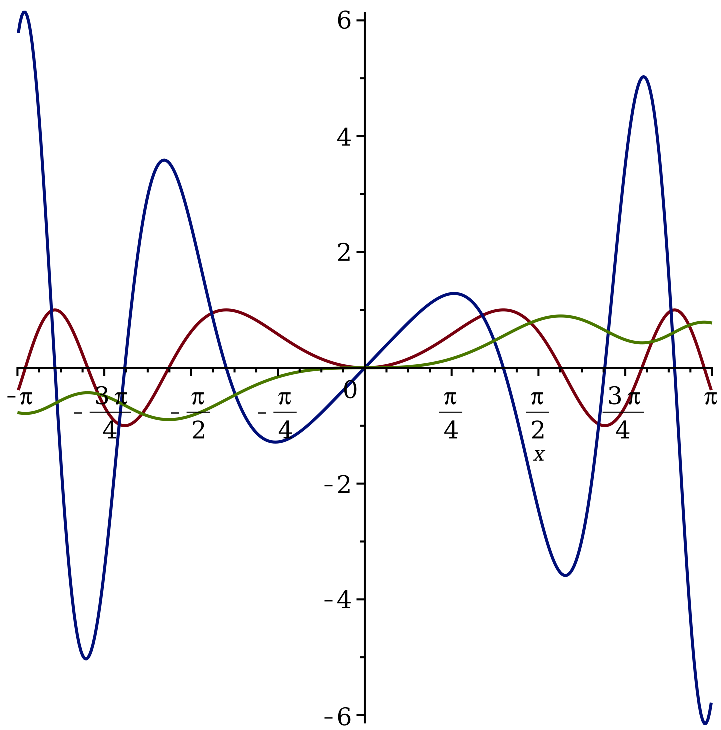


Figure 6.1: Plot an Expression Using the Context Panel

For information on customizing plots using the context menu, see *Context Panel Options (page 207)*.

The plot and plot3d Commands

The final method for creating plots is entering plotting commands.

The main advantages of using plotting commands are the availability of all Maple plot structures and the greater control over the plot output. Plot options are discussed in *Customizing Plots (page 206)*.

Table 6.1: The plot and plot3d Commands

plot(plotexpression, x=a..b, ...)
plot3d(plotexpression, x=a..b, y=a..b, ...)
• **plotexpression** - expression to be plotted
• **x=a..b** - name and horizontal range
• **y=a..b** - name and vertical range
Note: It's possible to not specify the ranges for the variables, in which case Maple determines a reasonable domain.

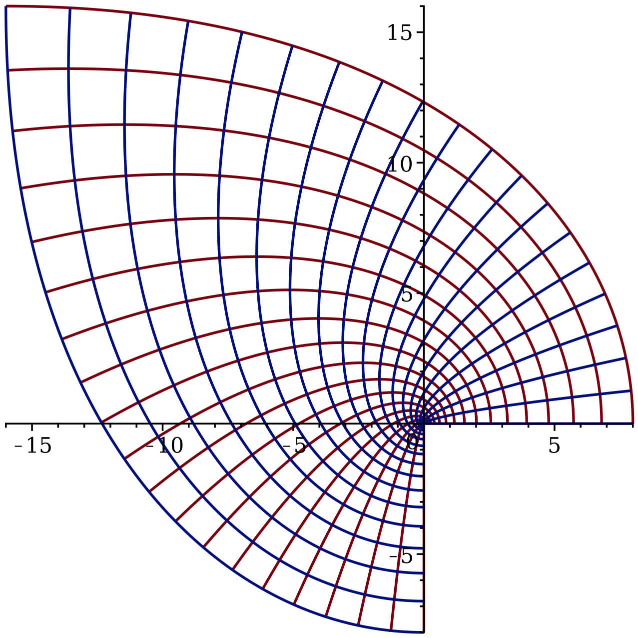
Maple commands from Creating Plots: Interactive Plot Builder

Example 1 - Display a plot of a single variable expression

> $plot3d\left(\frac{1+\sin(x\,y)}{x^2+y^2}, x=-2\,\pi\,..2\,\pi, y=-2\,\pi\,..2\,\pi, view=0\,..0.5, lightmodel=light1, shading=zgrayscale, style=patchnogrid, grid=[40, 40]\right)$

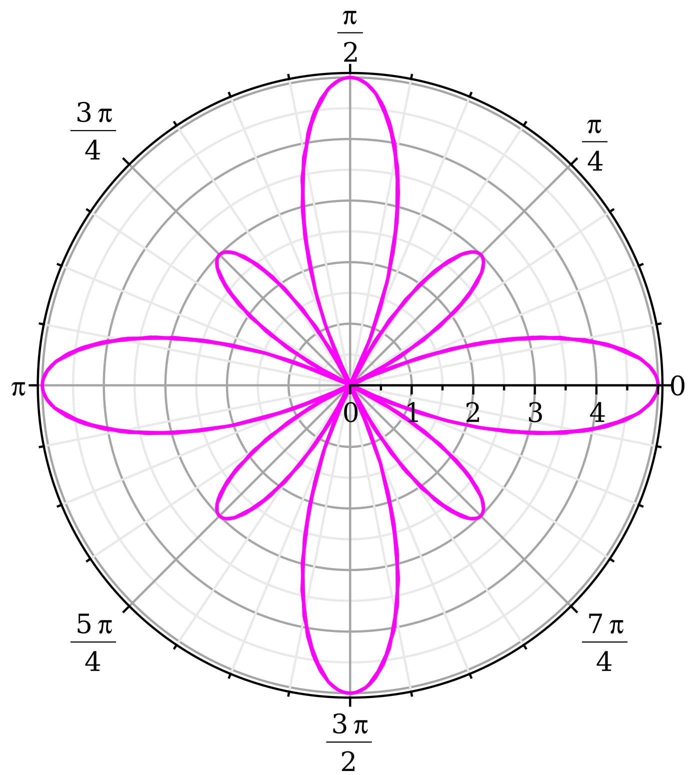
Example 2 - Display a plot of multiple expressions in 1 variable

To display multiple expressions in a plot, include the expressions in a list. To enter $plots[conformal](z^3, z = 0..2 + 2\,I, axes=normal, grid = [20, 20])$ and



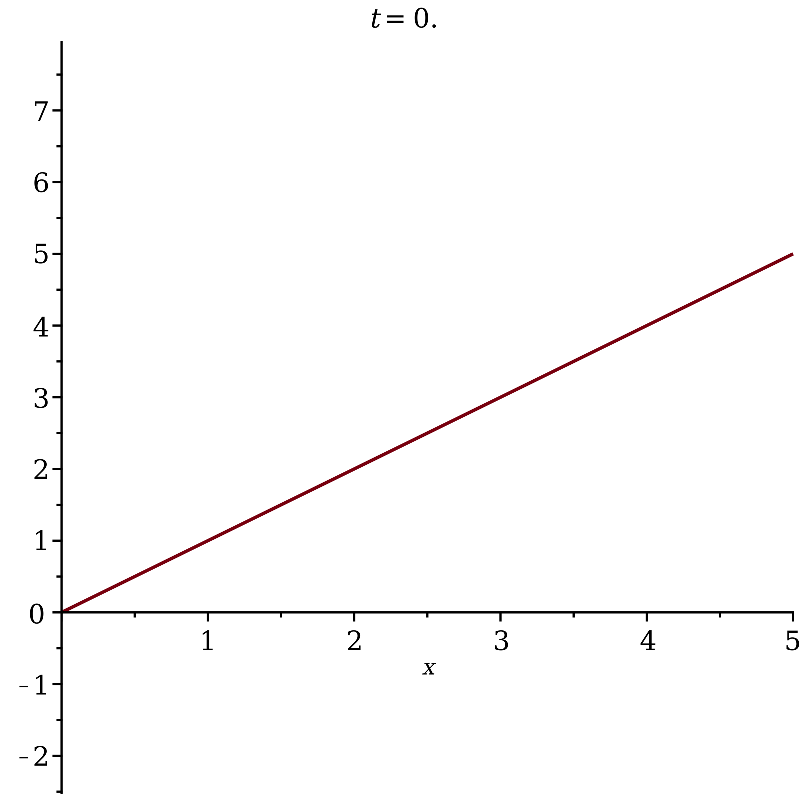
use the **Expression** palette. For more information, see *Palettes (page 15)*.

> *plots*[*polarplot*]$(1 + 4\cos(4\,\theta), \theta = 0\,..8\,\pi, \textit{color} = \textit{magenta})$



Example 3 - Display a plot of a multi-variable expression

> *plots*[*animate*](*plot*, [x+3 sin(x t), x = 0 ..5], t = 0 ..10)



Example 4 - Display a conformal plot

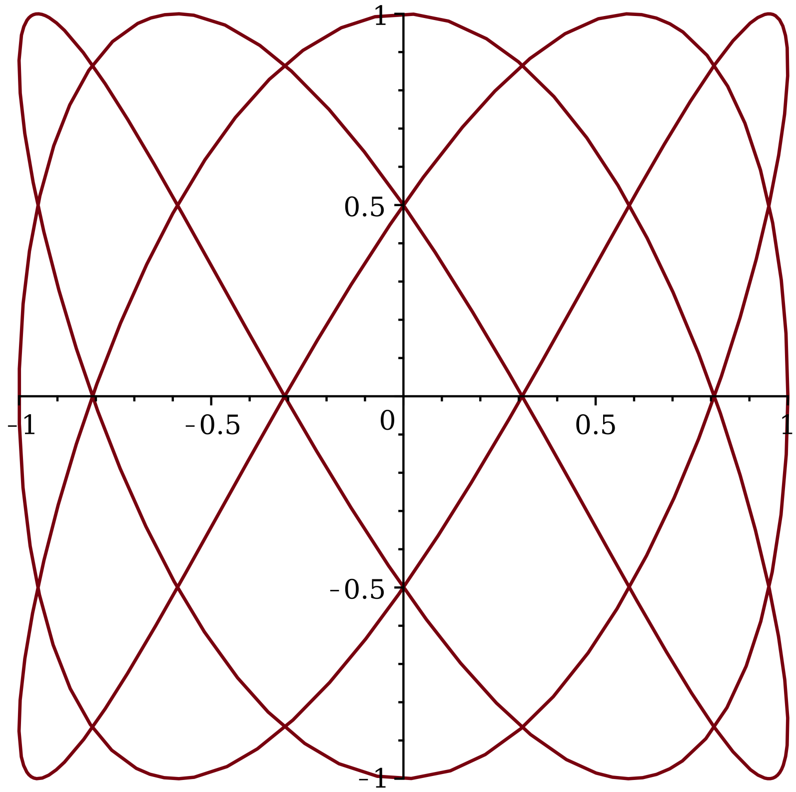
A collection of specialized plotting routines is available in the **plots** package. For access to a single command in a package, use the long form of the command.

>

$$y = f(x).$$

Example 5 - Display a plot in polar coordinates

> $plot([\cos(3\,t), \sin(5\,t), t = 0\,..2\,\pi])$



Example 6 - Interactive Plotting

> $plot3d\left(\frac{x\,y\,(x^2-y^2)}{x^2+y^2}, x=-2\,..2, y=-2\,..2, glossiness=0.5, style=patchnogrid, light=[100, 345, 0.4, 0.9, 0.7], ambientlight=[0.5, 0, 1]\right)$

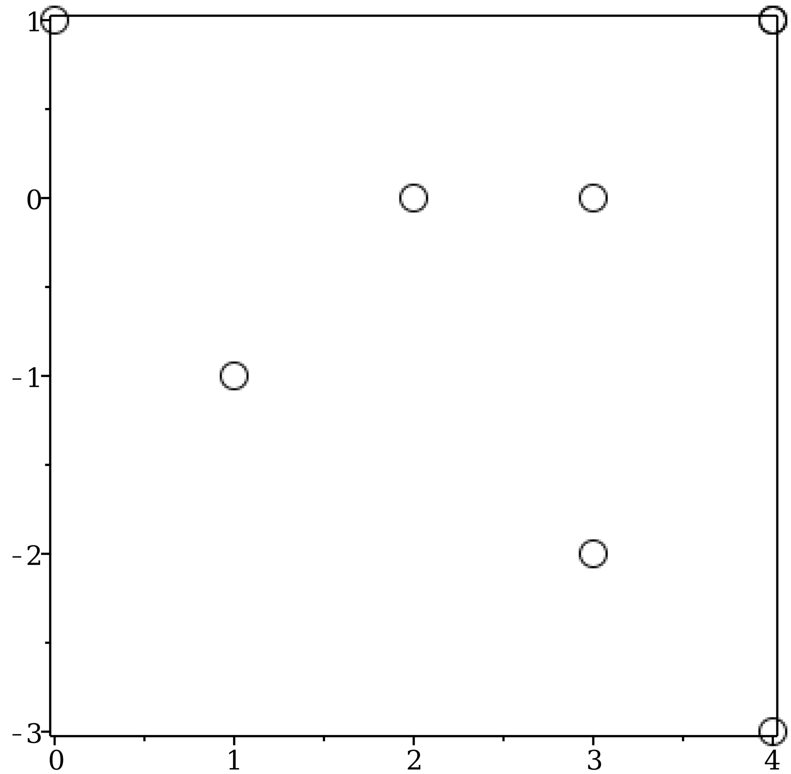
To play the animation, click the plot and the from the Animation Toolbar, select Play ( ). For information on playing the animation, see *Playing Animations (page 219)*.

For more information on the plot options used in this section, refer to the **plot/options** and **plot3d/options** help pages.

Display a Parametric Plot

Some graphs cannot be specified explicitly. In other words, you cannot write the dependent variable as a function of the independent variable, $[[x_1, y_1], [x_2, y_2], ..., [x_n, y_n]]$. One solution is to make both the x-coordinate and the y-coordinate depend on a parameter.

> *pointplot*([[0, 1], [1, −1], [3, 0], [4, −3], [2, 0], [4, 1], [3, −2], [4, 1]], *axes* = *BOXED*, *symbolsize* =25, *symbol*=*circle*)



Display a 3-D Plot

Maple can plot an expression of two variables as a surface in three-dimensional space. To customize the plot, include **plot3d** options in the calling sequence. For a list of plot options, see *The plot and plot3d Options (page 211)*.

> *with*(*LinearAlgebra*) :

$$A := HilbertMatrix(6)$$

The plots Package

The **plots** package contains numerous plot commands for specialized plotting. This package includes: **animate**, **contourplot**, **densityplot**, **fieldplot**, **odeplot**, **matrixplot**, **spacecurve**, **textplot**, **tubeplot**, and more. For details about this package, refer to the **plots** help page.

> $A := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \end{bmatrix}$

The pointplot Command

To plot numeric data, use the **pointplot** command in the **plots** package with the data organized in a list of lists structure of the form $B := ToeplitzMatrix([1, 2, 3, 4, 5, 6], symmetric)$ By default, Maple does not connect the points. To draw a line through the points, use the **style = line** option. For further analysis of data points, use the **Curve Fitting Assistant** (**Tools→Assistants→CurveFitting**), which fits and plots a curve through the points. For more information, refer to the **CurveFitting[Interactive]** help page.

> $B := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 2 & 3 \\ 5 & 4 & 3 & 2 & 1 & 2 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$

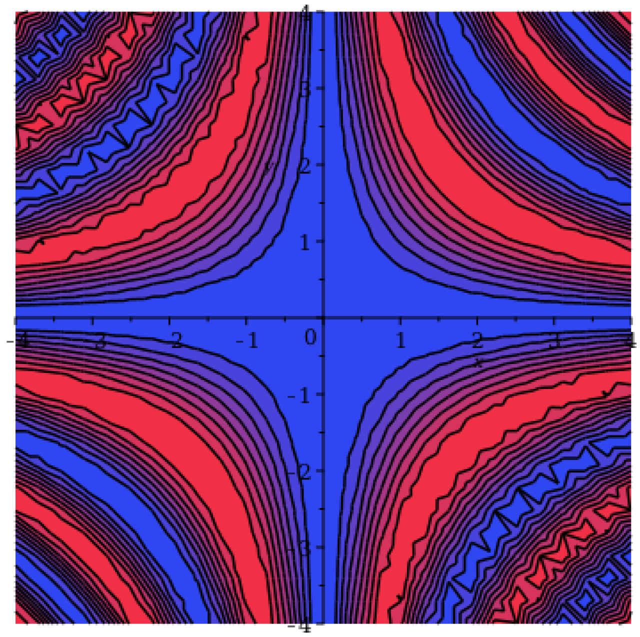
$matrixplot(A + B, heights = histogram, axes = normal, gap = 0.25, style = patch)$

The matrixplot Command

The **matrixplot** command plots the values of a plot object of type **Matrix**. The **matrixplot** command accepts options such as **heights** and **gap** to control the appearance of the plot. For more information on Matrices, see *Linear Algebra (page 116)*.

>

> *contourplot*(cos($x\,y$), $x = -4..4$, $y = -4..4$, *filled* = *true*, *numpoints* = 750)



> *plot*(sin(x), x)

$$\sin(2\,x)$$

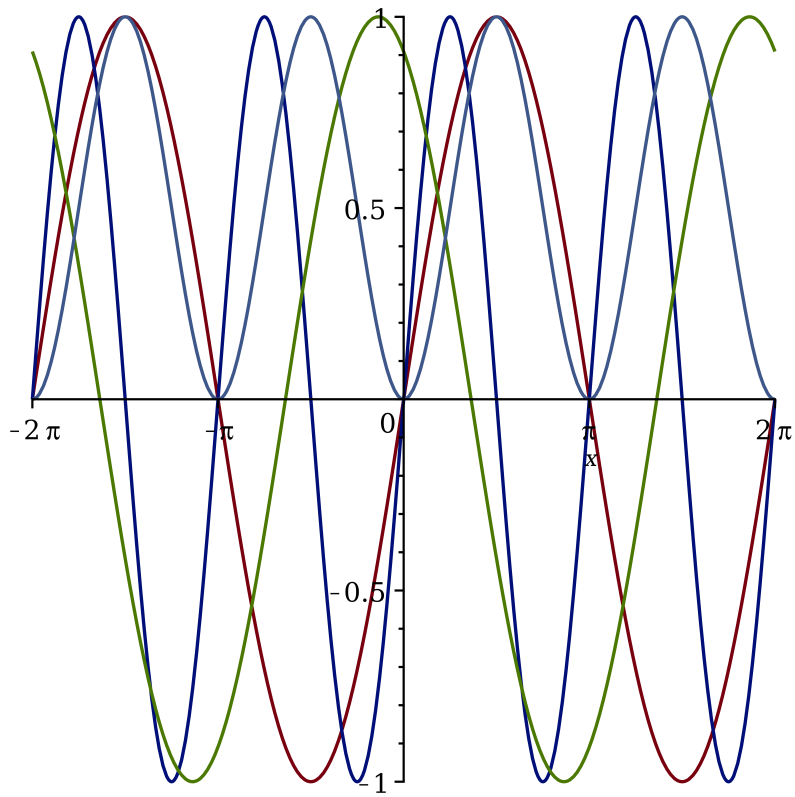
> sin(x)

$$\sin(x + 2)$$

The contourplot Command

The **contourplot** command generates a topographical map for an expression or function. To create a smoother and more precise plot, increase the number of points using the **numpoints** option.

> $\sin(x)^2$



Place your pointer over the graph to see the contour labels.

Multiple Plots in the Same Plot Region

Dragging to a Plot Region

To add an additional curve to an existing plot region, use the drag-and-drop method.

Advantages of the drag-and-drop method include the ease of adding and removing plots and the independence from plotting command syntax.

Example:

1. Enter the command $plot3d([\cos(5\,x)+\cos(5\,y), x^2+3\,y^2-4], x=-2..2, y=-1..1, shading=[zgrayscale, none], color=[default, grey], style=[patchnogrid, patch], lightmodel=light3, transparency=0.1)$ in an input region.
2. Execute.

3. Enter the expression in an input region.

4. Drag the expression onto the plot of $z := 10\left(x^2 + y^5 + \frac{x}{5}\right) e^{\left(-x^2 - y^2\right)}$:. When dragging an expression to a plot region, you can either make a copy of the expression from the input region or you can cut the expression, thereby removing it from the input region. To make a copy of the expression, select the full expression in the input region and press **Ctrl** (**Command**, Macintosh) while you drag the expression to the plot region. To cut the expression and paste it in the plot region, highlight the expression and drag it to the plot region.

4. Repeat steps 3 and 4 using the following expressions:
   hill := *plot3d*(*z*, *x* = −2 ..2, *y* = −2.5 ..2.5, *shading* = *zhue*, *style* = *patchnogrid*, *lightmodel* = *light3*, *orientation* = [−125, 60]) : and
   xt := cos(*t*) : .

$$yt := 2\sin(t) :$$

Tip: To remove an expression from the plot region, drag-and-drop the expression plot from the plot region to a Maple input region.

List of Expressions

To display multiple expressions in the same plot region, enter the expressions in a **list** data structure. To distinguish the surfaces, apply different shading options, styles, or colors to each surface.

> *curve* := *spacecurve*([*xt*, *yt*, 10], *t* = 0 ..10, *color* = *red*, *thickness* = 2) :

$$zt := subs(\{x = xt, y = yt\}, z) :$$

The display Command

To display different types of plots in the same plot region, use the **display** command in the **plots** package.

This example plots a curve over a hill with the shadow of the curve projected onto the hill.

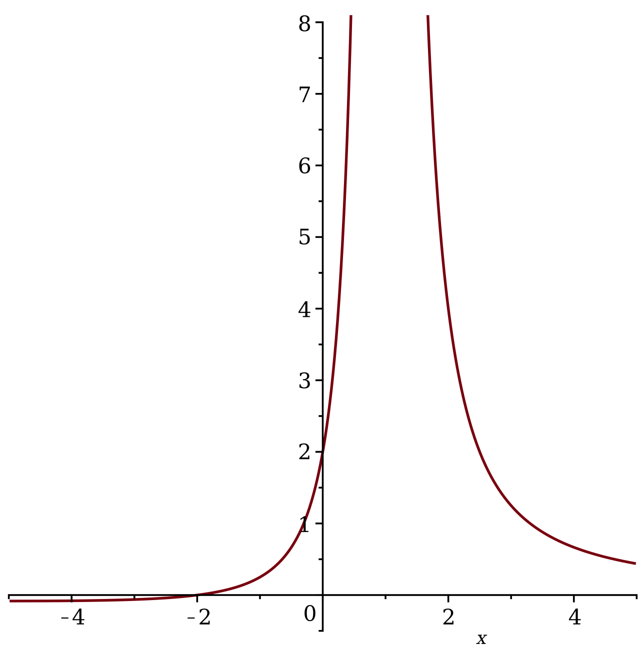
> *shadow* := *spacecurve*([*xt*, *yt*, *zt*], *t* = −π..π, *color* = *black*, *thickness* = 2):

> *display*(*hill*, *curve*, *shadow*)

>

> $plot\left(\frac{x+2}{(x-1)^2}, x=-5..5\right)$

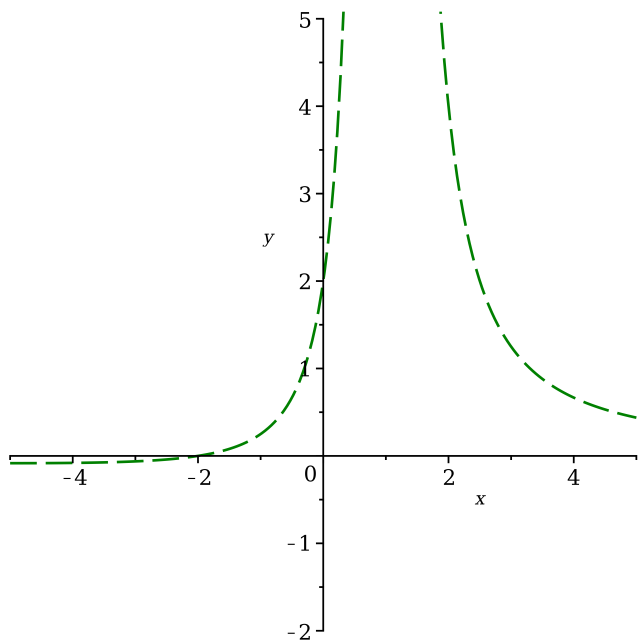
Maple can draw curves in three-dimensional space.



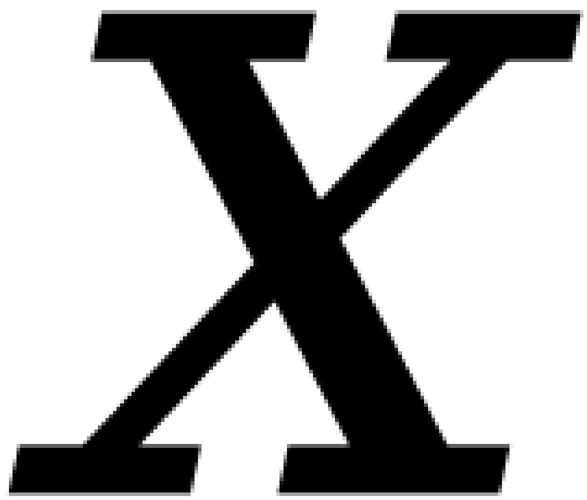
>

> x

> y


5
4
3
y
2
1
-4
-2
0
2
4
x
-1
-2

>



6.3 Customizing Plots

Maple provides many plot options to display the most aesthetically pleasing, illustrative results. Plot options include line styles, colors, shadings, axis styles, and titles where applicable. Plot options are applied using the **Interactive Plot Builder**, the context menus, or as options in the command syntax.

Interactive Plot Builder Options

The **Interactive Plot Builder** offers most of the plot options available in Maple in an easy-to-use interface.

Example:

Enter the expression, then launch the Plot Builder from the context panel:

1. Enter the expression **2*x^5-10*x^3+6*x-1**.
2. Launch the Plot Builder from the context panel. For information on interacting with the **Interactive Plot Builder**, see *Example 1 - Display a plot of a single variable expression (page 185)*.

Set the x-axis range:

3. Select 2-D Plot as the plot type.
4. Change the x-axis range to **-2 .. 2**.

In the Basic Options window:

5. From the **line style** list box, select **dot**.
6. From the **color** list box, select **blue**.
7. From the **axes style** list box, select **frame**.

In the Axes and Text window:

8. In the text field beside **title**, enter **My Plot**.

Context Panel Options

Using the options in the context panel, you can alter a plot by clicking the plot output. You can also access a large subset of plot options using the **Plot** toolbar and **Plot** menu options. These menus display when a plot region is selected. Regardless of the method used to insert a plot into Maple, you can use the context menu to apply different plot options. For a list of options available when plotting in two and three dimensions, see *The plot and plot3d Options (page 211)*.

2-D Plot Options

Consider this example:

> y

$$y = \frac{x+2}{(x-1)^2}$$

There is a singularity at x = 1. What are the $plot3d\left(\frac{x\,y}{x^2+y^2}, x=-10\,..10, y=-5\,..5\right)$ and

-intercepts? It's hard to see in the current view. If you

change the range, the locations of the intercepts are easier to see.

Alter the y-axis range:

1. Click the plot region. From the context panel, select **Axes**, and then **Properties**.
2. In the **Axes Properties** dialog, click the **Vertical** tab.
3. Clear the **Use data extents** check box and enter **-2** and **5** in the **Range min** and **Range max** text regions, respectively.
4. Click **Apply** to view the changes, or **OK** to return to the document.

Change the color:

5. Click on the curve. **Note:** The curve is selected when it becomes highlighted.
6. Select **Color**, and then **Green**.

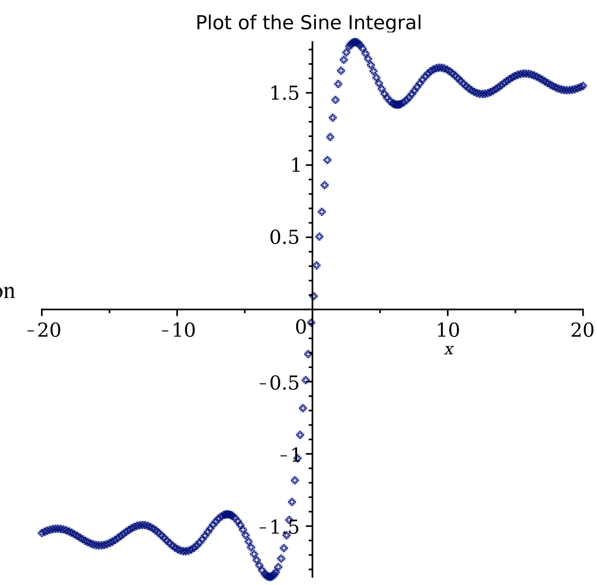
Note: The **Niagara** colors listed form the **default color palette** in Maple.

Change the line style:

7. Select **Line**, and then **Dash**.

The -intercept is at x=-2, and the -intercept is at y=2. This can be verified using the equation .

$plot(\mathrm{Si}(x), x=-20..20, title = \text{"Plot of the Sine Integral"}, titlefont = [HELVETICA, 12], color = \text{"Niagara 2"}, style = point)$



3-D Plot Options

By default, Maple displays the graph as a shaded surface with a wireframe and scales the plot to fit the window. To change these options, use the context menu.

$$> plot3d\left(\frac{x\,y^2}{x^2+y^4}, x=-10..10, y=-10..10, axes=boxed, numpoints=1500, lightmodel=light3, shading=zgrayscale, orientation=[160, 20], style=patchnogrid\right)$$

Maple has many preselected light source configurations.

Change the style:

1. Click the plot region. From the context panel select **Style** → **Surface**. The wireframe is removed.

Apply a light scheme:

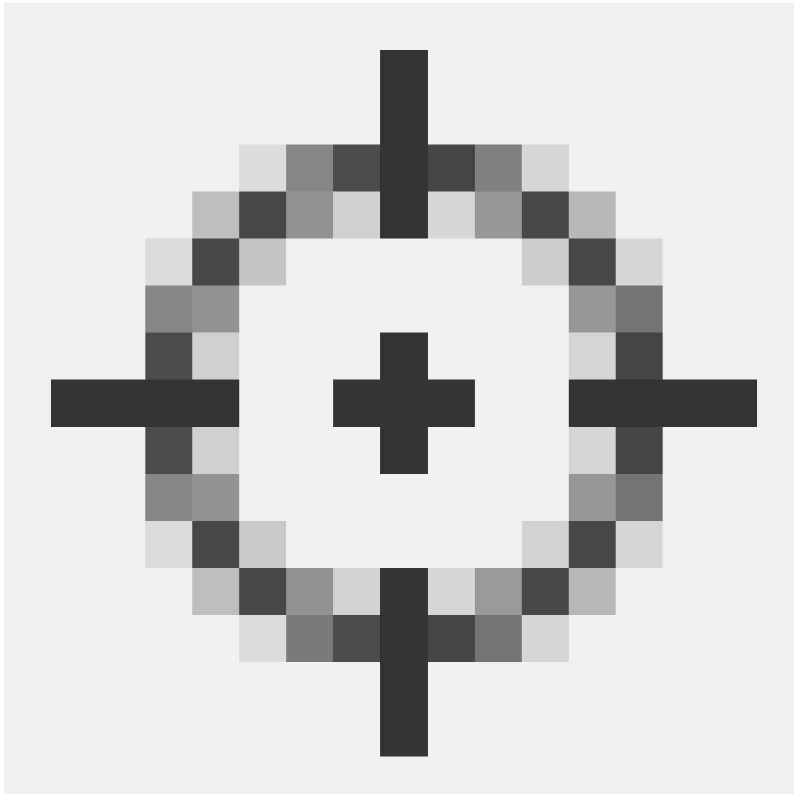
2. Select **Lighting** → **Light 1**.

Change the axes style:

3. Select **Axes** → **Boxed**.

Alter the glossiness:

4. Select **Glossiness** and then select **Set...**. Using the slider, adjust the level of glossiness.



The plot and plot3d Options

If you are using commands to insert a plot, you can specify plot options as arguments at the end of the calling sequence. You can specify the options in any order. Applying plot options in the command syntax offers a few more options and greater control than what is available in the **Interactive Plot Builder** and the context panel.

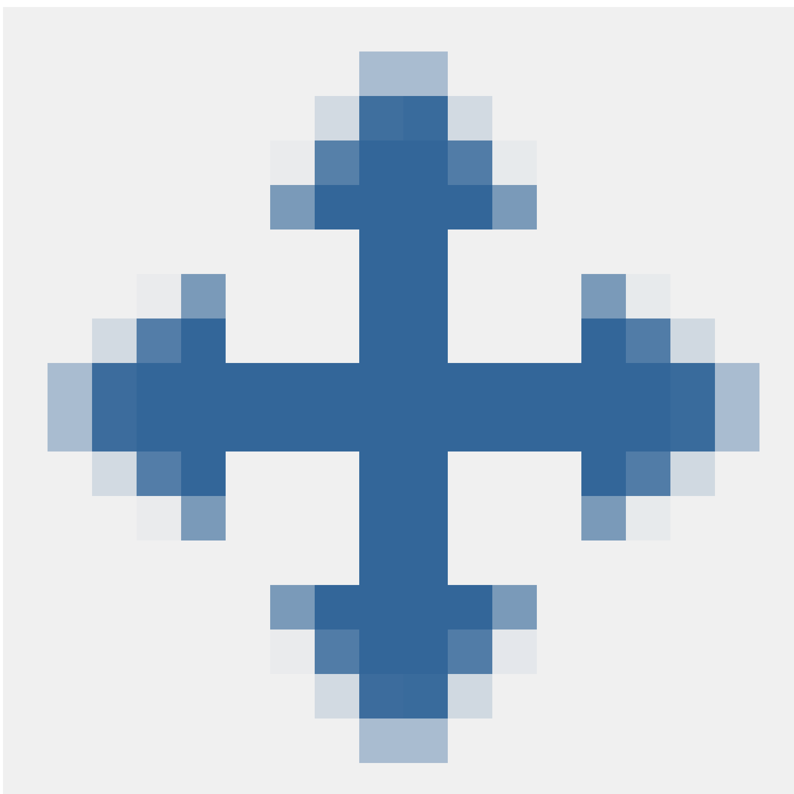
Table 6.2: Common Plot Options

Option	Description
axes	Defines the type of axes, one of: **boxed**, **frame**, **none**, or **normal**
caption	Defines the caption for the plot
color	Defines a color for the curves to be plotted
font	Defines the font for text objects in the plot
glossiness (3-D)	Controls the amount of light reflected from the surface
gridlines (2-D)	Defines gridlines in the plot
lightmodel (3-D)	Controls the light model to illuminate the plot, one of: **none**, **light1**, **light2**, **light3**, or **light4**
linestyle	Defines the dash pattern used to render lines in the plot, one of: **dot**, **dash**, **dashdot**, **longdash**, **solid**, **spacedash**, and **spacedot**
legend (2-D)	Defines a legend for the plot
numpoints	Controls the minimum total number of points generated
scaling	Controls the scaling of the graph, one of: **constrained** or **unconstrained**
shading (3-D)	Defines how the surface is colored, one of: **xyz**, **xy**, **z**, **zgrayscale**, **zhue**, or **none**

Option	Description
size (2-D)	Specifies the size (or ratio) of the plot window
style	Defines how the surface is to be drawn, one of: **line**, **point**, **pointline**, **polygon**, or **polygonoutline** for 2-D plots; **contour**, **point**, **pointline**, **surface**, **surfacecontour**, **surfacewireframe**, **wireframe**, or **wireframeopaque** for 3-D plots
symbol	Defines the symbol for points in the plot, one of: **asterisk**, **box**, **circle**, **cross**, **diagonalcross**, **diamond**, **point**, **solidbox**, **solidcircle**, or **soliddiamond** for 2-D plots; **asterisk**, **box**, **circle**, **cross**, **diagonalcross**, **diamond**, **point**, **solidsphere**, or **sphere** for 3-D plots
title	Defines a title for the plot
thickness	Defines the thickness of lines in the plot
transparency (3-D)	Controls the transparency of the plot surface
view	Defines the minimum and maximum coordinate values of the axes displayed on the screen

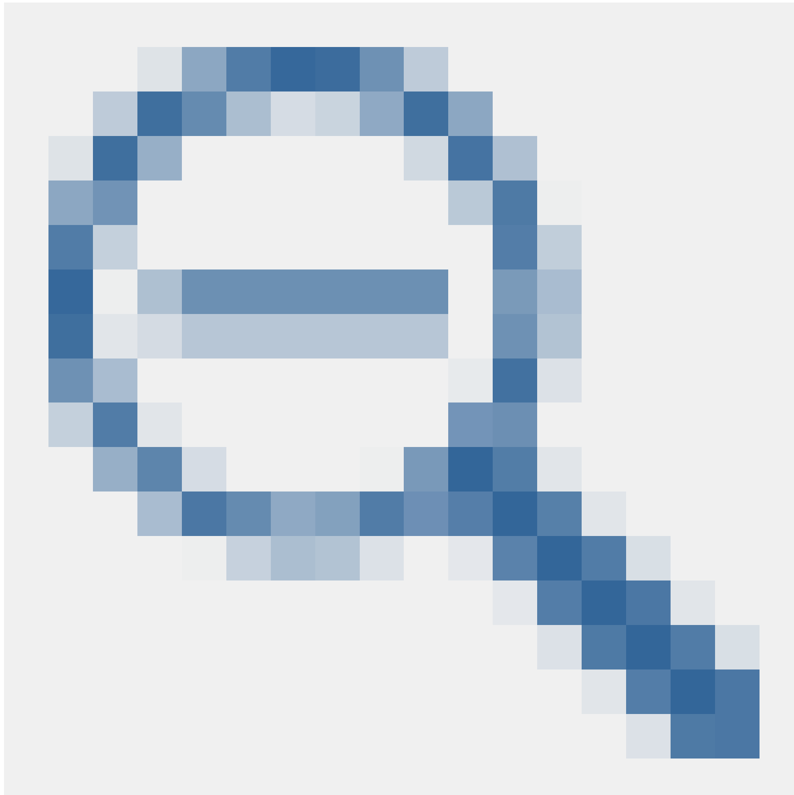
For a complete list of plot options, refer to the **plot/options** and **plot3d/options** help pages.

>



To create a smoother or more precise plot, calculate more points using the **numpoints** option.

> 



6.4 Analyzing Plots

Point Probe, Rotate, Pan, and Zoom Tools

To gain further insight into a plot, Maple offers various tools to analyze plot regions. These tools are available in the **Plot menu**, **Context Bar**, and in the context menu (under **Manipulator** and **Probe Info**) when the plot region is selected).

Table 6.3: Plot Context Bar Analysis Options

Name	Icon	Description
Point probe		Select a curve. For 2-D plots, select a coordinate type from the Selection Tool menu to display coordinates.
Selection Tool	*with*(*plots*):	Use the Selection Tool to select the information displayed in the point probe tool tooltip. You can choose to display coordinates derived from converted pixel coordinates or data points derived from the original data points.
Rotate (3-D)		Rotate a three-dimensional plot to see it from a different point of view.

Name	Icon	Description
Pan		Pan the plot by changing the view ranges for 2-D plots; plots are redrawn to reflect the new view. Change the position of the plot in the plot region for 3-D plots.
Zoom in		Zoom into the plot; plots are redrawn to reflect the new view. Also, scale the plot by placing the pointer over the plot and then rotating the wheel button. Zoom into a selected region by using a click-and-drag operation to select a rectangle.
Zoom out		Zoom out of the plot; plots are redrawn to reflect the new view. Also, scale the plot by placing the pointer over the plot and then rotating the wheel button.
Reset view		Reset the view to the default view of the plot.

6.5 Representing Data

The **Live Data Plots** palette has templates that allow you to represent your data in many different ways including:

- Area chart
- Bar chart
- Box plot
- Bubble plot
- Histogram
- Line chart
- Pie chart
- Scatter plot

After you select a type of plot, an interactive environment allows you to change a number of options to refine the look of your plot. As you refine your plot, Maple automatically updates the plot command with your options.

If the **Live Data Plots** palette is not displayed in the palette dock, from the main menu select **View → Palettes → Show Palette**, and then select **Live Data Plots**.

6.6 Creating Animations

Animations allow you to emphasize certain graphical behavior, such as the deformation of a bouncing ball, more clearly than in a static plot. A Maple animation is a number of plot frames displayed in sequence, similar to the action of movie frames. To create an animation, use the **Interactive Plot Builder** or commands.

Interactive Plot Builder

Creating Animations Using the Interactive Plot Builder:

Enter the expression, then launch the Plot Builder from the context panel:

1. Enter the expression **sin(i*sqrt(x^2+y^2)/10)**.
2. Select **Plot Builder** from the context panel.

In the Basic Options window:

3. Select the **Animation** radio button.
4. The default **x Axis** range is **-2*Pi** .. **2*Pi**. Change the **x Axis** range to **-6** .. **6**.
5. The default **y Axis** range is **-2*Pi** .. **2*Pi**. Change the **y Axis** range to **-6** .. **6**.
6. Select i as the animation parameter from the drop-down list and the range to **1** .. **30**.
7. From the **Style** group box, select **surface**.
8. From the **Color** group box, in the **Shading** drop-down menu, select **z (grayscale)**.

In the **3-D Options** window:

9. In the **Scaling** drop-down list, select **constrained**.

Build the Animation:

10. Click **Build Animation**
11. Click on the plot in the worksheet.
12. From the animation toolbar, click **Start/Resume playing the animation**.

For information on playing the animation, see *Playing Animations (page 219)*. To see the Maple syntax used to generate this plot, see *Maple Syntax for Creating Animations: Interactive Plot Builder Example (page 216)*.

The plots[animate] Command

You can also use the **animate** command, in the **plots** package, to generate animations.

Table 6.4: The animate Command

animate(plotcommand, plotarguments, t=a..b, ...)
animate(plotcommand, plotarguments, t=L, ...)
• **plotcommand** - Maple procedure that generates a 2-D or 3-D plot
• **plotarguments** - arguments to the plot command
• **t=a..b** - name and range of the animation parameter
• **t=L** - name and list of real or complex constants

To access the command, use the short form name after invoking the **with(plots)** command.

> $plot3d\left(1.3^{x}\sin(y),\ x=-1\,..2\,\pi,\ y=0\,..\pi,\ coords=spherical,\ style=patch,\ viewpoint=["circleleft"]\right)$

Maple Syntax for Creating Animations: Interactive Plot Builder Example

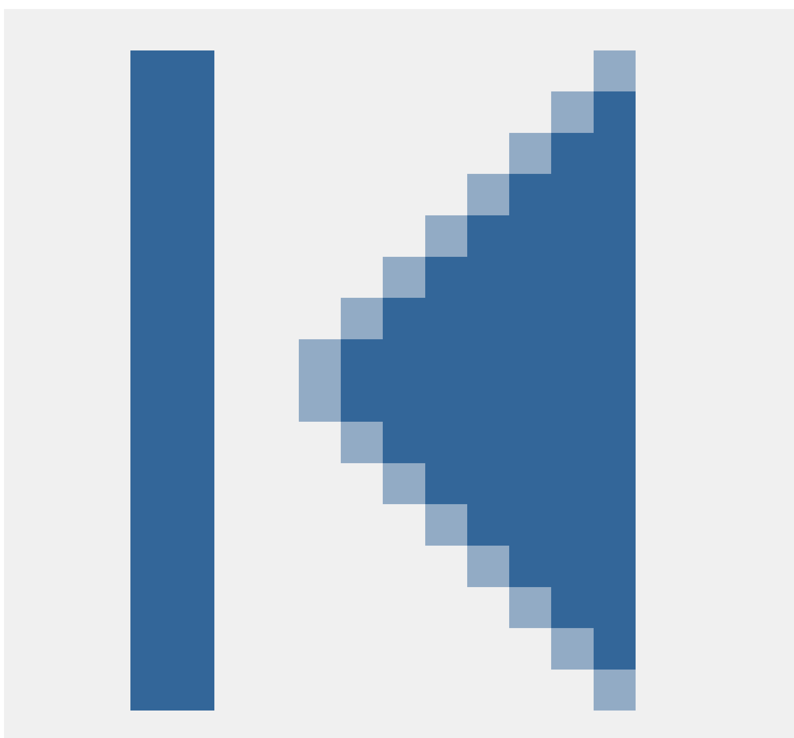
The following example shows the plotting command returned by the example in *Interactive Plot Builder (page 215)*.

>

$plot3d\left(\sin(x+y), x=-1..1, y=-1..1, shading=xyz, viewpoint=\left[path=\left[\left[50\,x, 90\cos(x), 100\sin(x)\right], x=-2\,\pi..\pi\right]\right]\right)$

Animate a 2-D plot

>



For more information on the **animate** command, refer to the **plots[animate]** help page.

The plot3d(...,viewpoint) Option

You can use the **viewpoint** option to create an animation in which the position from which you view a 3-D plot moves in all directions and in various angles around the plot surface based on coordinates and parameters you specify. This type of animation creates the effect of flying through, around, beside, towards, and away from a plot surface in three-dimensional space.

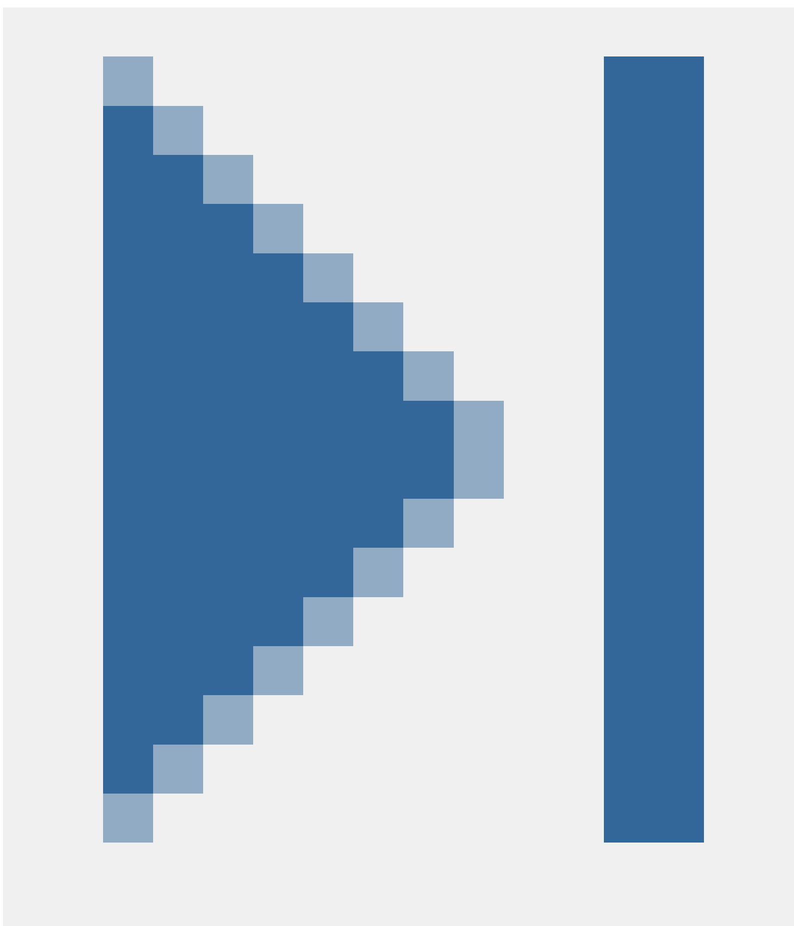
The moveable position from which you view the surface is called the *camera*. You can specify the orientation of the camera to view different sides of a surface, the path along which the camera moves throughout and around a surface, and the location of the camera in 3-D space in each animation frame. For example, you can specify coordinates to move the camera to specific points beside a surface; a pre-defined camera path to move the camera in a circle around the surface; and the range of view to move the camera close to or away from the surface. Refer to the **viewpoint** help page for more information.

To animate the following examples, click the plot object and then click the play button ( ) in the **Animation** context bar.

Example 1: Moving the Camera Around a 3-D Plot

In the following example, a pre-defined path **circleleft** moves the camera in a counter-clockwise circle around the plot surface.

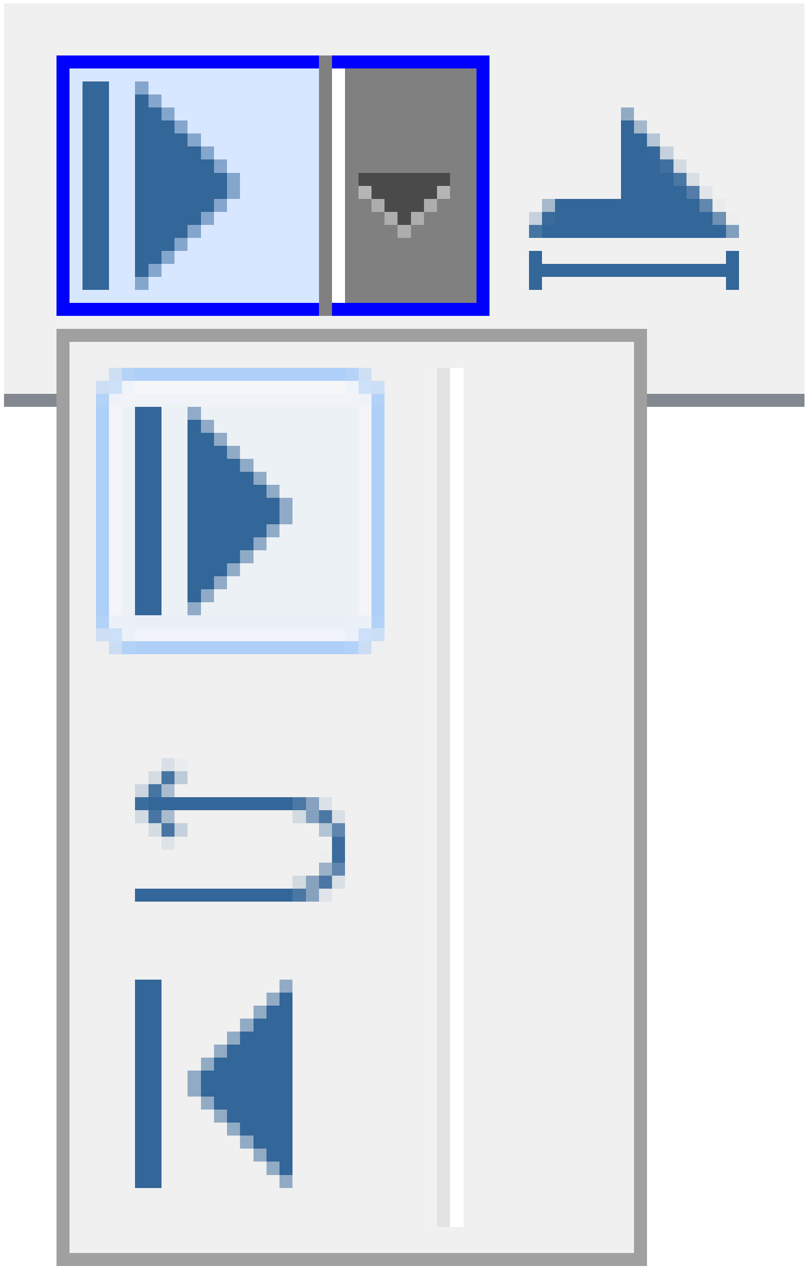
>



Example 2: Specifying a Path to Move the Camera Towards and Around a 3-D Plot

In the following example, a camera path is specified to zoom into and view different sides of the plot surface.

> 

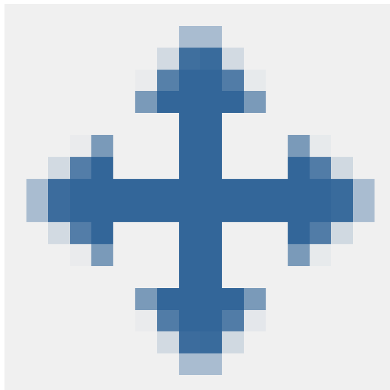
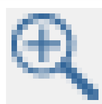
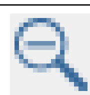
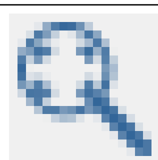


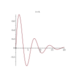
6.7 Playing Animations

Animation Context Bar

To run the animation, click the plot to display the **Animate** context bar.

Table 6.5: Animation Options

Name	Icon	Description
Previous Frame		View the previous frame in the animation.
Stop		Stop the animation.
Play		Play the selected animation.
Next Frame		View the next frame in the animation.
Current Frame		Slider control for viewing individual frames of an animated plot.
Forward Oscillate Backward		Forward - Play the animation forward. Oscillate - Play the animation forward and backward. Backward - Play the animation backward.
Single Continuous		Single - Run the animation in single cycle mode. The animation is displayed only once. Continuous - Run the animation in continuous mode. The animation repeats until you stop it.
Frames per second		Set the animation to play at a faster or slower speed.
Point probe		Select a curve. For 2-D plots, select a coordinate type from the Selection Tool menu to display coordinates.
Execute click and drag code (2-D)		In an embedded plot window, use this probe to execute the specified click and drag code, if any.
Pan		Pan the plot by changing the view ranges.

Name	Icon	Description
Zoom in		Zoom into the plot. Also, scale the plot by placing the pointer over the plot and then rotating the wheel button.
Zoom out		Zoom out of the plot. Also, scale the plot by placing the pointer over the plot and then rotating the wheel button.
Reset view		Reset the view to the default view of the 2-D plot.
Selection Tool (2-D)		Use the Selection Tool to select the information displayed in the point probe tool tooltip. You can choose to display coordinates derived from converted pixel coordinates or data points derived from the original data points. To copy the coordinate data into the clipboard, from the context panel select **Probe Info → Copy data**.
Rotate (3-D)		Rotate a three-dimensional plot to see it from a different point of view.

You can also run the animation using the context menu or the **Plot** menu.

6.8 Customizing Animations

The display options that are available for static plots are also available for Maple animations.

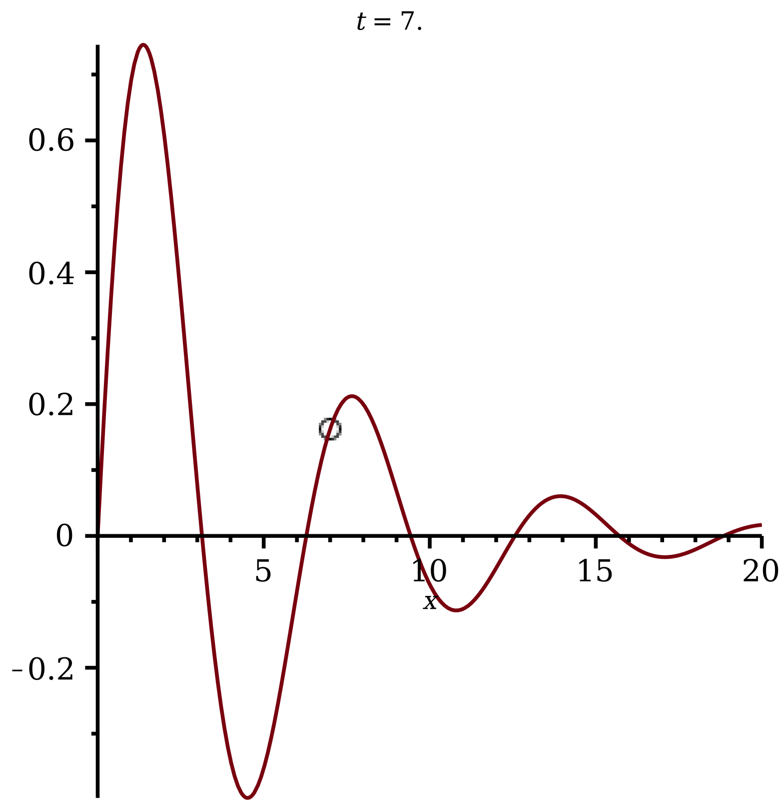
Interactive Plot Builder Animation Options

Using the **Interactive Plot Builder**, you can apply various plot options within the **Plot Options** window. See *Interactive Plot Builder (page 215)*.

Context Panel Options

As with static plots, you can apply plot options to the animation by clicking the animation output.

(6.2)



Customize the animation using the context panel:

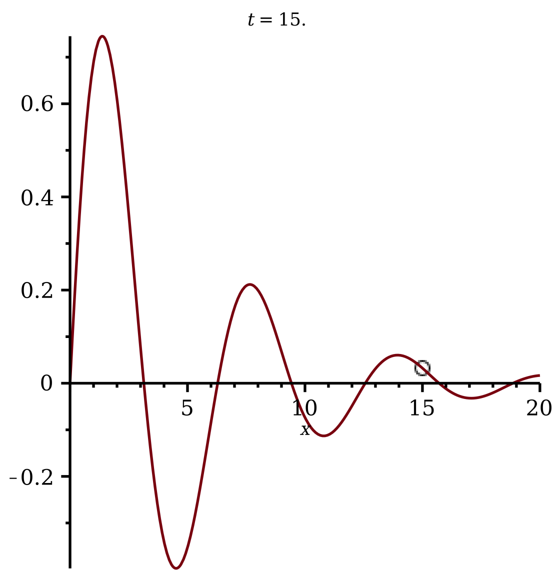
1. To change the line style, click the plot region. Select **Style** → **Point**.
2. To remove the axes, select **Axes** → **None**.

The animate Command Options

The **animate** command offers a few options that are not available for static plots. Refer to the **animate** help page for information on these additional options. By default, a two-dimensional animation consists of sixteen plots (frames) and a three-dimensional animation consists of eight plots (frames). To create a smoother animation, increase the number of frames using the **frames** option.

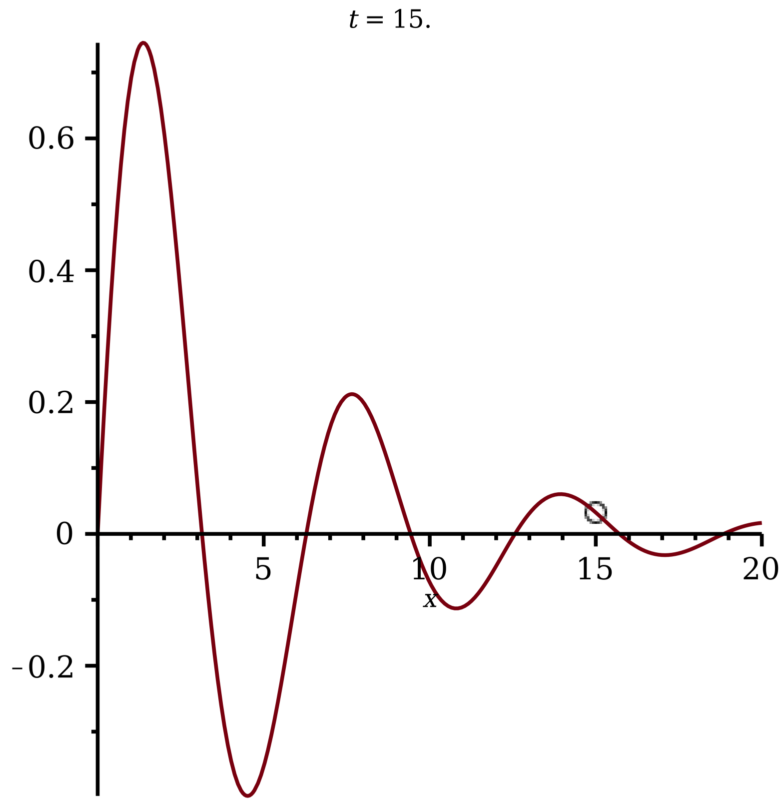
Note: Computing more frames increases time and memory requirements.

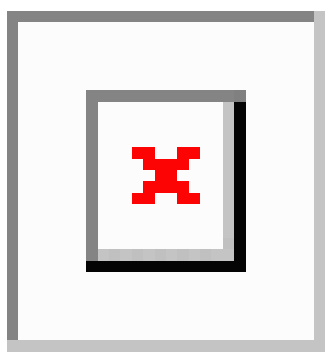
>

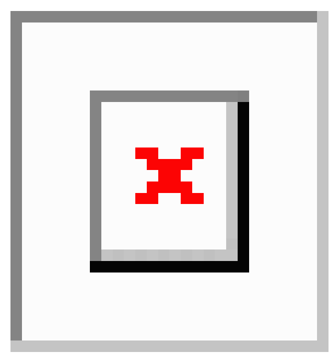

t = 15.
0.6
0.4
0.2
0
-0.2
5
10
15
20
x

>

>


t = 15.
0.6
0.4
0.2
0
-0.2
5
10
15
20
x





6.9 Exporting

You can export a generated plot or animation to an image in various file formats, including DXF and X3D (for 3-D plots), EPS, GIF, JPEG/JPG, POV, and Windows BMP. Exporting an animation to GIF produces an animated image file. The exported images can be included in presentations, web pages, Microsoft Word, or other software.

To export an image:

1. Click the plot region.
2. From the context panel, select **Export** and the file format.

Alternatively:

1. Click the plot.
2. From the **Plot** menu, select **Export**, and then the file format.

Maple has various plot drivers. By setting the **plotdevice**, a file can be automatically created without returning the image to the document. For more information, refer to the **plot,device** help page.

6.10 Code for Color Plates

Generating impressive graphics in Maple can require only a few lines of code, as shown by the examples in this chapter. However, other graphics require many lines of code. Code for the color plates is available at the Maple Application Center.

From the Help menu, select On the Web, User Resources, and then Application Center.

To access the color plate code:

1. Go to the Maple Application Center.
2. In the **Keyword or phrase** region, enter **Color Plate**.

7 Creating Mathematical Documents

Maple allows you to create powerful documents as business and education tools, technical reports, presentations, assignments, and handouts.

You can:

- Copy, cut, and paste information
- Format text for reports or course material
- Add headers and footers
- Insert images, tables, and symbols
- Generate 2-D and 3-D plots and animations
- Sketch in the document or on a plot
- Insert hyperlinks to other Maple files, websites, or email addresses
- Place instructions and equations side by side
- Bookmark specific areas
- Easily update, revise, and distribute your documents

In this chapter, we will create a document that demonstrates many of Maple's documentation features. For further examples, note that this guide was written using Maple.

7.1 In This Chapter

Section	Topics
Document Formatting (page 228) - Add various text formatting elements	• Copy and Paste • Quick Character Formatting • Quick Paragraph Formatting • Character and Paragraph Styles • Sections • Headers and Footers • Show or Hide Worksheet Content • Indentation and the Tab Key
Commands in Documents (page 241) - Format and display or hide commands in a document	• Document Blocks • Typesetting • Auto-Execute
Tables (page 245) - Create tables and modify their attributes	• Creating a table • Cell contents • Navigating table cells • Modifying Structural Layout • Modifying Physical Dimensions • Modifying Appearance • Printing Options • Execution Order

Section	Topics
Canvas (page 254) - Sketch an idea in the document by inserting a canvas	• Insert a Canvas • Drawing • Canvas Style • Inserting Images
Hyperlinks (page 258) and Bookmarks - Add hyperlinks to various sources	• Inserting a Hyperlink in the Document • Linking to an Email Address, Dictionary Topic, Help Page, Maplet Application, Webpage, or Document • Bookmarks
Embedded Components (page 263) - Insert buttons, sliders, and more in your document	• Overview of available components • Example using a task template
Spell Checking (page 265) - Verify text with the Maple spell checking utility	• How to Use the **Spellcheck** Utility • Selecting a Suggestion • User Dictionary

7.2 Document Formatting

To begin, create a new Maple document. From the **File** menu, select **New → Document Mode**. For this example, you can copy and paste text from any file. The example text below is from a Maple help page, **plot**, but the formatting has been removed for demonstration purposes.

Copy and Paste

You can cut, copy, and paste content within Maple documents, and from other sources.

To copy an expression, or part of an expression, to another location on the document:

1. Select the expression, or part of the expression, to copy. Alternatively, right-click and select **Copy**.
2. From the **Edit** menu, select **Copy**.
3. Place the cursor at the insertion point.
4. From the **Edit** menu, select **Paste**. Alternatively, right-click and select **Paste**.

Result:

plot - create a two-dimensional plot

Calling Sequence
plot(f, x)
plot(f, x=x0..x1)
plot(v1, v2)

Parameters
f - expression in independent variable x
x - independent variable
x0, x1 - left and right endpoints of horizontal range
v1, v2 - x-coordinates and y-coordinates

If you paste into a math input region, Maple interprets all the pasted content as input. If you paste into a text region, Maple interprets all the pasted content as text. However, note that 2-D Math retains its format in both input and text regions.

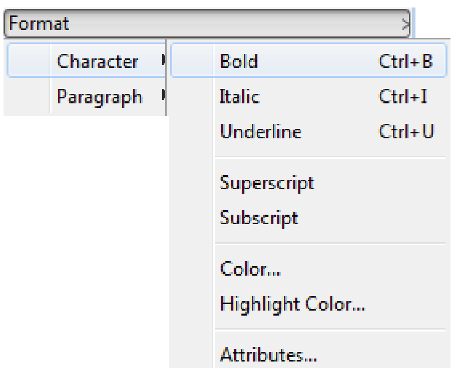
When you copy and paste to another application, in general, Maple retains the original structure.

Quick Character Formatting

The **Format→Character** menu provides access to the following quick formatting features: **Bold**, **Italic**, **Underline**, **Superscript**, **Subscript**, **Font Color**, and **Highlight Color**.

To modify text:

1. In the document, select the text to modify.
2. From the **Format** menu, select **Character**, and then the appropriate feature. Alternatively, select **Format→Character** from the context panel.
3. Select the quick formatting feature you want to apply (for example, bold or italic font).



Alternatively, use the context bar icons. For example, to apply a color to the parameters "f, x=x0..x1":

- Font Color Context Bar Icon
- Highlight Color Context Icon

For font and highlight colors, you can select from Swatches, a color wheel, RGB values, or choose a color using the eye dropper tool. See **Figure 7.1**.

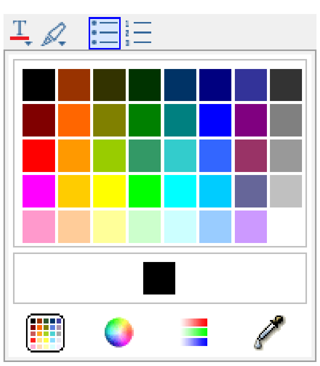


Figure 7.1: Select Color Dialog

In this example, choose a dark purple color, as in the help pages.

To format this text as bold, click the **Bold** toolbar icon, . Also, select the text "Calling Sequence" and format as bold.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(f, x)

plot(**f, x=x0..x1**)

plot(v1, v2)

Parameters

f - expression in independent variable x

x - independent variable

x0, x1 - left and right endpoints of horizontal range

v1, v2 - x-coordinates and y-coordinates

Attributes Submenu: Setting Fonts, Character Size, and Attributes

You can also change various character attributes such as font, character size, style, and color in one dialog.

To modify text:

1. In the document, select text to modify.
2. From the **Format** menu, select **Character**, and then **Attributes**. The **Character Style** dialog opens. See **Figure 7.2**.

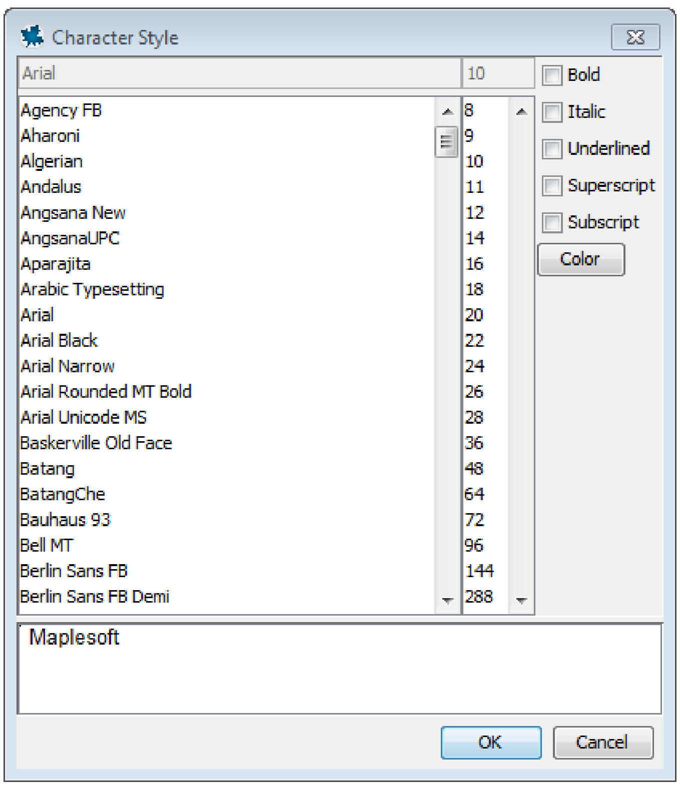


Figure 7.2: Character Style Dialog

Quick Paragraph Formatting

The **Format→Paragraph** menu provides access to the following quick alignment features: **Align Left**, **Center**, and **Align Right**.

To modify a paragraph:

1. In the document, select the paragraph to modify.
2. From the **Format** menu, select **Paragraph**, and then the appropriate feature.

Attributes Submenu: Spacing, Indent, Alignment, Bullets, Line Break, and Page Break

You can change various paragraph attributes in one dialog.

- From the **Format** menu, select **Paragraph**, and then **Attributes**. The **Paragraph Style** dialog opens. See **Figure 7.3**.
- When changing spacing, you must indicate units (inches, centimeters, or points) in the **Units** drop-down list.

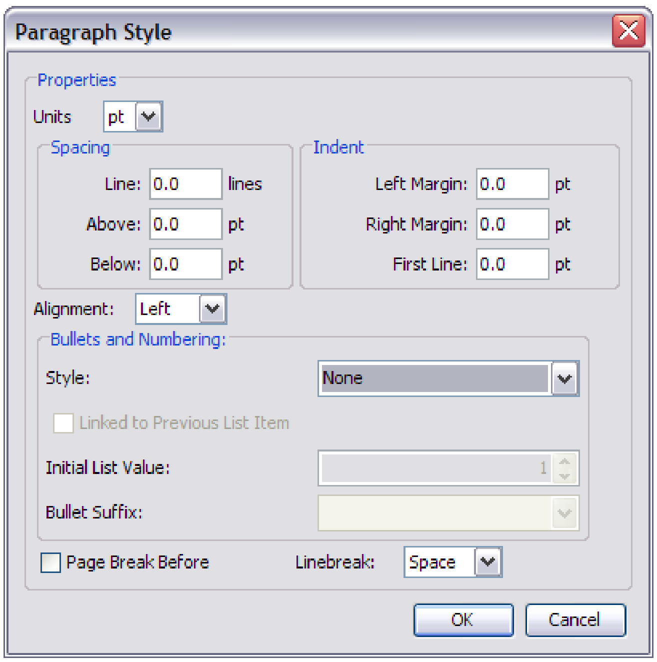


Figure 7.3: Paragraph Style Dialog

For example, in the pasted text, select all of the items under "Parameters", then open the **Paragraph Style** dialog. Notice that the spacing has already been set.

In the **Indent** section, change the **Left Margin** indent to 10.0 pt.

In the **Bullets and Numbering** section, click the **Style** drop-down and select **Dash**. Click **OK** to close the dialog and apply the styles.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(f, x)

plot(**f, x=x0..x1**)

plot(v1, v2)

Parameters

- f - expression in independent variable x
- x - independent variable
- x0, x1 - left and right endpoints of horizontal range
- v1, v2 - x-coordinates and y-coordinates

For more information, refer to the **paragraphmenu** help page.

Character and Paragraph Styles

Maple has predefined styles for characters and paragraphs. A style is a set of formatting characteristics that you can apply to text in your document to change the appearance of that text. When you apply a style, you apply a group of formats in one action.

- A **character style** controls text font, size, color, and attributes such as bold and italic. To override the character style within a paragraph style, you must apply a character style or character formatting.
- A **paragraph style** controls all aspects of a paragraph's appearance, such as text alignment, line spacing, and indentation. In Maple, each paragraph style includes a character style.

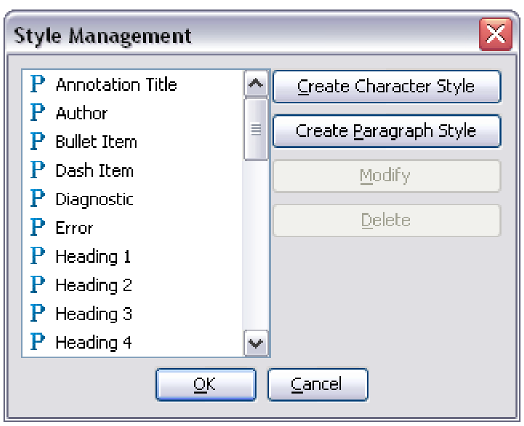


Figure 7.4: Style Management Dialog

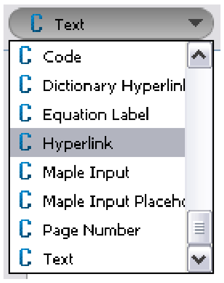
Applying Character Styles

By using the drop-down list in the document context bar, you can apply:

- Existing Maple character styles.
- New styles that you have created through the **Style Management** (**Figure 7.4**) and **Character Style** (**Figure 7.5**) dialogs.

To apply a character style to text in your document:

1. Select the text to modify.
2. In the styles drop-down list in the context bar of your document, select an appropriate character style. All character styles are preceded by the letter **C**. The selected text now reflects the attributes of the character style you have chosen.



3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

Creating and Modifying Character Styles

You can create custom character styles to apply to text, or change existing character styles. New styles are automatically added to the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See **Figure 7.4**.

To create a character style:

- Click **Create Character Style**. The **Character Style** dialog opens. See **Figure 7.5**.
- In the first row of the dialog, enter a style name in the blank text region.

To modify a character style:

- From the style list, select the character style to modify. Recall that all character styles are preceded by the letter **C**, while paragraph styles are preceded by the letter **P**.
- Click **Modify**. The **Character Style** dialog opens with the current attributes displayed. See **Figure 7.5**.

For either action, continue:

2. Select the properties for the new character style, such as font, size, attributes, and color. In the font attributes, the **Superscript** and **Subscript** check boxes are mutually exclusive. When you select one of the two check boxes, the other is disabled. You must clear one before selecting the other.

Note: A preview of the style is displayed in the last row of the **Character Style** dialog.

3. To save the style, click **OK** or to abandon, click **Cancel**. If you have modified a style, all text in your document that uses the altered style is updated to reflect the changes.

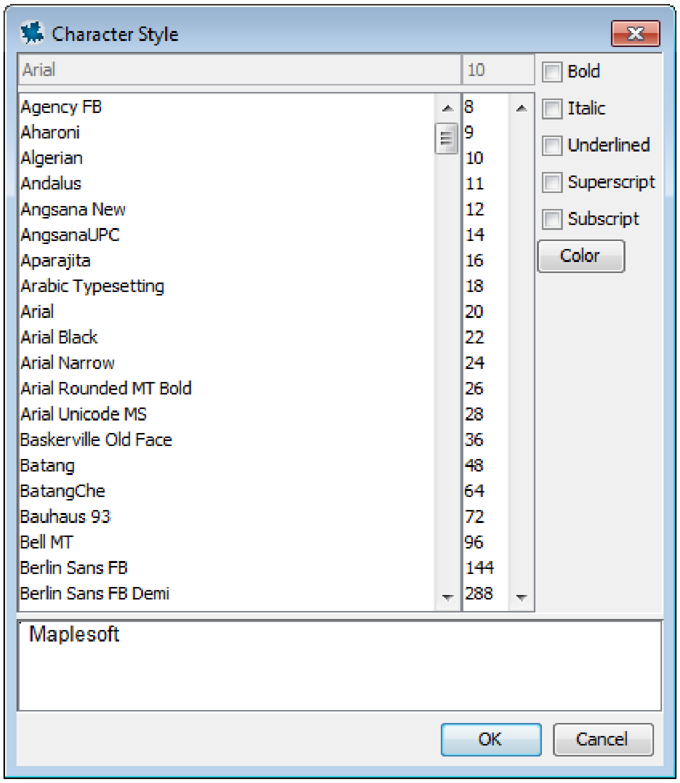
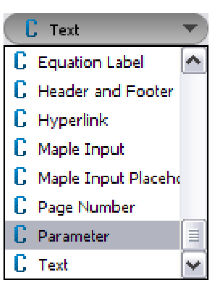


Figure 7.5: Defining a Character Style

For example, in the pasted text, suppose we want to create a character style for the bold, purple parameter.

- From the **Format** menu, select **Styles**, then click **Create Character Style**.
- Enter the style name, "Placeholder", and then select the character attributes. In this case, click the **Bold** check box. Then click the **Color** button and choose a dark purple. Click **OK** to create the character style.

Now you can apply the style to any text. Under **Calling Sequences**, select each list of parameters inside the command. To apply the style, from the **Styles** drop-down menu in the toolbar, select **Parameter**.



Result:

plot - create a two-dimensional plot

Calling Sequence

```
plot(f, x)
plot(f, x=x0..x1)
plot(v1, v2)
```

Parameters

f	-	expression in independent variable x
x	-	independent variable
x0, x1	-	left and right endpoints of horizontal range
v1, v2	-	x-coordinates and y-coordinates

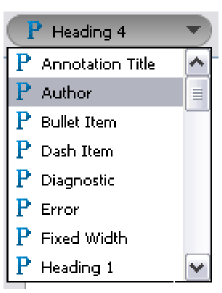
Applying Paragraph Styles

By using the drop-down list in the document context bar, you can apply:

- Existing Maple paragraph styles.
- New styles that you have created through the **Style Management** (**Figure 7.4**) and **Defining a Paragraph Style** (**Figure 7.6**) dialogs.

To apply a Maple paragraph style to text in your document:

1. Select the text to modify.
2. In the styles drop-down list in the context bar of your document, select an appropriate paragraph style. All Maple paragraph styles are preceded by the letter **P**. The selected text now reflects the attributes of the paragraph style you have chosen.



For example, to format the title of the pasted text as a title, first select the line: "plot - create a two-dimensional plot". In the Styles drop-down, select **Title**.

Result:

plot - create a two-dimensional plot

Calling Sequence

```
plot(f, x)
plot(f, x=x0..x1)
plot(v1, v2)
```

Parameters

f	-	expression in independent variable x
x	-	independent variable
x0, x1	-	left and right endpoints of horizontal range
v1, v2	-	x-coordinates and y-coordinates

3. (Optional) If necessary, you can remove this style. From the **Edit** menu, select **Undo**.

Creating and Modifying Paragraph Styles

You can create custom paragraph styles to apply to text, or change existing paragraph styles. New styles are automatically added to the styles drop-down list in the context bar of your document.

1. From the **Format** menu, select **Styles**. The **Style Management** dialog opens. See **Figure 7.4**.

To create a paragraph style:

- Click **Create Paragraph Style**. The **Paragraph Style** dialog opens. See **Figure 7.6**.
- In the first row of the dialog, enter a style name in the blank text field.

To modify a paragraph style:

- Select a paragraph style to modify. Recall that all paragraph styles are preceded by the letter **P**.
- Click **Modify**. The **Paragraph Style** dialog opens with the current attributes displayed.

For either action, continue:

4. In the **Units** drop-down menu, select the units used to determine spacing and indentation. Select from inches (**in**), centimeters (**cm**), or points (**pt**).
5. Select the properties to use for this paragraph style, such as **Spacing**, **Indent**, **Alignment**, **Bullets and Numbering**, **Page Break Before**, and **Linebreak**.
6. To add or modify a font style, click **Font**. The **Character Style** dialog opens. For detailed instructions, see *Creating and Modifying Character Styles (page 233)*.
7. To save the style, click **OK**, or to abandon, click **Cancel**. If you are modifying an existing style, all text in your document that uses the altered style is updated to reflect the changes.

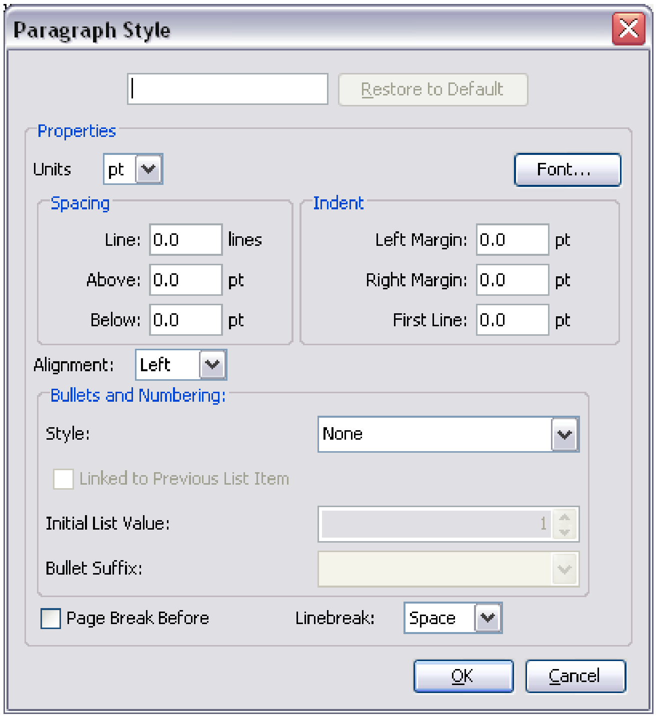


Figure 7.6: Defining a Paragraph Style

Style Set Management: Saving Styles for Future Use

You can use the style set of a particular document as the default style for all documents.

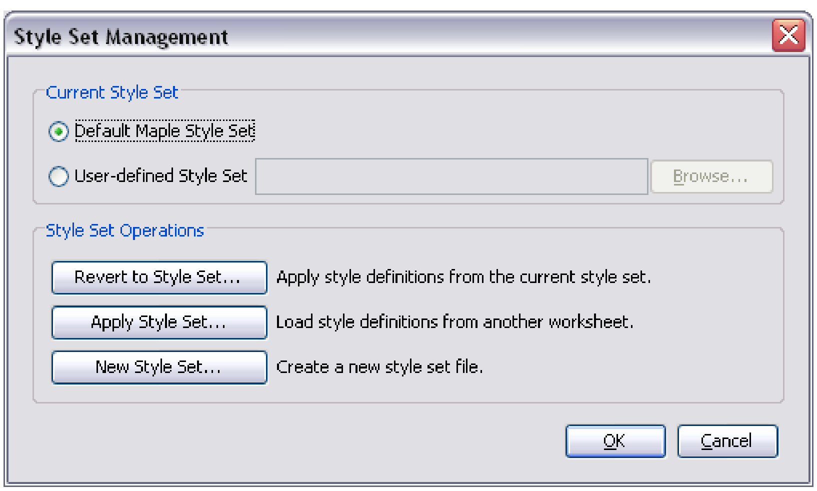
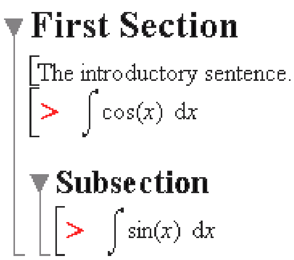


Figure 7.7: Style Set Management Dialog

For information on creating and managing style sets, see the **worksheet/documenting/styles** help page.

Sections

You can organize your document into sections, either before or after the text has been entered.



Using the Insert Menu to Add Sections

1. Place the cursor in the paragraph or execution group above the location at which you want to insert a new section.

- If the cursor is inside a section, Maple inserts the new section after the current section.
- If the cursor is in an execution group, Maple inserts the new section after the execution group.

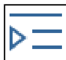
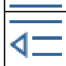
2. From the **Insert** menu, select **Section**. An arrow marks the start of the section.
3. Move the cursor to the text area beside the arrow. Enter the section heading.
4. Move the cursor to the body of the section. Enter the content.

Tips for Adding Sections and Subsections

- If you are in a section and select **Insert→Section**, a subsection is created.
- To insert a subsection immediately after the current subsection, use **Edit→Split or Join→Split Section**. The newly created section is at the same indentation level as the existing section.
- To outdent, use **Edit→Remove Section**. This decreases the indentation of a worksheet element by one level by removing one subsection that encloses the worksheet element.

Using the Indent and Outdent Toolbar Icons

You can shift sections to create or remove subsections.

	Enclose the selection in a section or subsection
	Outdent the selection to the next section level, if possible.

For example, to create two sections containing the two categories of information in the pasted text:

1. Select "Parameters" and all of the items under it.
2. Click the Indent toolbar item.
3. Cut and paste "Parameters" from inside the section to its title.
4. Similarly, create a section with the title "Calling Sequence", containing the items under that heading.

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f, x**)
plot(**f, x=x0..x1**)
plot(**v1, v2**)

Parameters

- f - expression in independent variable x
- x - independent variable
- x0, x1 - left and right endpoints of horizontal range
- v1, v2 - x-coordinates and y-coordinates

Note: The section titles are automatically formatted as section titles, but you can change the formatting through the **Paragraph Style** dialog.

Headers and Footers

You can add headers and footers to your document that will appear at the top and bottom of each page when you print the document.

To add or edit headers and footers:

From the **Insert** menu, select **Header Footer**. The **Header Footer** dialog appears. See **Figure 7.8**.

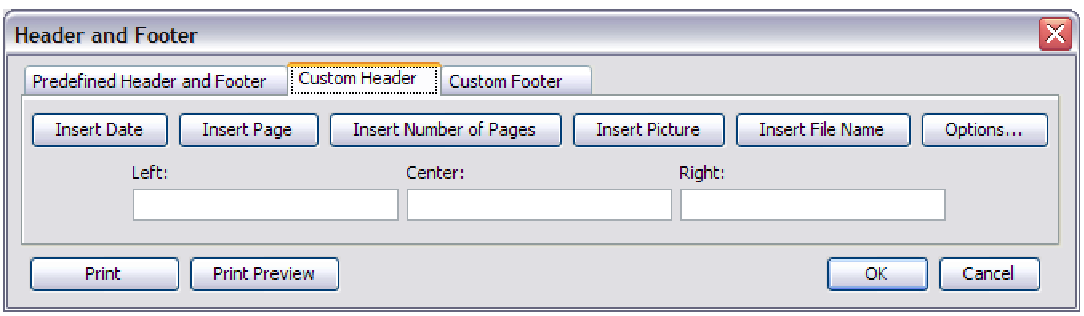


Figure 7.8: Header and Footer Dialog - Custom Header

The available elements include the current date, page number, number of pages, an image, the filename, or any plain text. These elements can be placed in the left or right corner or the center of the page.

You can choose one of the predefined header or footer styles in the **Predefined Header and Footer** tab, or create your own by clicking the **Custom Header** or **Custom Footer** tab.

For more information on header and footer options, refer to the **headerfooter** help page.

Show or Hide Worksheet Content

You can hide document elements of a specific type so that they are not visible. This does not delete them, but hides them from view. Hidden elements are not printed or exported, but they can be copied and pasted.

In a document, use the **Show Contents** dialog to hide all spreadsheets, input, output, or graphics, plus markers for section boundaries, execution group boundaries, hidden table borders on mouse pointer roll over, and annotations. The dialog is accessed from the **View→Show/Hide Contents** menu.

Using the Show Contents Dialog

A check mark beside the item indicates that all document elements of that type are displayed for the current document. See **Figure 7.9**.

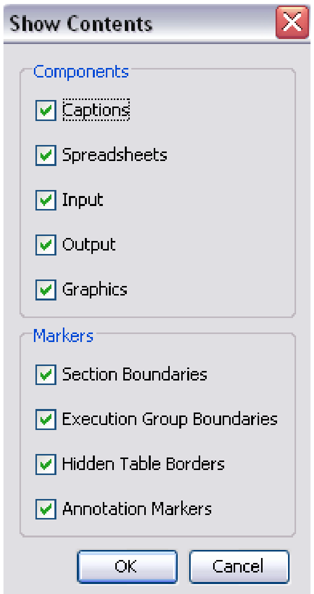


Figure 7.9: Show Contents Dialog

1. From the **View** menu, select **Show/Hide Contents**. The **Show Contents** dialog opens with all items selected for display.
2. Clear the check box associated with the document components or markers to hide them.

Note: By clearing the **Input** check box, Maple Input (1-D Math) and 2-D Math input are hidden. However, this will not hide Text input in Document mode. Clearing the **Graphics** check box ensures that a plot, an image, or the **Canvas** inserted in the document by using the **Insert** menu option is also hidden.

Command Output Versus Inserted Content

Output is considered an element that results from executing a command. Inserted components are not considered output.

Consider the following examples.

The plot resulting from executing the **plot(sin)** command is considered output.

- To show a plot from the **plot(sin)** command, select both the **Output** and **Graphics** check boxes in the **Show Contents** dialog.

Inserted images and the **Canvas** are not considered output. As such, they are not hidden if you clear the **Output** check box.

- To hide an inserted image or canvas, clear the **Graphics** check box in the **Show Contents** dialog.

Indentation and the Tab Key

The Tab key has a few different uses:

- Indentation
- Navigation through a document or between table cells

- Navigation between placeholders within a math expression

Control the behavior of the Tab key using the **Tab Navigation** feature in the **Format** menu. For more information, refer to the tabkey help page.

7.3 Commands in Documents

Document Blocks

With document blocks, you can create documents that present text and math in formats similar to those found in business and education documents.

In a document block, an input prompt or execution group is not displayed.

By hiding Maple input such that only text and results are visible, you create a document with better presentation flow. Before using document blocks, it is recommended that you display **Markers**. A vertical bar is displayed along the left pane of the document. Icons representing document blocks are displayed in this vertical bar next to associated content.

To activate Markers:

- From the **View** menu, select **Markers**.

For further details on document blocks, see *Document Blocks (page 35)* in Chapter 1.

Working with Document Blocks

In document mode, each time you press **Enter**, a new document block appears. Documents consist of a series of document blocks.

1. Create a new document block after the last section of the pasted example, either by pressing **Enter**, or by selecting, from the **Edit→Document Blocks** menu, **Create Document Block**.
2. Enter text and an expression to evaluate. For example, enter "Plot the expression $\sin(x)$ and its derivative, $\frac{d}{dx}\sin(x)$ ". For detailed instructions on entering this phrase, see *Example 6 - Enter Text and 2-D Math in the Same Line Using Toolbar Icons (page 22)* in Chapter 1.
3. Select the expression to display the context panel.
4. From the context panel, click **Evaluate and Display Inline**. The expression is evaluated.
5. Check that the input mode is **Text**, then enter the rest of the sentence: ", in the same plot." See **Figure 7.10**.

Before

After

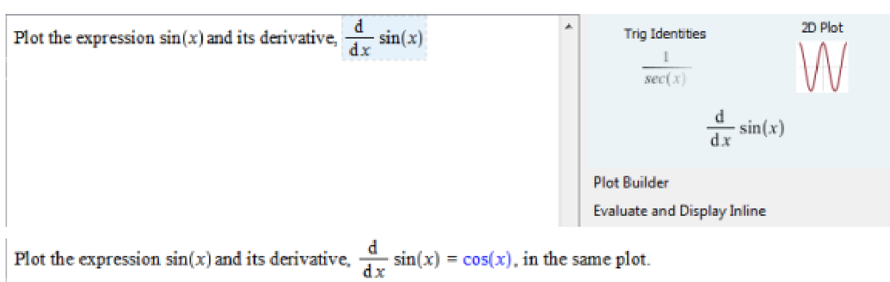


Figure 7.10: Working with Document Blocks

Result:

plot - create a two-dimensional plot

Calling Sequence

plot(**f**, **x**)
plot(**f**, **x=x0..x1**)
plot(**v1**, **v2**)

Parameters

- f - expression in independent variable x
- x - independent variable
- x0, x1 - left and right endpoints of horizontal range
- v1, v2 - x-coordinates and y-coordinates

Plot the expression $\sin(x)$ and its derivative, $\frac{d}{dx}\sin(x) = \cos(x)$, in the same plot.

Inline Document Output

Document blocks can display content inline, that is, text, input, and output in one line as presented in business and education documents.

To display content inline:

1. Place the cursor in the document block.
2. From the **Edit→Document Blocks** menu, select **Inline Document Output**.

View Document Code

To view the contents, that is, all code and expanded execution groups within a document block, you must expand the document block.

1. Place the cursor in the document block region.
2. From the **Edit→Document Blocks** menu, select **Show Command**.

Plot the expression $\sin(x)$ and its integral,

> $\int \sin(x)\, dx$

output redirected...

=

```
> print((1)); # input placeholder
```

$-\cos(x)$

, in the same plot.

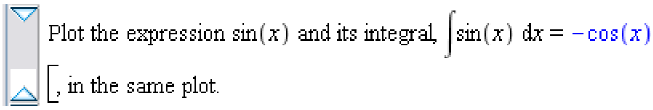
3. To hide code again, clear the check box beside **Show Command**.

Expand an Execution Group within a Document Block

An execution group is a grouping of Maple input with its corresponding Maple output. It is distinguished by a large square bracket at the left called a group boundary.

As document blocks can contain many execution groups, you can select to expand an execution group within a document block.

1. Place the cursor near the end of the document block region.
2. From the **Edit→Document Blocks** menu, select **Show Execution Group**.



3. To hide the group, clear the check box beside **Show Execution Group**.

Switch between Input and Output

1. Place the cursor in the document block region.
2. From the **Edit→Document Blocks** menu, select **Toggle Input/Output Display**.

Input from any executable math or commands is displayed in one instance, or only output is displayed.

Typesetting

You can control typesetting and 2-D Math equation parsing options in the Standard Worksheet interface. Extended typesetting uses a customizable set of rules for displaying expressions.

The rule-based typesetting functionality is available when the **Typesetting level** is set to **Extended** (the default). You can set the typesetting level under **Tools→Options→Display** tab). This parsing functionality applies to 2-D Math editing (**Math** mode) and output.

For example, you can change the display of derivatives to suit the content and audience of your document.

> $\frac{d}{dx} f(x)$

Tools→Options→Display tab: Typesetting level = Extended.

$$f'(x)$$

> $\frac{d}{dx} f(x)$

Tools→Options→Display tab: Typesetting level = Maple Standard.

$$\frac{d}{dx} f(x)$$

To specify rules, use the **Typesetting Rule Assistant**.

- From the **View** menu, select **Typesetting Rules**. The **Typesetting Rule Assistant** dialog opens.

For more information, see the **Typesetting**, **TypesettingRuleAssist**, and **OptionsDialogDisplay** help pages.

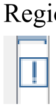
Auto-Execute

The **Autoexecute** feature allows you to designate regions of a document for automatic execution. These regions are executed when the document opens or when the **restart** For more information, refer to the **restart** help page. command is executed. This is useful when sharing documents. Important commands can be executed as soon as the user opens your document. The user is not required to execute all commands.

Setting the Auto-Execute Feature

1. Select the region to be automatically executed when the document opens.
2. From the **Evaluate** menu, select **Set Selection to Autoexecute**.

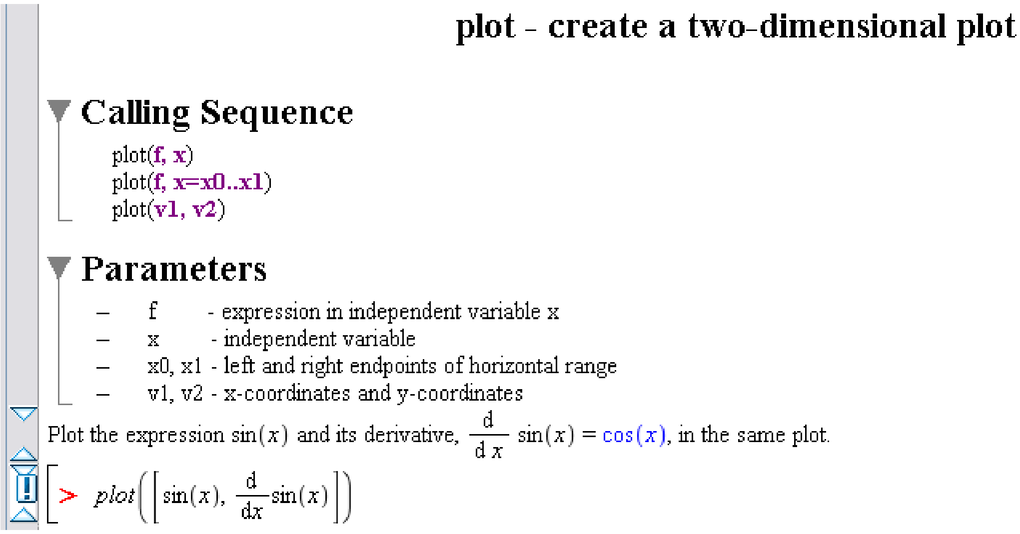
Regions set to **Autoexecute** are denoted by exclamation mark symbols in the Markers region (**View → Markers**),


.

For example, to display a plot in your document without saving the plot, making your document use less memory, you can set a plot command to autoexecute.

1. After the plot instruction, enter a Maple prompt (**Insert** → **Execution Group** → **After Cursor**).
2. Enter the plot command: $plot\left(\left[\sin(x), \int \sin(x)\,dx\right]\right)$ and press **Enter** to execute.
3. Select the plot, then select **Evaluate** → **Remove Output from Selection**.
4. Place the cursor in the plot command, then select **Evaluate** → **Set Selection to Autoexecute**.
5. Save and close the document; on reopening, the command is re-executed.

Result:



Removing the Auto-Execute Setting

To remove the setting in a region:

1. Select the region.
2. From the **Evaluate** menu, select **Clear Selection from Autoexecute**.

To remove all autoexecuted regions from a document:

- From the **Evaluate** menu, select **Clear All Autoexecute**.

Repeating Auto-Execution

To execute all marked groups:

- From the **Evaluate** menu, select **Repeat Autoexecute**.

Security Levels

By default, Maple prompts the user before automatically executing the document.

To set security levels for the autoexecute feature, use the **Security** tab in the **Options** dialog. For details, refer to the **OptionsDialogSecurity** help page.

7.4 Tables

Tables allow you to organize content in a document.

Creating a Table

To create a table:

1. From the **Insert menu**, select **Table**.
2. Specify the number of rows and columns in the table creation dialog.
3. Click **OK**.

The default properties for the table include visible borders and auto-adjustment to 100% of the document width. These options, as well as the table dimensions, can be modified after table creation.

Create a table with 4 rows and 2 columns at the end of your document. In document mode, the input mode is set to **Math** by default; in worksheet mode, the default is **Text** mode.

Cell Contents

Any content that can be placed into a document can also be placed into a table cell, including other sections and tables. Table cells can contain a mix of:

- Input commands
- 2-D Math
- Embedded components: buttons, sliders, check boxes, and more
- Plots
- Images

Enter a heading in both columns of the first row, in 2-D Math. You can use any text formatting features within each cell; for example, bold and center the headings.

$\boldsymbol{f(x)}$	$\frac{\mathbf{d}}{\mathbf{d}\boldsymbol{x}}\boldsymbol{f(x)}$

Navigating Table Cells

Use the **Tab** key to move to the next cell. Ensure that **Format → Tab Navigation** is selected.

Tab Navigation selected	Allows you to move between cells using the **Tab** key.
Tab Navigation not selected	Allows you to indent in the table using the **Tab** key.

Tab between the cells of the table and enter the following expressions in the first column. For each function, from the context panel, select **Differentiate → With respect to → x**. Cut and paste the resulting expression into the second column.

$f(x)$	$\frac{d}{dx} f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2 \left(1+\frac{1}{x}\right)^2 x^2}$
$\sin(\omega\, x)\, e^{(-5\, x)}$	$\cos(\omega\, x)\, \omega\, e^{-5\, x} - 5 \sin(\omega\, x)\, e^{-5\, x}$
$\frac{d^2}{d\, x^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$

Modifying the Structural Layout of a Table

The number of rows and columns in a table are modified using the **Insert** and **Delete** submenus in the **Table** menu or by using the **Cut** and **Copy/Paste** tools. The **Table** menu is found under the **Format** menu. The same table menu options are available in the Context Panel when the cursor resides in a table.

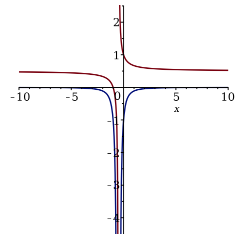
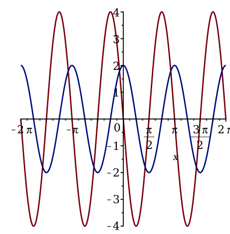
Inserting Rows and Columns

Row and column insertion is relative to the table cell that currently contains the cursor. If the document has an active selection, insertion is relative to the selection boundaries.

- Column insertion can be to the left or right of the document position marker or selection.
- Row insertion can be above or below the marker or selection.

In your table, add a third column on the right to display the plots of these expressions. Add the heading, and insert a blank plot region in each cell below it, by selecting **Insert → Plot → 2-D** (or **3-D** for the second expression). Then **Ctrl**-drag (**Control**-drag for Macintosh) each expression in the row into its plot region to display it. For details on this procedure, see *Plots and Animations (page 183)*.

Resize the plots and table as desired.

$f(x)$	$\frac{d}{dx} f(x)$	Plot of $f(x)$ and $\frac{d}{dx} f(x)$
$\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}$	$-\dfrac{1}{\left(1+\dfrac{1}{1+\dfrac{1}{x}}\right)^2 \left(1+\dfrac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\, \omega\, e^{-5x} - 5 \sin(\omega x)\, e^{-5x}$	
$\dfrac{d^2}{d x^2} \sin^2(x)$	$-8 \sin(x) \cos(x)$	

Deleting Rows and Columns

With deleting operations using the **Delete** key, the **Delete Table Contents** dialog opens allowing you to specify the desired behavior. For example, you can delete the selected rows, or delete the contents of the selected cells. See **Figure 7.11**.

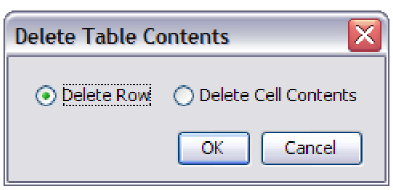


Figure 7.11: Delete Table Contents Verification Dialog

Pasting

Pasting a table subselection into a table may result in the creation of additional rows or columns, overwriting existing cell content, or the insertion of a subtable within the active table cell. When there is a choice, the **Table Paste Mode** dialog opens, allowing you to choose. See **Figure 7.12**.

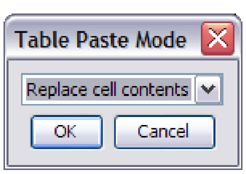


Figure 7.12: Table Paste Mode Selection Dialog

Merging Cells

To merge adjacent cells in a table, select the cells you would like to merge. From the **Table** menu, select **Merge Cells**. You can merge cells across row or column borders. See **Figure 7.13**. The resulting cell must be rectangular. The contents of the individual cells in the merge operation are concatenated in execution order. See **Figure 7.14**. For details on cell execution order, see *Execution Order Dependency (page 251)*.

> $a+b$ $a+b$	> $c+d$ $c+d$

Figure 7.13: Two Cells

> $a+b$ $a+b$ > $c+d$ $c+d$

Figure 7.14: Merged Cells

Modifying the Physical Dimensions of a Table

The overall width of the table can be controlled in several ways.

The most direct way is to press the left mouse button (press mouse button, for Macintosh) while hovering over the left or right table boundary and dragging the mouse left or right. Upon release of the mouse button, the table boundary is updated. This approach can also be used to resize the relative width of table columns.

Alternatively, the size of the table can be controlled from the **Table Properties** dialog. The Table Properties dialog is found in the Context Panel or through the **Format** menu under **Table** and then **Properties**. Two sizing modes are supported.

1. **Fixed percentage of page width**. Using this option, the table width adjusts whenever the width of the document changes. This option is useful for ensuring that the entire content of the table fits in the screen or printed page.
2. **Scale with zoom factor**. This option is used to preserve the size and layout of the table regardless of the size of the document window or the zoom factor. If the table exceeds the width of the document window, the horizontal scroll bar can be used to view the rightmost columns. **Note**: Using this option, tables may be incomplete when printed.

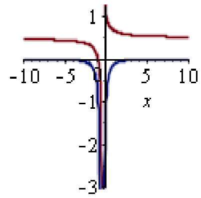
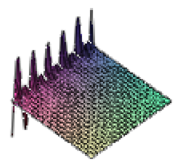
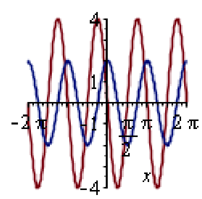
Modifying the Appearance of a Table

Table Borders

The style of exterior and interior borders is set using the **Table Properties** dialog.

- You can set all, none, or only some of the borders to be visible in a table. Exterior borders are controlled separately.
- You can control the visibility of interior borders by using the Group submenu of the Table menu; grouping rows or columns suppresses interior borders, provided that the interior border style is set by row and column group.

For example, group the columns together, and group rows 2 to 4 together. Then in the **Table Properties** dialog, select **Exterior Borders**: **Top and bottom**, and **Interior Borders**: **By row and column group**.

$f(x)$	$\frac{d}{dx}f(x)$	Plot of $f(x)$ and $\frac{d}{dx}f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2\left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\,\omega\, e^{-5x} - 5\sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2}\sin^2(x)$	$-8\sin(x)\cos(x)$	

- Hidden borders are visible when the mouse hovers over a table. **Note**: You can hide the visibility of lines on mouse pointer roll over by using the **View→Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box. This setting applies to all tables in the worksheet. You can also set controls for an individual table from the **Table Properties→Show hidden borders** option. Using this option, borders can be hidden in a table even if they are set to visible on roll over in the **Show/Hide Contents dialog**.

Alignment Options

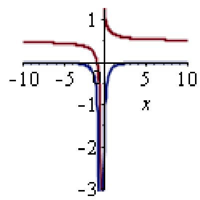
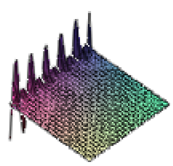
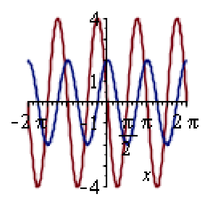
The table alignment tools control the horizontal alignment of columns and vertical alignment of rows.

For column alignment, the current selection is expanded to encompass all rows in the selected columns. The alignment choice applies to all cells within the expanded selection. If the document does not contain a selection, the cursor position is used to identify the column.

Similarly, the selection is expanded to include all columns in the selected rows for vertical alignment options. The following table illustrates the vertical alignment options. The baseline option is useful for aligning equations across multiple cells within a row of a table.

Top	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Center	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Bottom	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$
Baseline	$x^{\left(\frac{1}{y}\right)}$	$\frac{1}{x^{\left(\frac{1}{y}\right)}}$

For example, set the **Row** alignment to **Baseline** for all rows, and set the **Column** alignment to **Center** for all columns.

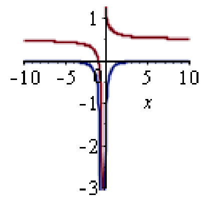
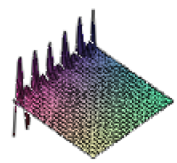
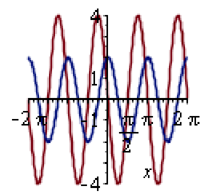
$f(x)$	$\frac{d}{dx}f(x)$	Plot of $f(x)$ and $\frac{d}{dx}f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2\left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\,\omega\, e^{-5x} - 5\sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2}\sin^2(x)$	$-8\sin(x)\cos(x)$	

Cell Color

You can set the background color of any cell or collection of cells to be any color. This coloring is independent of any highlighting or text color that may also be applied.

To change the color of a cell, place the cursor in the cell, then from the **Table** menu, select **Cell Color...**. In the **Select A Color** dialog, choose a color from the swatches, the color wheel, or RGB. See the **DrawingTools** help page for details on color selection.

For example, select the first row of the table and apply a light blue color. This sets the header off from the content below.

$f(x)$	$\frac{d}{dx}f(x)$	**Plot of** $f(x)$ **and** $\frac{d}{dx}f(x)$
$\frac{1}{1+\frac{1}{1+\frac{1}{x}}}$	$-\frac{1}{\left(1+\frac{1}{1+\frac{1}{x}}\right)^2\left(1+\frac{1}{x}\right)^2 x^2}$	
$\sin(\omega x)\, e^{(-5x)}$	$\cos(\omega x)\,\omega\, e^{-5x} - 5\sin(\omega x)\, e^{-5x}$	
$\frac{d^2}{dx^2}\sin^2(x)$	$-8\sin(x)\cos(x)$	

Controlling the Visibility of Cell Content

The **Table Properties** dialog includes two options to control the visibility of cell content. These options allow control over the visibility of Maple input and execution group boundaries. Thus, these elements can be hidden in a table even if they are set to visible for the document in the **View→Show/Hide Contents** dialog.

Printing Options

The **Table Properties** dialog contains options to control the placement of page breaks when printing. You can fit a table on a single page, allow page breaks between rows, or allow page breaks within a row.

Execution Order Dependency

The order in which cells are executed is set in the **Table Properties** dialog. The following tables illustrate the effect of execution order.

Row-wise execution order	
`> x:=1;` $x := 1$ (7.1)	`> x:=x+1;` $x := 2$ (7.2)
`> x:=x+1;` $x := 3$ (7.3)	`> x:=x+1;` $x := 4$ (7.4)

Column-wise execution order	
`> x:=1;` $x := 1$ (7.5)	`> x:=x+1;` $x := 3$ (7.6)
`> x:=x+1;` $x := 2$ (7.7)	`> x:=x+1;` $x := 4$ (7.8)

Editable Tables

Tables can be marked as editable or non-editable. The editable property for tables is independent of the document editability, though if a document is marked as non-editable, tables cannot be edited. After a table has been marked as non-editable, any content stored in the table cannot be modified. It is not possible to add any new content such as embedded components or to run computations in execution groups or document blocks. Existing interactive embedded components inside of a table will continue to work.

A table can be marked as editable or non-editable in its **Table Properties** using either the **DocumentTools:-SetProperty** command or the Context Panel. To make a table editable or non-editable using the Context Panel, in the Table properties, select or clear the **Editable** check box

Additional Examples

For more practice creating and manipulating tables, try creating the following tables at the end of your document.

Table of Values

This example illustrates how to set the visibility options for cell contents to display a table of values.

$$> y := t \rightarrow \frac{1}{2} t^2 :$$

Create a table with 2 rows and 7 columns. Enter the values as below, and then select all table cells. In the **Table → Alignment** menu, select **Columns**, and then **Center**.

t seconds	0	1	2	3	4	5	6
$y(t)$ meters	$y(0)$ 0	$> y(1)$ $\frac{1}{2}$	$> y(2)$ 2	$> y(3)$ $\frac{9}{2}$	$> y(4)$ 8	$> y(5)$ $\frac{25}{2}$	$> y(6)$ 18

Table settings:

In the **Table Properties** dialog:

1. Set **Table Size Mode** to **Scale with zoom factor**.
2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

t seconds	0	1	2	3	4	5	6
$y(t)$ meters	0	$\frac{1}{2}$	2	$\frac{9}{2}$	8	$\frac{25}{2}$	18

Formatting Table Headers

The following table uses cell merging for formatting row and column headers, and row and column grouping to control the visibility of cell boundaries.

By default, invisible cell boundaries are visible on mouse pointer roll over. You can hide the visibility of lines on mouse pointer roll over by using the **View→Show/Hide Contents** dialog, and clearing the **Hidden Table Borders** check box.

		Parameter 2	
		Low	High
Parameter 1	Low	13	24
	High	18	29

Table settings:

1. Insert a table with 4 rows and 4 columns and enter the information shown above.

Using the **Table** menu:

2. **Merge** the following sets of (**R**ow,**C**olumn) cells: (R1,C1) to (R2,C2), (R1,C3) to (R1,C4), and (R3,C1) to (R4,C1).
3. **Group** columns 1 and 2, and columns 3 and 4.
4. **Group** rows 1 and 2, and rows 3 and 4.

In the **Properties** dialog:

5. Set **Exterior Borders** to **None**.
6. (Optional) Change **Table Size Mode** size option to **Scale with zoom factor**.

Using the **Table** menu:

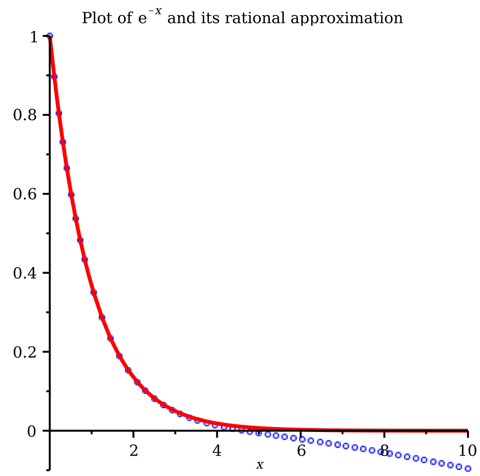
7. Set **Alignment** of columns 3 and 4 to **Center**.

2-D Math and Plots

The following example illustrates the use of tables to display 2-D Math and plots side by side.

Approximating exp(-x) as a rational polynomial using a 3^{rd} order Padé approximation.

$$e^{-x} \approx \frac{1 - \frac{1}{2}x + \frac{1}{10}x^2 - \frac{1}{120}x^3}{1 + \frac{1}{2}x + \frac{1}{10}x^2 + \frac{1}{120}x^3}$$



Insert a table with 1 row and 2 columns. Enter the information in text and executable 2-D Math to create the calculation and plot, as shown.

Table Settings:

In the **Properties** dialog:

1. Set **Exterior** and **Interior Borders** to **None**.
2. Hide Maple input and execution group boundaries: Clear the **Show input** and **Show execution group boundaries** check boxes.

Using the **Table** menu:

3. Change row **Alignment** to **Center**.

7.5 Canvas

Using the drawing tools, you can sketch an idea in a canvas, draw on plots, and draw on images. See **Figure 7.15**. For details about the drawing feature, refer to the **DrawingTools** help page.

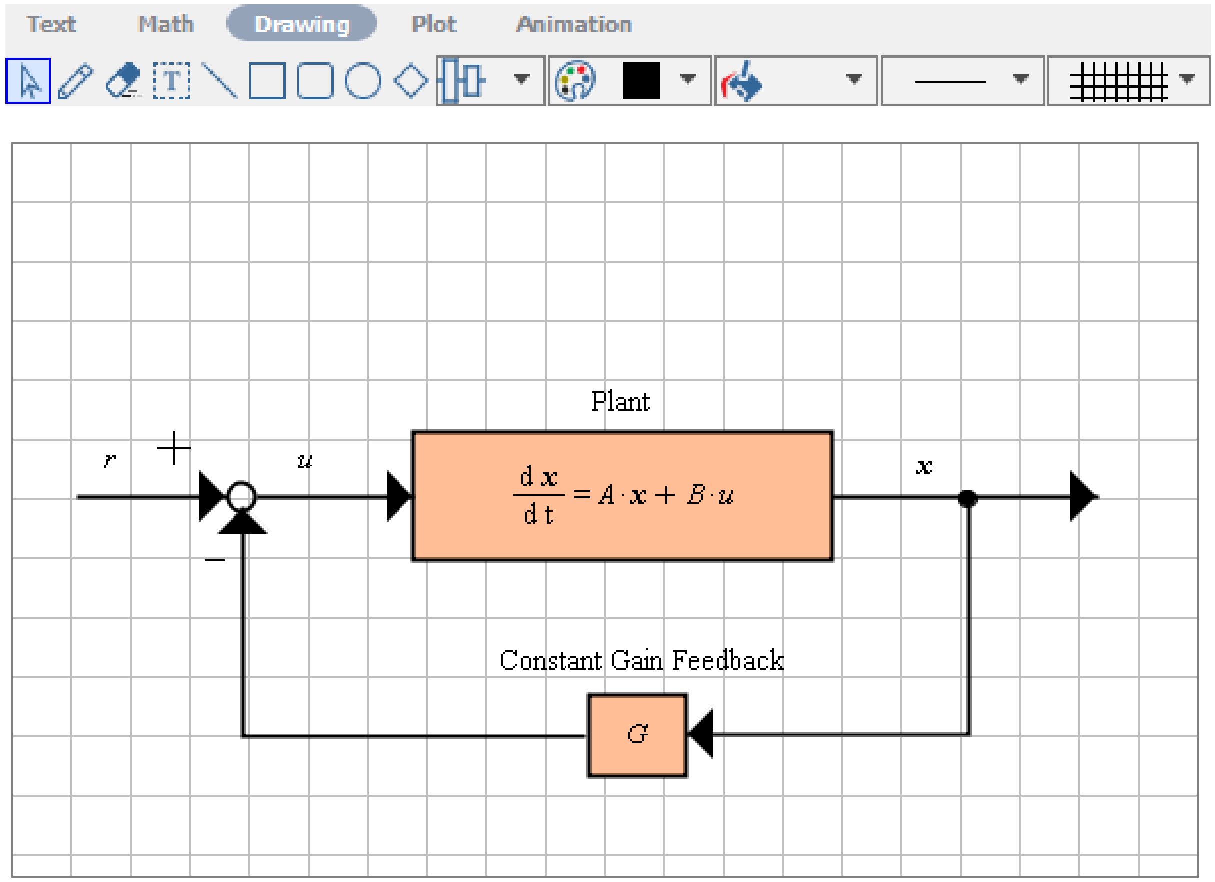


Figure 7.15: Drawing Tools and Canvas

Insert a Canvas

To insert a canvas:

1. Place the cursor where the canvas is to be inserted.
2. From the **Insert** menu, select **Canvas**. A canvas with grid lines appears in the document at the insertion point. The **Drawing** icon is available and associated context bar icons are displayed.

The tools include the following: selection tool, pencil (free style drawing), eraser, text insert, straight line, rectangle, rounded rectangle, oval, diamond, alignment, drawing outline, drawing fill, drawing linestyle, and drawing canvas properties.

Drawing

To draw with the pencil tool in the canvas:

1. From the **Drawing** icons, select the pencil icon.
2. Click and drag your mouse in the canvas to draw lines. Release the mouse to complete the drawing.

To adjust the color of drawing tools:

1. From the **Drawing** icons, select the **Drawing Outline** icon. See **Figure 7.16**.
2. Select one of the color swatches available or select the color wheel, RGB ranges, or eye dropper icon at the bottom of the dialog and customize the color to your preference.

3. After selecting a new color, draw on the canvas using the pencil icon and notice the new color.

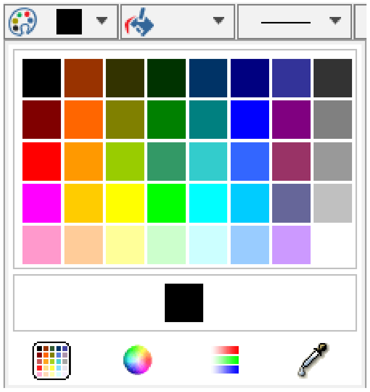
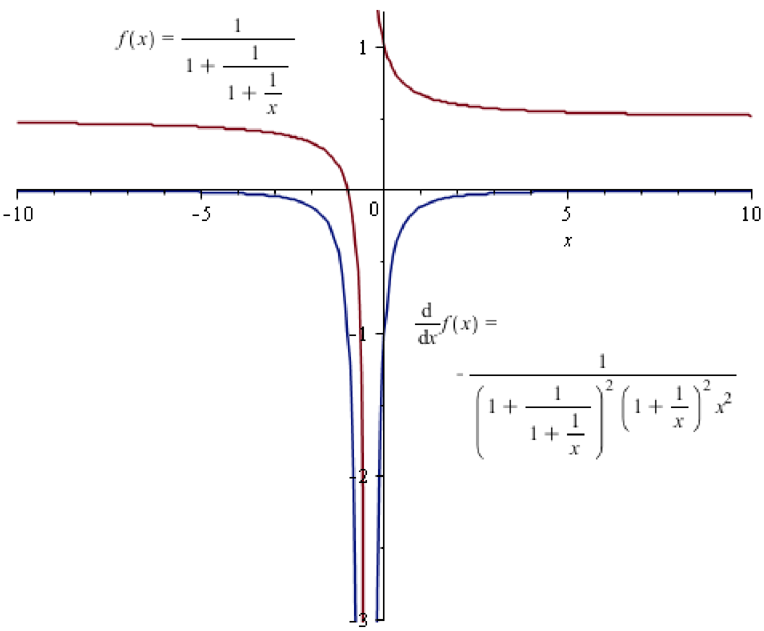


Figure 7.16: Drawing Outline Color Icon

In your document, there are three plots, two of which are 2-D plots that can be drawn on. All of the information in the table you made in the previous section could be drawn onto the plot, putting the information in a more concise layout.

Consider one of the plots from the table:



Click on the plot, and notice that the **Plot** toolbar is open. However, the **Drawing** toolbar is also available. Click on **Drawing** to see the toolbar.

Select the **Text** icon, T, and click on the plot. Enter the expression $f(x)$ in one text area, and its derivative in another, as shown. You can move the text areas around on the plot so that they indicate the correct lines.

For details on the rest of the drawing features, refer to the **DrawingTools** help page.

Canvas Style

You can alter the **Canvas** in the following ways:

- Add a grid of horizontal and/or vertical lines. By default, the canvas opens with a grid of horizontal and vertical lines.

- Change the grid line color.
- Change the spacing between grid lines.
- Change the background color.

These options can be changed in the **Drawing Properties Canvas Icon**. See **Figure 7.17**.

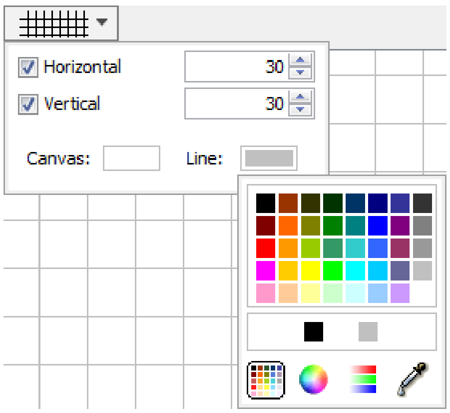


Figure 7.17: Drawing Properties Canvas Icon - Change the Gridline Color

Inserting Images

You can insert images in these file formats into your document.

- Graphics Interchange Format - gif
- Joint Photographic Experts Group - jpe, jpeg, jpg
- Portable Network Graphics - png
- Bitmap Graphics - bmp
- Tagged Image File Format - tif, tiff, jfx
- Portable aNyMap - pnm
- Kodak FlashPix - fpx

To insert an image into the document at the cursor location:

1. From the **Insert** menu, select **Image**. The **Load Image** dialog opens.
2. Specify a path or folder name.
3. Select a filename.
4. Click **Open**. The image is displayed in the document.

If the source file is altered, the embedded image does not change because the original object is pasted into the document.

To resize an inserted image:

1. Click the image. Resizing anchors appear at the sides and corners of the image.
2. Move the mouse over the resize anchor. Resizing arrows appear.
3. Click and drag the image to the desired size.

Note: To constrain the proportions of the image as it is resized, press and hold the **Shift** key as you drag.

You can also draw on images in the same way as the **drawing canvas** For more information, refer to the **worksheet/documenting/drawingtools** help page..

ImageTools Package

You can manipulate image data using the **ImageTools** package. This package is a collection of utilities for reading and writing common image file formats, and for performing basic image processing operations within Maple.

Within Maple, images are represented as dense, rectangular Arrays of 64-bit hardware floating-point numbers. Grayscale images are 2-D, whereas color images are 3-D (the third dimension representing the color channels).

In addition to the commands in the **ImageTools** package, many ordinary **Array** and **Matrix** operations are useful for image processing.

For details about this feature, refer to the **ImageTools** help page.

7.6 Hyperlinks

Use a hyperlink in your document to access any of the following.

- Webpage (URL)
- Email
- Worksheet
- Help Topic
- Task
- Dictionary Topic
- Maplet

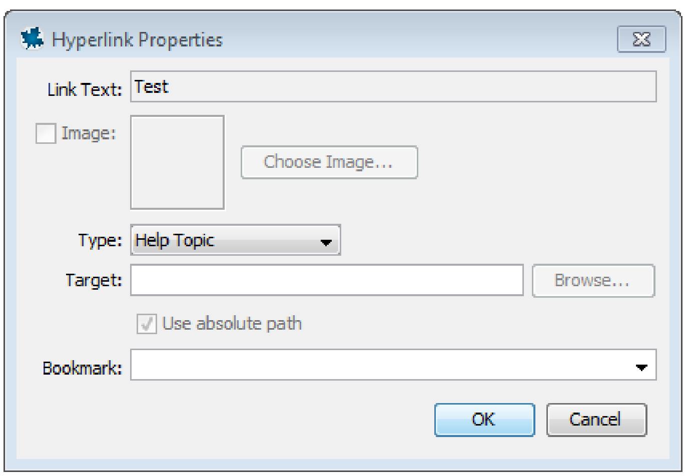


Figure 7.18: Hyperlink Properties Dialog

Inserting a Hyperlink in a Document

To create a hyperlink from existing text in the document:

1. Highlight the text that you want to make a hyperlink.
2. From the **Insert** menu, select **Hyperlink**. Alternatively, from the Context Panel for the highlighted text, select **Convert To→Hyperlink.**

3. In the **Hyperlink Properties** dialog box, the **Link Text** field is dimmed since the text region you highlighted is used as the link text. This is demonstrated in **Figure 7.18**. The highlighted text region, Diff is dimmed.
4. Specify the hyperlink **Type** and **Target** as described in the appropriate following section.

To insert a text or image hyperlink into the document:

1. From the **Insert** menu, select **Hyperlink**.
2. In the **Hyperlink Properties** dialog box, enter the **Link Text.**

Optionally, use an image as the link. Select the **Image** check box and click **Choose Image** for the file. In **.mw** files, the image appears as the link. You can resize the image as necessary. Click and drag from the corners of the image to resize.

3. Specify the hyperlink **Type** and **Target** as described in the appropriate following section.

Linking to a Webpage

To link to a webpage:

1. In the **Type** drop-down list, select **URL**.
2. In the **Target** field, enter the full URL, for example, **http://www.maplesoft.com**.
3. Click **OK**.

Linking to an Email Address

To link to an email address:

1. In the **Type** drop-down list, select **Email**.
2. In the **Target** field, enter the email address.
3. Click **OK**.

Linking to a Worksheet

To link to a Maple worksheet or document:

1. In the **Type** drop-down list, select **Worksheet**.
2. In the **Target** field, enter the path and filename of the document or click **Browse** to locate the file. (Optional) In the **Bookmark** drop-down list, enter or select a bookmark.

Note: To link within a single Maple document, leave the **Target** field blank and choose the bookmark from the **Bookmark** drop-down list.

Tip: When linking to another document, the default is to use a relative path. When sharing documents that contain hyperlinks, ensure that target documents are in the same directory, or use a ZIP file to preserve the directory structure if you are sharing a large collection of interlinking documents..

3. Click **OK**.

Linking to a Help Page

To link to a help page:

1. In the **Type** drop-down list, select **Help Topic**.
2. In the **Target** field, enter the topic of the help page. (Optional) In the **Bookmark** drop-down list, enter or select a bookmark.
3. Click **OK**.

Linking to a Task

To link to a task:

1. In the **Type** drop-down list, select **Task**.
2. In the **Target** field, enter the topic name of the task template (see the status bar at the bottom of the Task Browser window).
3. Click **OK**.

Linking to a Dictionary Topic

To link to a Dictionary topic:

1. In the **Type** drop-down list, select **Dictionary Topic**.
2. In the **Target** field, enter a topic name. Dictionary topics begin with the prefix **Definition/**, for example, **Definition/dimension**.
3. Click **OK**.

Linking to a Maplet Application

To link to a Maplet application:

1. In the **Type** drop-down list, select **Maplet**.
2. In the **Target** field, enter the local path to a file with the **.maplet** extension. Optionally, click **Browse** to locate the file.

If the Maplet application exists, clicking the link launches the Maplet application. If the Maplet application contains syntax errors, then error messages are displayed in a popup window.

When sharing documents that contain links to Maplet applications, ensure that target Maplet applications are in the same directory, or use a ZIP file to preserve the directory structure if you are sharing a large collection.

3. Click **OK**.

Note: To link to a Maplet application available on a MapleNet™ webpage, use the URL hyperlink type to link to the webpage. For information on MapleNet, see *Embedded Components and Maplets (page 311)*.

Linking to a Workbook Attachment

Similar to attaching to a worksheet, you can link to workbook content by directly entering the workbook file **URI**, or by browsing to the target workbook file.

If you want to enter the URI directly into Target field, you should copy the URI first.

To copy the URI of the content you want to link to:

1. In the Workbook Navigator palette, right-click on the file you want to link to.
2. From the context menu, select **Copy Path**.
3. The URI of the target file is now copied to the clipboard.

To link to a workbook attachment:

4. Select **Workbook Attachment** from the **Type** drop-down list.
5. In the **Target** field, enter the URI of the worksheet, obtained in the above instructions, or click **Browse** to locate the workbook file.
6. Click **OK**.

Linking to a Workbook File

You can link to a workbook file (instead of an attachment inside the workbook) using the hyperlink properties dialog.

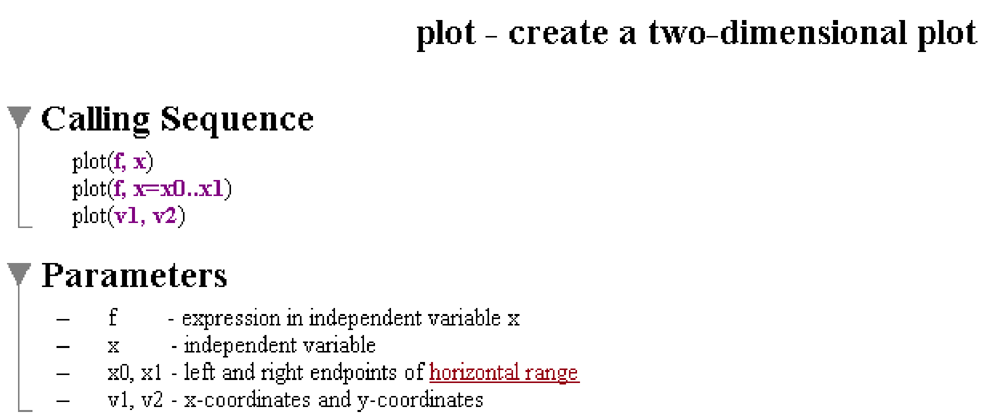
1. Select **Worksheet** from the **Type** drop-down list.
2. In the **Target** field, enter the path and filename of the workbook (.maple) file or click **Browse** to locate the file.
3. Click **OK**.

Example

For this example, link the text "horizontal range" to the dictionary page for domain. As indicated in the section for Linking to a Dictionary Topic, select **Dictionary Topic** in the **Type** drop-down list, and then enter **Definition/domain** in the **Target** field.

Links to dictionary topics appear underlined and in red.

Result:



Bookmarks

Use a bookmark to designate a location in an active document. This bookmark can then be accessed from other regions in your document or by using hyperlinks in other documents.

To display bookmark formatting icons, activate the **Marker** feature.

- From the **View** menu, select **Markers**.

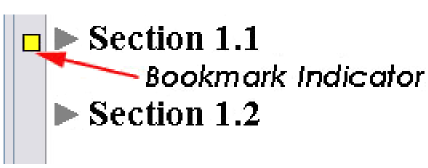


Figure 7.19: Bookmark Indicator

Note: You can display bookmark properties by holding the pointer over a bookmark indicator. See **Figure 7.19**.

Inserting, Renaming, and Deleting a Bookmark

To insert a bookmark:

1. Place the cursor at the location at which to place the bookmark. For example, place the cursor in the **Parameters** section title.
2. From the **Format** menu, select **Bookmarks**. The **Bookmark** dialog opens, listing existing bookmarks in the document.

3. Click **New**. The **Create Bookmark** dialog opens. See **Figure 7.20**. Enter a bookmark name, "parameters", and click **Create**.

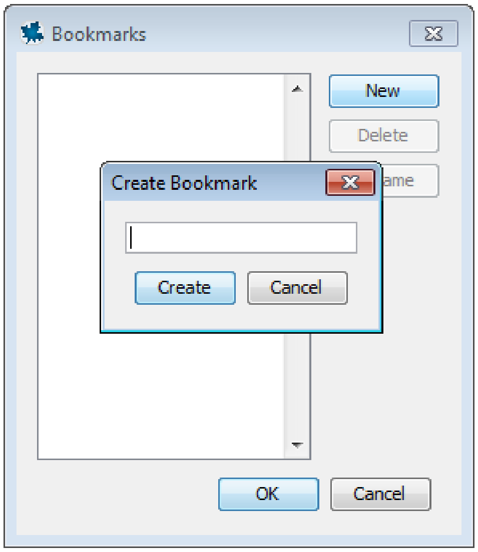


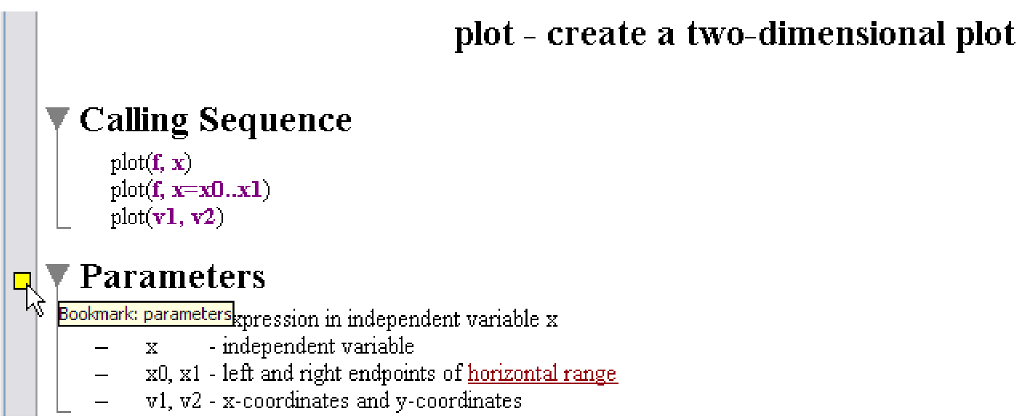
Figure 7.20: Create Bookmark Dialog

4. The new bookmark appears in the **Bookmark** dialog list. Click **OK**.

Note: You can also rename and delete bookmarks using the **Bookmark** dialog.

Alternatively, right-click on a particular bookmark to rename or delete it.

Result:



Go to a Bookmark

You can automatically move the cursor to the location of the bookmark in the active document.

1. From the **Edit** menu, select **Go To Bookmark**. The **Go To Bookmark** dialog opens with the current bookmarks listed.
2. Select the bookmark "parameters" and click **OK**. The cursor moves to the bookmark, at the beginning of the **Parameters** section.

For more information, refer to the **bookmarks** help page.

7.7 Embedded Components

You can embed simple graphical interface components, such as a button, in your document. These components can then be associated with actions that are to be executed. For example, the value of a slider component can be assigned to a document variable, or a text field can be used to input an equation.

Adding Graphical Interface Components

The graphical interface components can be inserted by using the **Components** palette (**Figure 7.21**) or by cutting/copying and pasting existing components to another area of the document. Although copied components have most of the same characteristics, they are distinct.

By default, palettes are displayed when you launch Maple. If palettes are not visible, use the following procedure:

1. From the **View** menu, select **Palettes**.
2. Select **Expand Dock**.
3. If the **Components** palette is not displayed, right-click (**Control**-click, for Macintosh) the palette dock. From the context menu, select **Show Palette**, and then **Components**.

For more information, see *Palettes (page 15)*.

You can embed the following items:

- Button, Toggle Button
- Combo Box, Check Box, List Box, Radio Button
- Text Area, Label
- Slider, Plot, Mathematical Expression
- Dial, Meter, Rotary Gauge, Volume Gauge
- Data Table
- Video Player
- Shortcut Component

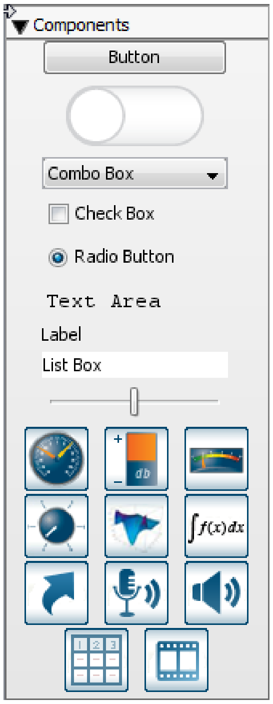


Figure 7.21: Components Palette

Task Template with Embedded Components

In your document, you can add components that have already been configured to work together, by using a task template. Here, we use the Interactive Application template. For details on how to create and modify components, see *Creating Embedded Components (page 314)*.

To insert the task template, from the **Tools** menu, select **Tasks** → **Browse**. In the table of contents, expand **Document Templates**, and select **Interactive Application**. Click **Insert Minimal Content**. The following is inserted into your document.

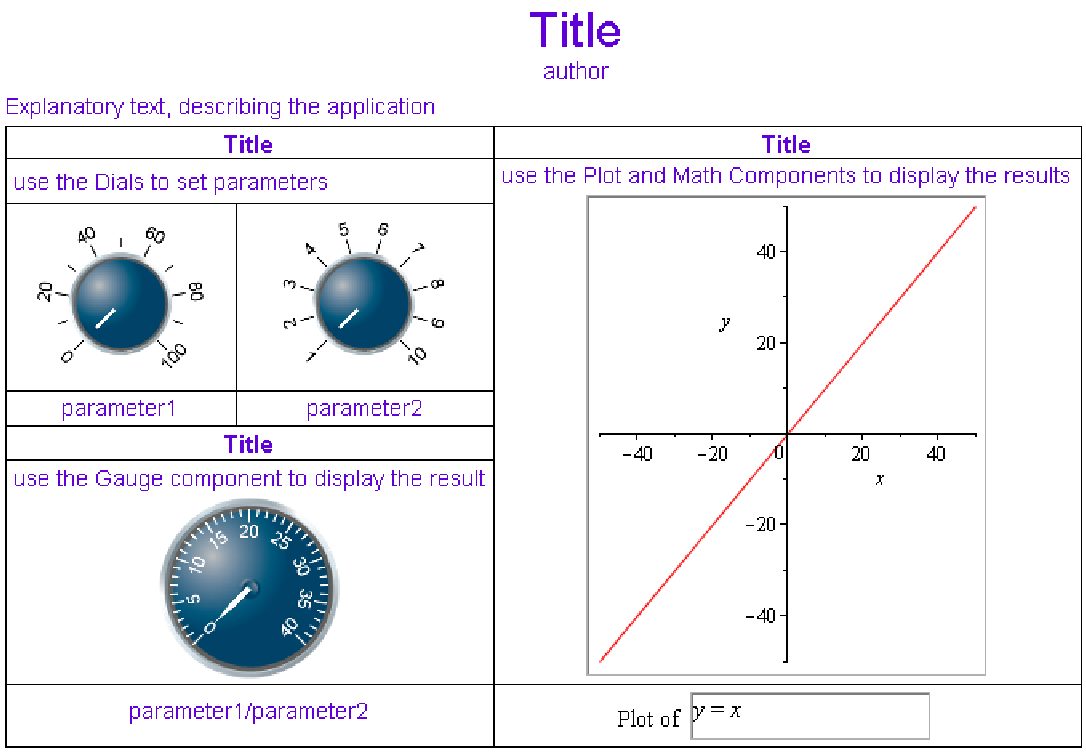


Figure 7.22: Interactive Application Task Template

This configuration of components plots a linear function with slope and y-intercept given respectively by the two dials $parameter2$ and $parameter1$, and displays the function $\frac{parameter2}{parameter1}$ on a gauge. For details on how these components work together, see *Embedded Components and Maplets (page 311)*.

7.8 Spell Checking

The **Spellcheck** utility examines all designated text regions of your document for potential spelling mistakes, including regions that are in collapsed sections. It does not check input, output, text in execution groups, or math in text regions. See **Figure 7.23**.

Note: The **Spellcheck** utility uses American spelling.

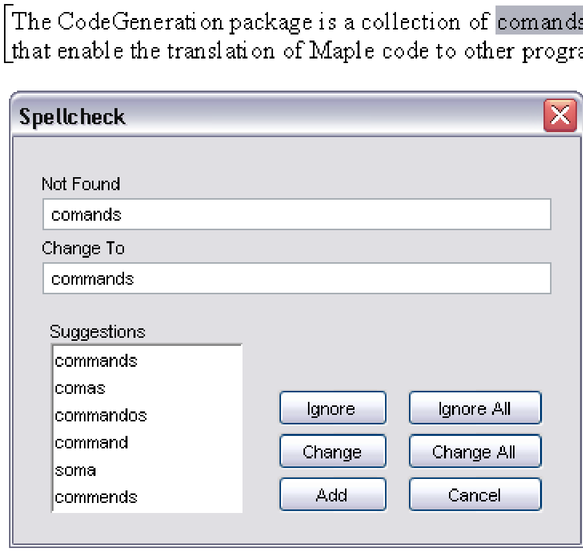


Figure 7.23: Spellcheck Dialog

How to Use the Spellcheck Utility

1. From the **Tools** menu, select **Spellcheck**. Alternatively, press **F7**. The **Spellcheck** dialog appears. It automatically begins checking the document for potential spelling mistakes.
2. If the **Spellcheck** utility finds a word that it does not recognize, that word is displayed in the **Not Found** text box.

You have six choices:

- To ignore the word, click **Ignore**.
- To ignore all instances of the word, click **Ignore All**.
- To change the word, that is, accept the suggested spelling that is in the **Change To** text box, click **Change**.
- To change all instances of the word, that is, accept the suggested spelling to replace all instances of the word, click **Change All**.
- To add the word to your dictionary, click **Add**. For details, see the following **User Dictionary** section.
- To close the **Spellcheck** dialog and stop the spelling check, click **Cancel**.

3. When the **Spellcheck** is complete, a dialog containing the message "The spelling check is complete" appears. Click **OK** to close this dialog.

Note: when using the **Spellcheck** utility, you can fix spelling errors in the dialog, but you cannot change the text in document. The **Spellcheck** utility does not check grammar.

Selecting a Suggestion

To select one of the suggestions as the correct spelling, click the appropriate word from the list in the **Suggestions** text box.

If none of the suggestions are correct, highlight the word in the **Change To** text box and enter the correct spelling. Click **Change** to accept this new spelling.

User Dictionary

You can create and maintain a custom dictionary that works with the Maple **Spellcheck** utility.

Properties of the Custom Dictionary File

- It must be a text file, that is, have the file extension **.txt**. For example, **mydictionary.txt.**
- It is a list of words, one word per line.
- It is case sensitive. This means that integer and Integer require individual entries in the dictionary file.
- It does not require manual maintenance. You build your dictionary file by using the **Add** functionality of the **Spellcheck**. However, you can manually edit the file.

To specify a custom dictionary to be used with the Maple Spellcheck utility:

1. Create a **.txt** file in a directory/folder of your choice.
2. In Maple, open the **Options** dialog, **Tools → Options**, and select the **General** tab.
3. In the **User Dictionary** field, enter the path and name of the **.txt** file you created, or click **Browse** to select the location and filename.
4. To ignore Maple words that are command and function names, clear the **Use Maple words in spellchecker** check box.
5. Click **Apply to Session** or **Apply Globally** to save the settings, or **Cancel** to discard.

Adding a Word to Your Dictionary

When running the spellcheck, if the word in the **Not Found** text box is correct, you can add the word to your dictionary.

1. Click the **Add** button. If this is the first time you are adding a word, the **Select User Dictionary** dialog opens.
2. Enter or select the custom dictionary (**.txt** file) you created. See *User Dictionary (page 266)*.
3. Click **Select**. The word is automatically added to your custom dictionary file.

Note: Specifications in the **Options** dialog determine whether this word is recognized in your next Maple session. If you set your custom dictionary and clicked **Apply to Session**, then this word will *not* be recognized in a new Maple session. If you set your custom dictionary and clicked **Apply Globally**, then this new word will be recognized.

8 Maple Expressions

This chapter provides basic information on using Maple expressions, including an overview of the basic data structures. Many of the commands described in this chapter are useful for programming. For information on additional Maple programming concepts, such as looping, conditional execution, and procedures, see *Basic Programming (page 295)*.

8.1 In This Chapter

Section	Topics
Creating and Using Data Structures (page 269) - How to define and use basic data structures	• Expression Sequences • Sets • Lists • Tables • Arrays • Matrices and Vectors • Functional Operators • Strings
Working with Maple Expressions (page 278)- Tools for manipulating and controlling the evaluation of expressions	• Low-Level Operations • Manipulating Expressions • Evaluating Expressions

8.2 Creating and Using Data Structures

Constants, data structures, mathematical expressions, and other objects are Maple expressions. For more information on expressions, refer to the Maple Help System.

This section describes the key data structures:

- Expression sequences
- Sets
- Lists
- Tables
- Arrays
- Matrices and Vectors
- Functional operators
- Strings

Expression Sequences

The fundamental Maple data structure is the *expression sequence*. It is a group of expressions separated by commas.

> $S := 2, y, \sin(x^2), I:$

Accessing Elements

To access one of the expressions:

- Enter the sequence name followed by the position of the expression enclosed in brackets(**[ ]**).

For example:

> $S[2]$

$$5$$

Using negative integers, you can select an expression from the end of a sequence.

> $S[-2]$

$$\sin(x^2)$$

You can select multiple expressions by specifying a range using the **range operator** (**..**).

> $S[2..-2]$

$$5, \sin(x^2)$$

Note: This syntax is valid for most data structures.

Sets

A set is an expression sequence enclosed in curly braces (**{ }**).

> $\left\{4, 12\, i, \sin\left(\frac{2}{3}\right)\right\}$:

A Maple set has the basic properties of a mathematical set.

- Each element is unique. Repeated elements are stored only once.
- The order of elements is not stored.

For example:

> $\{c, a, a, a, b, c, a\}$

$$\{a, b, c\}$$

Using Sets

To perform mathematical set operations, use the set data structure.

> $\{2, 6, 5, 1\} \cup \{2, 8, 6, 7\}$

$$\{1, 2, 5, 6, 7, 8\}$$

Note: The union operator is available in 1-D Math input as **union**. For more information, refer to the **union** help page.

For more information on sets, refer to the **set** help page.

Lists

A list is an expression sequence enclosed in brackets (**[ ]**).

> $L := [2, 3, 3, 1, 0]$

$$L := [2, 3, 3, 1, 0]$$

Note: Lists preserve both the **order** and **repetition** of elements.

Accessing Entries

To refer to an element in a list:

- Use square brackets.

For example:

> $L[-2..-1]$

$$[1, 0]$$

For more information, see *Accessing Elements (page 269)*.

Using Lists

Some commands accept a list (or set) of expressions.

For example, you can solve a list (or set) of equations using the context panel or the **solve** command.

> $solve([x - y^2 = -2, x + y = 0])$

For more information, see *Solving Equations and Inequations (page 80)*.

For more information on sets and lists, refer to the **set** help page.

Arrays

Conceptually, the **Array** data structure is a generalized list. Each element has an index that you can use to access it.

The two important differences are:

- The indices can be any integers.
- The dimension can be greater than one.

Creating and Using Arrays

To define an Array, use the **Array** constructor.

Standard **Array** constructor arguments are:

- Expression sequences of ranges - Specify the indices for each dimension
- Nested lists - Specify the contents

For example:

> $a := Array(1..3, 1..3, [[1, 2, 3], [4, 5, 6], [7, 8, 9]])$

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

> $b := Array(1..2, 2..5, [[1.2, 4.9, 6.3, 7.1], [9.2, 5.5, 2.4, 1.7]])$

$$b := \begin{bmatrix} 1.2 & 4.9 & 6.3 & 7.1 \\ 9.2 & 5.5 & 2.4 & 1.7 \end{bmatrix}$$

1 .. 2 × 2 .. 5 Array

To access entries in an Array, use either square bracket or round bracket notation.

Square bracket notation respects the actual index of an Array, even when the index does not start at 1.

> $a[1, 1]$

$$1$$

> $a[2, 3]$

$$6$$

> $b[2, 3]$

$$5.5$$

> $b[1, 1]$

```
Error, Array index out of range
```

Round bracket indexing normalizes the dimensions to begin at 1. Since this method is relative, you can access the end of the array by entering -1.

> $a(-1, 2)$

$$8$$

> $b(1, 1)$

$$1.2$$

Arrays can have more than one or two dimensions. For example, the following is a simple three-dimensional Array.

> $A := Array(1..2, 1..2, 1..2, fill = 3)$

$$A := \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \tag{8.1}$$

slice of 2 × 2 × 2 Array

The **Array** constructor supports other syntaxes. It also supports many options. For more information on the **Array** constructor and the Array data structure, refer to the **Array** help page. For more information on indexing methods, refer to the **rtable_indexing** help page.

Large Arrays

Larger Arrays than can be practically displayed in the document display with a summary format. For example, the multidimensional Array in **(8.1)** is displayed in summary format. The following 100-entry Array is as well. In both cases, double-clicking on the placeholder output opens a browser for the entire data structure.

> $Array(0..100)$

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,..., ⋯ 1 Array(0 .. 100) entries not shown]

To view large Arrays:

- Double-click the placeholder output.

The **Matrix Browser** displays the Array.

In the case of a multidimensional array, you can change what slice you are viewing in the Options tab.

For more information, see *Viewing Large Matrices and Vectors (page 118)*.

Tables

Tables are conceptually an extension of the Array data structure, but the table data structure is implemented using hash tables. Tables can be indexed by any values, not only integers.

Defining Tables and Accessing Entries

> $Greek := table([a = \alpha, b = \beta, c = \gamma])$:

> $Greek[b]$

$$\beta$$

You can also assign anything, for example, a list, to each element.

> $Translation := table([one = [un, uno], two = [deux, dos], three = [trois, tres]])$:

> $Translation[two]$

$$[deux, dos]$$

For more information on tables, refer to the **table** help page.

Matrices and Vectors

Matrices and Vectors are specialized data structures used in linear algebra and vector calculus computations.

> $M := \begin{bmatrix} 12 & 33 \\ 83 & 12 \end{bmatrix}$: $v := <2, 14>$:

For information on defining Matrices and Vectors, see *Creating Matrices and Vectors (page 116)*.

> $M.v$

$$\begin{bmatrix} 486 \\ 334 \end{bmatrix}$$

> $v^{\%T}.M$

$$\begin{bmatrix} 1186 & 234 \end{bmatrix}$$

> M^{-1}

$$\begin{bmatrix} -\frac{4}{865} & \frac{11}{865} \\ \frac{83}{2595} & -\frac{4}{865} \end{bmatrix}$$

For more information on these data structures, including how to access entries and perform linear algebra computations, see *Linear Algebra (page 116)*.

Functional Operators

A functional operator is a mapping $f: x \to y(x)$. The value of $f(x)$ is the result of evaluating $y(x)$.

Using functional operators, you can define mathematical functions.

Defining a Function

To define a function of one or two variables:

1. In the **Expression** palette, click one of the function definition items. See **Figure 8.1**. Maple inserts the function definition.
2. Replace the placeholders, using **Tab** to move to the next placeholder. **Note:** If pressing the **Tab** key indents the text, click the Tab icon in the toolbar. This allows you to move between placeholders.
3. Press **Enter**.

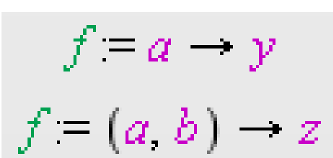


Figure 8.1: Function Definition Palette Items

For example, define a function that adds 1 to its input.

> $add1 := x \to x + 1:$

Note: Instead of using the palettes, you can type the definition. To insert the right arrow, you can enter the characters **->**. In 2-D Math, Maple replaces -> with the right arrow symbol $\to$. In 1-D Math, the characters are not replaced.

You can evaluate the function **add1** with symbolic or numeric arguments.

> $add1(12);\ add1(x+y)$

$$13$$

$$x+6$$

Distinction between Functional Operators and Other Expressions

Note: The expression $x+1$ is different from the functional operator $x \to x+1$.

Assign the functional operator $x \to x+1$ to *f*.

> $f := x \to x+1:$

Assign the expression $x+1$ to *g*.

> $g := x+1:$

To evaluate the functional operator f at a value of x:

- Specify the value as an argument to f.

> $f(22)$

$$23$$

To evaluate the expression g at a value of x:

- You **must** use the **eval** command.

The following is not meaningful:

> $g(22)$

$$x(22)+1$$

Evaluating g at $x=22$ gives the desired result.

> $eval(g, x=22)$

$$23$$

For more information on the **eval** command, and on using palettes and the context panel to evaluate an expression at a point, see *Substituting a Value for a Subexpression (page 285)*.

Multivariate and Vector Functions

To define a multivariate or vector function:

- Enclose coordinates or coordinate functions in parentheses (**( )**).

For example, a multivariate function:

> $f := (x, y) \to \frac{x^3}{y^2+1}:$

> $f(0, 0);\ f(-2.1, 1.9)$

$$0$$

$$-2.008893709$$

A vector function:

> $g := t \rightarrow (\sin(t), \cos(t), t):$

> $g(0);\ g\left(\frac{\pi}{2}\right)$

$$0, 1, 0$$

$$1, 0, \frac{\pi}{2}$$

Using Operators

To perform an operation on a functional operator, specify arguments to the operator. For example, for the operator *f*, specify *f*(*x*), which Maple evaluates as an expression. See the following examples.

Plotting:

Plot a three-dimensional operator as an expression using the **plot3d** command.

> $unassign('y'):$

> $h := (x, y) \rightarrow x^2 \cos(y):$

> $plot3d(h(x, y), x = -2..2, y = -2\pi..2\pi)$

For information on plotting, see *Plots and Animations (page 183)*.

Integration:

Integrate a function using the **int** command.

> $k := x \rightarrow \sin(\cos(x)\pi)$:

> $int\left(k(t), t = 0..\frac{\pi}{2}\right)$

$$\frac{\pi \, \mathrm{StruveH}(0, \pi)}{2}$$

The result uses the **Struve** function $\mathrm{StruveH}(v, x)$.

For information on integration and other calculus operations, see *Calculus (page 128)*. For information on mathematical functions, including accessing detailed information on the properties of a function, see *Mathematical Functions (page 58)* and the **FunctionAdvisor** help page.

Strings

A string is a sequence of characters enclosed in double quotes (**" "**).

> *S* := "This is a sequence of characters."

S := "This is a sequence of characters."

Accessing Characters

You can access characters in a string using brackets.

> *S*[11 ..−2]

"sequence of characters"

Using Strings

The **StringTools** package is an advanced set of tools for manipulating and using strings.

> *with*(*StringTools*):

> *Random*(9, '*alnum*')

"MJoY2Qs0F"

> *Stem*("impressive")

"impress"

> *Split*("Create a list of strings from the words in a string")

["Create", "a", "list", "of", "strings", "from", "the", "words", "in", "a", "string"]

8.3 Working with Maple Expressions

This section describes how to manipulate expressions using commands. Topics covered include testing the expression type, accessing operands of an expression, and evaluating an expression.

Low-Level Operations

Expression Types

A Maple *type* is a broad class of expressions that share common properties. Maple contains over 200 types, including:

- \`+\`
- **boolean**
- **constant**
- **integer**
- **Matrix**
- **trig**
- **truefalse**

For more information and a complete list of Maple types, refer to the **type** help page.

The type commands return **true** if the expression satisfies the type check. Otherwise, they return **false**.

Testing the Type of an Expression

To test whether an expression is of a specified type:

- Use the **type** command.

> $type(sin(x), 'trig')$

$$true$$

> $type(sin(x) + \cos(x), 'trig')$

$$false$$

For information on enclosing keywords in right single quotes ('), see *Delaying Evaluation (page 291)*.

Maple types are not mutually exclusive. An expression can be of more than one type.

> $type(3, 'constant')$

$$true$$

> $type(3, 'integer')$

$$true$$

For information on converting an expression to a different type, see *Converting (page 284)*.

Testing the Type of Subexpressions

To test whether an expression has a subexpression of a specified type:

- Use the **hastype** command.

> $hastype(sin(x) + \cos(x), 'trig')$

$$true$$

Testing for a Subexpression

To test whether an expression contains an instance of a specified subexpression:

- Use the **has** command.

> $has(\sin(x+y), x)$

$$true$$

> $has(\sin(x+y), x+y)$

$$true$$

> $has(\sin(x+y), \sin(x))$

$$false$$

The **has** command searches the structure of the expression for an exactly matching subexpression.

For example, the following calling sequence returns **false**.

> $has(x + y + z, x + z)$

$$false$$

To return all subexpressions of a particular type, use the **indets** command. For more information, see *Indeterminates (page 281)*.

Accessing Expression Components

Left and Right-Hand Side

To extract the left-hand side of an equation, inequality, or range:

- Use the **lhs** command.

To extract the right-hand side of an equation, inequality, or range:

- Use the **rhs** command.

For example:

> $y = x + 1$

$$y = x + 1 \quad (8.2)$$

> $lhs(\textbf{(8.2)})$

$$y \quad (8.3)$$

> $rhs(\textbf{(8.2)})$

$$x + 1 \quad (8.4)$$

For the following equation, the left endpoint of the range is the left-hand side of the right-hand side of the equation.

> $x = 3\,..5$

$$x = 3\,..5 \quad (8.5)$$

> $lhs(rhs(\textbf{(8.5)}))$

$$3 \quad (8.6)$$

Numerator and Denominator

To extract the numerator of an expression:

- Use the **numer** command.

To extract the denominator of an expression:

- Use the **denom** command.

> $e := \dfrac{1 + \sin(x)^3 - \dfrac{y}{x}}{y^2 - 1 + x}:$

If the expression is not in normal form, Maple normalizes the expression before selecting the numerator or denominator. (For more information on normal form, refer to the **normal** help page.)

> *numer*(*e*)

$$\sin(x)^3 x + x - y$$

> *denom*(*e*)

$$x\left(y^2 + x - 1\right)$$

> *denom*(*denom*(*e*))

$$1$$

The expression can be any algebraic expression. For information on the behavior for non-rational expressions, refer to the **numer** help page.

Components of an Expression

The components of an expression are called its *operands*.

To count the number of operands in an expression:

- Use the **nops** command.

For example, construct a list of solutions to an equation.

> $solutions := \left[solve\left(6\,x^3 - x^2 + 7, x\right)\right]$

$$solutions := \left[-1, \frac{7}{12} - \frac{I\sqrt{119}}{12}, \frac{7}{12} + \frac{I\sqrt{119}}{12}\right]$$

Using the **nops** command, count the number of solutions.

> *nops*(*solutions*)

$$3$$

For more information on the **nops** command and operands, refer to the **nops** help page.

Indeterminates

To find the indeterminates of an expression:

- Use the **indets** command.

The **indets** command returns the indeterminates as a set. Because the expression is expected to be rational, functions such as sin(x), f(x), and sqrt(x) are considered to be indeterminate.

> $indets\left((3 + \pi)\, x^2 \sin\left(\sqrt{1 + y}\right)\right)$

$$\left\{x, y, \sqrt{1 + y}, \sin\left(\sqrt{1 + y}\right)\right\}$$

To return all subexpressions of a particular type, specify the type as the second argument. For information on types, see *Testing the Type of an Expression (page 279)*.

> $indets\left((3 + \pi)\, x^2 \sin\left(\sqrt{1 + y}\right), 'radical'\right)$

$$\left\{\sqrt{1 + y}\right\}$$

To test whether an expressions has subexpressions of a specific type (without returning them), use the **has** command. For more information, see *Testing for a Subexpression (page 279)*.

Manipulating Expressions

This section introduces the most commonly used manipulation commands. For additional manipulation commands, see *Iterative Commands (page 303)*.

Simplifying

To simplify an expression:

- Use the **simplify** command.

The **simplify** command applies simplification rules to an expression. Maple has simplification rules for various types of expressions and forms, including trigonometric functions, radicals, logarithmic functions, exponential functions, powers, and various special functions. You can also specify custom simplification rules using a set of *side relations*.

> $simplify\left(5 + 32 - 8^{\left(\frac{1}{3}\right)}\right)$

$$35$$

> $simplify\left(\sin(x)^2 + \ln(2\,y) + \cos(x)^2\right)$

$$1 + \ln(2) + \ln(y)$$

To limit the simplification, specify the type of simplification to be performed.

> $simplify\left(\sin(x)^2 + \ln(2\,y) + \cos(x)^2,\ '\,trig'\right)$

$$1 + \ln(2\,y)$$

> $simplify\left(\sin(x)^2 + \ln(2\,y) + \cos(x)^2,\ '\ln'\right)$

$$\sin(x)^2 + \ln(2) + \ln(y) + \cos(x)^2$$

You can also use the **simplify** command with *side relations*. See *Substituting a Value for a Subexpression (page 285)*.

Factoring

To factor a polynomial:

- Use the **factor** command.

> $factor\left(x^6 - x^5 - 9\,x^4 + x^3 + 20\,x^2 + 12\,x\right)$

$$x\,(x-2)\,(x-3)\,(x+2)\,(x+1)^2$$

> $factor\left(x^3\,y + x^2\,y^2 - 3\,x^3 - x^2\,y + 2\,x\,y^2 - 6\,x^2 - 5\,x\,y + y^2 - 3\,x - 3\,y\right)$

$$(y-3)\,(x+1)^2\,(x+y)$$

Maple can factor polynomials over the domain specified by the coefficients. You can also factor polynomials over algebraic extensions. For details, refer to the **factor** help page.

For more information on polynomials, see *Polynomial Algebra (page 110)*.

To factor an integer:

- Use the **ifactor** command.

> *ifactor*(196911)

$$(3)^4 (11) (13) (17)$$

For more information on integers, see *Integer Operations (page 76)*.

Expanding

To expand an expression:

- Use the **expand** command.

The **expand** command distributes products over sums and expands expressions within functions.

> *expand*$\left((y-3)\,(x+1)^2\,(x+y)\right)$

$$x^3\,y+x^2\,y^2-3\,x^3-x^2\,y+2\,x\,y^2-6\,x^2-5\,x\,y+y^2-3\,x-3\,y$$

> *expand*$(\sin(x+y))$

$$\sin(x)\,\cos(y)+\cos(x)\,\sin(y)$$

Combining

To combine subexpressions in an expression:

- Use the **combine** command.

The **combine** command applies transformations that combine terms in sums, products, and powers into a single term.

> *combine*$(\sin(x)\,\cos(y)+\cos(x)\,\sin(y))$

$$\sin(x+y)$$

Recall that a was previously assigned to represent a two-dimensional array (see *Creating and Using Arrays (page 271)*).

> *combine*$\left((x^a)^2\,x\right)$

$$\begin{bmatrix} x^3 & x^5 & x^7 \\ x^9 & x^{11} & x^{13} \\ x^{15} & x^{17} & x^{19} \end{bmatrix}$$

The **combine** command applies only transformations that are valid for all possible values of names in the expression.

> *combine*$(4\ln(x)-\ln(y))$

$$4\ln(x)-\ln y$$

To perform the operation under assumptions on the names, use the **assuming** command. For more information about assumptions, see *Assumptions on Variables (page 104)*.

> $combine(4\ln(x) - \ln(y))$ assuming $x > 0, y > 0$

$$\ln\left(\frac{x^4}{y}\right)$$

Converting

To convert an expression:

- Use the **convert** command.

The **convert** command converts expressions to a new form, type (see *Expression Types (page 278)*), or in terms of a function. For a complete list of conversions, refer to the **convert** help page.

Convert a measurement in radians to degrees:

> $convert(\pi, '\,degrees')$

$$180\ degrees$$

To convert measurements that use units, use the Unit Converter or the **convert/units** command.

> $convert(450.2\text{kg}, 'units', lb)$

$$992.5211043\ \text{lb}$$

For information on the Unit Converter and using units, see *Units (page 93)*.

Convert a list to a set:

> $convert([b, c, d], 'set')$

$$\{c, d, Array(1..2, 2..5, \{(1, 2) = 1.2, (1, 3) = 4.9, (1, 4) = 6.3, (1, 5) = 7.1, (2, 2) = 9.2, (2, 3) = 5.5, (2, 4) = 2.4, (2, 5) = 1.7\})\}$$

Maple has extensive support for converting mathematical expressions to a new function or function class.

> $convert(\cos(x), \exp)$

$$\frac{e^{I x}}{2} + \frac{e^{-I x}}{2}$$

Find an expression equivalent to the inverse hyperbolic cotangent function in terms of **Legendre** functions.

> $convert(\text{arccoth}(z), Legendre)$

$$\text{LegendreQ}\left(0, \frac{1}{5}\right) + \frac{\pi\sqrt{-16}}{8}$$

For more information on converting to a class of functions, refer to the **convert/to_special_function** help page.

Normalizing

To normalize an expression:

- Use the **normal** command.

The **normal** command converts expressions into *factored normal form*.

> $normal\left(\frac{x^2-y^2}{(x-y)^3}\right)$

$$\frac{x+y}{(x-y)^2}$$

You can also use the **normal** command for zero recognition.

> $normal\left(x^3+1-(x+1)^3+3\,x\,(1+x)\right)$

$$0$$

To expand the numerator and denominator, use the **expanded** option.

> $normal\left(\frac{x^2-y^2}{(x-y)^3}, 'expanded'\right)$

$$\frac{x+y}{x^2-2\,x\,y+y^2}$$

> $normal\left(\sin\left(1+\frac{1}{x}\right)\right)$

$$\sin\left(\frac{x+1}{x}\right)$$

Sorting

To sort the elements of an expression:

- Use the **sort** command.

The **sort** command orders a list of values or terms of a polynomial.

> $sort([4, 3, 2.1, -4, 43, 0])$

$$[-4, 0, 2.1, 3, 4, 43]$$

> $sort\left(x+4\,x^5-7\,x^2+1+9\,x^4-5\,x^3\right)$

$$4\,x^5+9\,x^4-5\,x^3-7\,x^2+x+1$$

> $sort\left(x\,y-6\,y^2\,x+2\,y^3+5\,x-1\right)$

$$-6\,x\,y^2+2\,y^3+x\,y+5\,x-1$$

For information on sorting polynomials, see *Sorting Terms (page 111)*.

For more information on sorting, refer to the **sort** help page.

Evaluating Expressions

Substituting a Value for a Subexpression

To evaluate an expression at a point, you must substitute a value for a variable.

To substitute a value for a variable using the context panel:

1. Select the expression.
2. From the context panel, select **Evaluate at a Point**. The **Evaluate at a Point** dialog is displayed. See **Figure 8.2**.

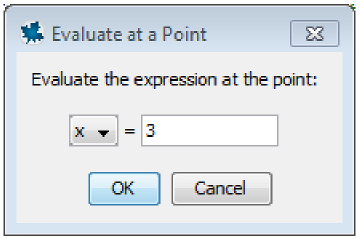


Figure 8.2: Evaluate at a Point

3. In the drop-down list, select the variable to substitute.
4. In the text field, enter the value to substitute for the variable. Click **OK**.

In Worksheet mode, Maple inserts the **eval** command calling sequence that performs the substitution. This is the most common use of the **eval** command.

For example, substitute $x = 3$ in the following polynomial.

> $x^3 + 4\,x^2 - 7\,x + 2$

$$x^3 + 4\,x^2 - 7\,x + 2$$

> $eval\left(x^3 + 4\,x^2 - 7\,x + 2, [x = 3]\right)$

$$44$$

To substitute a value for a variable using palettes:

1. In the **Expression** palette, click the evaluation at a point item $f(x)\big|_{x=a}$.
2. Specify the expression, variable, and value to be substituted.

For example:

> $\sqrt{x^2 - x - 3}\,\Big|_{x=5}$

$$\sqrt{17}$$

Substitutions performed by the **eval** function are *syntactical*, not the more powerful *algebraic* form of substitution.

If the left-hand side of the substitution is a name, Maple performs the substitution.

> $unassign('a') : unassign('b') :$

> $eval\left(\cos(a\,b\,c),\ a = \frac{\pi}{6}\right)$

$$\cos\left(\frac{\pi\,b\,c}{6}\right)$$

If the left-hand side of the substitution is not a name, Maple performs the substitution only if the left-hand side of the substitution is an operand of the expression.

> $eval\left(\cos(a\,b),\ a\,b = \frac{\pi}{6}\right)$

$$\frac{\sqrt{3}}{2}$$

> $eval\left(\cos(a\,b\,c),\ a\,b = \frac{\pi}{6}\right)$

$$\cos(a\,b\,c)$$

Maple did not perform the evaluation because $a\,b$ is not an operand of $\cos(a\,b\,c)$. For information on operands, refer to the **op** help page.

For algebraic substitution, use the **algsubs** command, or the **simplify** command with side relations.

> $algsubs\left(a\,b = \frac{\pi}{6},\ \cos(a\,b\,c)\right)$

$$\cos\left(\frac{c\,\pi}{6}\right)$$

> $simplify\left(\cos(a\,b\,c),\ \left\{a\,b = \frac{\pi}{6}\right\}\right)$

$$\cos\left(\frac{c\,\pi}{6}\right)$$

Numerical Approximation

To compute an approximate numerical value of an expression:

- Use the **evalf** command.

The **evalf** command returns a floating-point (or complex floating-point) number or expression.

> $evalf\left(\cos\left(\frac{\pi}{6}\right)\right)$

$$0.8660254040$$

> $evalf\left(\frac{17}{\sqrt{3}}x^2 + x - e^{\pi}\right)$

$$9.814954579\, x^2 + x - 23.14069264$$

> $evalf(\pi)$

$$3.141592654$$

By default, Maple calculates the result to ten digits of accuracy, but you can specify any number of digits as an index, that is, in brackets (**[ ]**).

> $evalf[40](\pi)$

$$3.141592653589793238462643383279502884197$$

For more information, refer to the **evalf** help page.

See also *Numerically Computing a Limit (page 129)* and *Numeric Integration (page 136)*.

Evaluating Complex Expressions

To evaluate a complex expression:

- Use the **evalc** command.

If possible, the **evalc** command returns the output in the canonical form **expr1** + **i expr2**.

In 2-D Math input, you can enter the imaginary unit using the following two methods.

- In the **Common Symbols** palette, click the **i** or **j** item. See *Palettes (page 15)*.
- Enter *i* or *j*, and then press the symbol completion key. See *Symbol Names (page 21)*.

> $evalc\left(\sqrt{1 + \mathrm{i}}\right)$

$$\frac{\sqrt{2 + 2\sqrt{2}}}{2} + \frac{\mathrm{I}\sqrt{-2 + 2\sqrt{2}}}{2}$$

> $evalc(\sin(3 + 5\,\mathrm{j}))$

$$\sin(3)\cosh(5) + \mathrm{I}\cos(3)\sinh(5)$$

In 1-D Math input, enter the imaginary unit as an uppercase i (**I**).

```
> evalc(2^(1 + I));
```

$$2\cos(\ln(2)) + 2\,\mathrm{I}\sin(\ln(2))$$

Evaluating Boolean Expressions

To evaluate an expression involving relational operators ($=$, $\neq$, $>$, $<$, $\leq$, and $\geq$):

- Use the **evalb** command.

Note: In 1-D Math input, enter $\neq$, $\leq$, and $\geq$ using the **<>**, **<=**, and **>=** operators.

The **evalb** command uses a three-valued logic system. The return values are **true**, **false**, and **FAIL**. If evaluation is not possible, an unevaluated expression is returned.

> $evalb(x = x)$

$$true$$

> $evalb(x = y)$

$$false$$

> $evalb(3 + 2\,I < 2 + 3\,I)$

$$FAIL$$

Important: The **evalb** command does not perform arithmetic for inequalities involving $<$, $\leq$, $>$, or $\geq$, and does not simplify expressions. Ensure that you perform these operations before using the **evalb** command.

> $evalb(\Re(x) < \Re(x + 1))$

$$\Re(x) < 1 + \Re(x)$$

> $evalb(\Re(x) - \Re(x + 1) < 0)$

$$true$$

Applying an Operation or Function to All Elements in a List, Set, Table, Array, Matrix, or Vector

You can use the tilde character (~) to apply an operation or function to all of the elements in a list, set, table, Array, Matrix, or Vector.

In the following example, each element in the Matrix M is multiplied by 2 by adding a tilde character after the multiplication operator($\cdot$).

> $M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \tag{8.7}$$

> $M \cdot\sim 2$

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix} \tag{8.8}$$

In the following example, the function **sin** is applied to each element in the Matrix M.

> sin~(M)

$$\begin{bmatrix} \sin(1) & \sin(2) & \sin(3) \\ \sin(4) & \sin(5) & \sin(6) \\ \sin(7) & \sin(8) & \sin(9) \end{bmatrix} \quad (8.9)$$

The tilde character can also be used to apply a function to multiple data sets, for example,

> *unassign*('*z*') :

> $diff\sim\left(z \cdot x^2 + x \cdot y^2, [x, x, y, y, z, z], [y, z, x, z, x, y]\right);$

$$[2\,y, 2\,x, 2\,y, 0, 2\,x, 0] \quad (8.10)$$

You can use values in one data structure type to compute values in another data structure type, as long as both data structures are dimensional and contain the same number of elements. In the following example, the values in an Array are compared to the values in a Matrix that contains the same number of elements.

> $[12, 88, 20] >\sim \langle 3, 100, 25 \rangle$

$$\begin{bmatrix} 3 < 12 \\ 100 < 88 \\ 25 < 20 \end{bmatrix} \quad (8.11)$$

For more information, refer to the **elementwise** help page.

Levels of Evaluation

In a symbolic mathematics program such as Maple, you encounter the issue of *levels of evaluation.* If you assign **y** to **x**, **z** to **y**, and then 5 to **z**, what is the value of **x**?

At the top-level, Maple *fully evaluates* names. That is, Maple checks if the name or symbol has an assigned value. If it has a value, Maple substitutes the value for the name. If this value has an assigned value, Maple performs a substitution, recursively, until no more substitutions are possible.

For example:

> $x := y$:

> $y := z$:

> $z := 5$:

Maple fully evaluates the name **x**, and returns the value 5.

> x

$$5$$

To control the level of evaluation of an expression:

- Use the **eval** command with an integer second argument.

If passed a single argument, the **eval** command fully evaluates that expression. If you specify an integer second argument, Maple evaluates the expression to that level.

> $eval(x)$

$$5$$

> $eval(x, 1)$

$$y$$

> $eval(x, 2)$

$$z$$

> $eval(x, 3)$

$$5$$

For more details on levels of evaluation, refer to the **lastnameevaluation**, **assigned**, and **evaln** help pages.

Delaying Evaluation

To prevent Maple from immediately evaluating an expression:

- Enclose the expression in right single quotes (**' '**).

Because right single quotes delay evaluation, they are referred to as *unevaluation quotes*.

> $i := 4:$

> i

$$4$$

> $'i'$

$$i$$

Using an Assigned Name as a Variable or Keyword

If you use an assigned name as a variable, Maple evaluates the name to its value, and passes the value to the command. In this example, that causes Maple to return an error message.

> $\sum_{i=1}^{n} i^2$

```
Error, (in sum) summation variable previously assigned, second argument evaluates to 4 = 1
.. n
```

Note: In general, it is recommended that you unassign a name to use it as a variable. See *Unassigning a Name Using Unevaluation Quotes (page 292)*.

To use an assigned name as a variable:

- Enclose the name in unevaluation quotes. Maple passes the name to the command.

> $\sum_{'i'=1}^{n} 'i'^2$

$$\frac{(n+1)^3}{3} - \frac{(n+1)^2}{2} + \frac{n}{6} + \frac{1}{6}$$

Important: It is recommended that you enclose keywords in unevaluation quotes.

For example, if you enclose the keyword **left** in unevaluation quotes, Maple uses the name, not its assigned value. We can also unassign the definition of **'x'** with unevaluation quotes.

> $x := 'x'$:

> $left := 3$:

> $limit\left(\frac{1}{x}, x = 0, 'left'\right)$

$$-\infty$$

Full Evaluation of an Expression in Quotes

Full evaluation of a quoted expression removes one set of right single quotes.

> $i := 4$:

> $''i' + 1'$

$$'i' + 1 \quad (8.12)$$

> **(8.12)**

$$i + 1 \quad (8.13)$$

> **(8.13)**

$$5 \quad (8.14)$$

For information on equation labels and equation label references, see *Equation Labels (page 68)*.

Enclosing an expression in unevaluation quotes delays evaluation, but does not prevent automatic simplification.

> $'q - i + 3\,q'$

$$4\,q - i \quad (8.15)$$

Unassigning a Name Using Unevaluation Quotes

To unassign a name:

- Assign the name enclosed in unevaluation quotes to itself.

> $i := 'i'$:

> i

$$i$$

You can also unassign a name using the **unassign** command. For more information, see *Unassigning Names right single (page 67)*.

9 Basic Programming

You have used Maple interactively in the previous chapters, sequentially performing operations such as executing a single command. Because Maple has a complete programming language, you can also use sophisticated programming constructs.

In Maple, you can write programs called procedures, and save them in modules. These modules can be used and distributed in the same way as Maple packages.

Important: It is strongly recommended that you use the Worksheet mode and 1-D Math input when programming or using programming commands. Hence, all input in this chapter is entered as 1-D Math.

9.1 In This Chapter

Section	Topics
Flow Control (page 295) - Basic programming constructs: if-then statements and loops	• Conditional Execution (**if** Statement) • Repetition (**for** Statement)
Iterative Commands (page 303) - Specialized, efficient iterative commands	• Creating a sequence • Adding and Multiplying Expressions • Selecting Expression Operands • Mapping a Command over a Set or List • Mapping a Binary Command over Two Lists or Vectors
Procedures (page 305) - Maple programs	• Defining and Running Simple Procedures • Procedures with Inputs • Procedure Return Values • Displaying Procedure Definitions • Displaying Maple Library Procedure Definitions • Modules
Programming in Documents (page 308) - Display methods for Maple code	• Code Edit Region • Startup Code • Document Blocks

9.2 Flow Control

Two basic programming constructs in Maple are the **if** statement, which controls the conditional execution of statement sequences, and the **for** statement, which controls the repeated execution of a statement sequence (a loop).

Conditional Execution (if Statement)

You can specify that Maple perform an action only if a condition holds. You can also perform an action, from a set of many, depending on which conditions hold.

Using the **if** statement, you can execute one statement from a series of statements based on a boolean (**true**, **false**, or **FAIL**) condition. Maple tests each condition in order. When a condition is satisfied, Maple executes the corresponding statement, and then exits the **if** statement.

Syntax

The **if** statement has the following syntax.

```
> if conditional_expression1 then
     statement_sequence1
  elif conditional_expression2 then
     statement_sequence2
  elif conditional_expression3 then
     statement_sequence3
  ...
  else
     statement_sequenceN
  end if;
```

The conditional expressions (*conditional_expression1*, *conditional_expression2*, ...) can be any **boolean expression**. You can construct boolean expressions using:

- Relational operators - <, <=, =, >=, >, <>
- Logical operators - **and**, **or**, **xor**, **implies**, **not**
- Logical names - **true**, **false**, **FAIL**

The statement sequences (*statement_sequence1*, *statement_sequence2*, ..., *statement_sequenceN*) can be any sequence of Maple statements, including **if** statements.

The **elif** clauses are optional. You can specify any number of **elif** clauses.

The **else** clause is optional.

Simple if Statements

The simplest **if** statement has only one conditional expression.

```
> if conditional_expression then
     statement_sequence
  end if;
```

If the conditional expression evaluates to **true**, the sequence of statements is executed. Otherwise, Maple immediately exits the **if** statement.

For example:

```
> x := 1173:
```

```
> if not isprime(x) then
     ifactor(x);
  end if;
```

$$(3)\,(17)\,(23)$$

else Clause

In a simple **if** statement with an **else** clause, if the evaluation of the conditional expressions returns **false** or **FAIL**, Maple executes the statement sequence in the **else** clause.

For example:

```
> if false then
     "if statement";
  else
     "else statement";
  end if;
```

"else statement"

elif Clauses

In an **if** statement with **elif** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple exits the **if** statement.

The keyword **elif** stands for **else if**.

```
> x := 11:
> if not type(x, integer) then
      printf("%a is not an integer.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  end if;
```

```
11 is an integer with more than one digit.
```

Order of elif Clauses: An **elif** clause's statement sequence is executed only if the evaluation of all previous conditional expressions returns **false** or **FAIL**, and the evaluation of its conditional expression returns **true**. This means that changing the order of **elif** clauses may change the behavior of the **if** statement.

In the following **if** statement, the **elif** clauses are in the **wrong order**.

```
> if not(type(x, integer)) then
      printf("%a is not an integer.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  end if;
```

```
11 is an integer with one digit.
```

elif and else Clauses

In an **if** statement with **elif** and **else** clauses, Maple evaluates the conditional expressions in order until one returns **true**. Maple executes the corresponding statement sequence, and then exits the **if** statement. If no evaluation returns **true**, Maple executes the statement sequence in the **else** clause.

```
> x := -12:
> if not type(x, integer) then
      printf("%a is not an integer.", x);
  elif x >= 10 then
      printf("%a is an integer with more than one digit.", x);
  elif x >= 0 then
      printf("%a is an integer with one digit.", x);
  else
      printf("%a is a negative integer.", x);
  end if;
```

```
-12 is a negative integer.
```

For more information on the **if** statement, refer to the **if** help page.

Repetition (for Statement)

Using **repetition** statements, you can repeatedly execute a statement sequence. You can repeat the statements in three ways.

- Until a counter variable value exceeds a limit (**for/from** loop)
- For each operand of an expression (**for/in** loop)
- Until a boolean condition does not hold (**while** loop or **until** loop)

for/from Loop

The **for/from** loop statement repeats a statement sequence until a counter variable value exceeds a limit.

Syntax

The **for**/**from** loop has the following syntax.

```
> for counter from initial by increment to final do
      statement_sequence
  end do;
```

The behavior of the **for/from** loop is:

1. Assign the *initial* value to the name **counter**.
2. Compare the value of **counter** to the value of *final*. If the **counter** value **exceeds** the *final* value, exit the loop. (This is the *loop bound test*.)
3. Execute the *statement_sequence*.
4. Increment the **counter** value by the value of *increment*.
5. Repeat steps 2 to 4, until Maple exits the loop.

The **from**, **by**, and **to** clauses are optional and can be in any order between the **for** clause and the **do** keyword. **Table 9.1** lists the default clause values.

Table 9.1: Default Clause Values

Clause	Default Value
from *initial*	1
by *increment*	1
to *final*	**infinity** (∞)

Examples

The following loop returns the square root of the integers 1 to 5 (inclusive).

```
> for n to 5 do
     evalf(sqrt(n));
  end do;
```

$$1.$$

$$1.414213562$$

$$1.732050808$$

$$2.$$

$$2.236067977$$

When the value of the counter variable **n** is **strictly greater than 5**, Maple exits the loop.

```
> n;
```

$$6$$

The previous loop is equivalent to the following **for/from** statement.

```
> for n from 1 by 1 to 5 do
     evalf(sqrt(n));
  end do;
```

$$1.$$

$$1.414213562$$

$$1.732050808$$

$$2.$$

$$2.236067977$$

The **by** value can be negative. The loop repeats until the value of the counter variable is **strictly less than** the **final** value.

```
> for n from 10 by -1 to 3 do
     if isprime(n) then
        print(n);
     end if;
  end do;
```

$$7$$

$$5$$

$$3$$

```
> n;
```

$$2$$

for/in Loop

The **for/in** loop statement repeats a statement sequence for each component (*operand*) of an expression, for example, the elements of a list.

Syntax

The **for/in** loop has the following syntax.

```
> for variable in expression do
      statement_sequence
  end do;
```

The **for** clause must appear first.

The behavior of the **for/in** loop is:

1. Assign the first operand of *expression* to the name *variable*.
2. Execute the *statement_sequence*.
3. Assign the next operand of *expression* to *variable*.
4. Repeat steps 2 and 3 for each operand in *expression*. If there are no more operands, exit the loop. (This is the *loop bound test*.)

Example

The following loop returns a floating-point approximation to the **sin** function at the angles (measured in degree) in the list **L**.

```
> L := [23.4, 87.2, 43.0, 99.7]:
> for i in L do
      evalf(sin(i*Pi/180));
  end do;
```

$$0.3971478907$$

$$0.9988061374$$

$$0.6819983602$$

$$0.9857034690$$

while Loop and until Loop

The **while** loop repeats a statement sequence until a boolean expression does not hold.

The **until** loop also repeats a statement sequence until a boolean expression does not hold, but it tests the terminating condition *at the end of each iteration* of the loop, instead of the beginning.

Syntax

The **while** loop has the following syntax.

```
> while conditional_expression do
      statement_sequence
  end do;
```

A **while** loop repeats until its **boolean expression** *conditional_expression* evaluates to **false** or **FAIL**. For more information on boolean expressions, see *Conditional Execution (if Statement) (page 295)*.

The **until** loop has the following syntax.

```
> do
      statement_sequence
  until conditional expression;
```

Note the difference in syntax: the until clause appears in place of **end do**

Example

Compare the following loops. In the first one, the terminating condition is met the first time it is tested, and the loop terminates immediately. In the second one, which uses the **until** clause, the body of the loop is performed before the condition is tested, and the loop is used.

```
> a:=1;
```

$$a := 1 \tag{9.1}$$

```
> while a mod 7 <>1 do
     a:=a+1;
  end do:
> a;
```

$$1 \tag{9.2}$$

```
> a:=1;
```

$$a := 1 \tag{9.3}$$

```
> do
     a:=a+1;
  until a mod 7=1:
> a;
```

$$8 \tag{9.4}$$

The following loop computes the digits of 872,349 in base 7 (in order of *increasing* significance).

```
> x := 872349:
> while x > 0 do
     irem(x, 7);
     x := iquo(x, 7);
  end do;
```

$$2$$
$$x := 124621$$
$$0$$
$$x := 17803$$
$$2$$
$$x := 2543$$
$$2$$
$$x := 363$$
$$6$$
$$x := 51$$
$$2$$
$$x := 7$$
$$0$$
$$x := 1$$
$$1$$
$$x := 0$$

To perform such conversions efficiently, use the **convert/base** command.

```
> convert(872349, base, 7);
```

$$[2, 0, 2, 2, 6, 2, 0, 1]$$

For information on non-base 10 numbers, see *Non-Base 10 Numbers (page 78)*.

General Loop Statements

You can include a **while** statement in a **for/from** or **for/in** loop.

The general **for**/**from** loop has the following syntax.

```
> for counter from initial by increment to final
  while conditional_expression do
      statement_sequence
  end do;
```

The general **for**/**in** loop has the following syntax.

```
> for variable in expression
  while conditional_expression do
      statement_sequence
  end do;
```

After testing the loop bound condition at the beginning of each iteration of the **for** loop, Maple evaluates *conditional_expression*.

- If *conditional_expression* evaluates to **false** or **FAIL**, Maple exits the loop.
- If *conditional_expression* evaluates to **true**, Maple executes *statement_sequence*.

The equivalent general **until** loops have the same form but with **end do** replaced with **until** *conditional_expression*.

Infinite Loops

You can construct a loop for which there is no exit condition, for example, a **while** loop in which the *conditional_expression* always evaluates to **true**. This is called an *infinite loop*. Maple indefinitely executes an infinite loop unless it executes a **break**, **quit**, or **return** statement or you interrupt the computation using the interrupt icon in the toolbar (in worksheet versions). For more information, refer to the **break**, **quit**, **return**, and **interrupt** help pages.

Additional Information

For more information on the **for** statement and looping, refer to the **do** help page and the Loops section of the *Maple Statements* chapter of the *Maple Programming Guide*.

9.3 Iterative Commands

Maple has commands that perform common selection and repetition operations. These commands are more efficient than similar algorithms implemented using library commands. **Table 9.2** lists the iterative commands.

Table 9.2: Iterative Commands

Command	Description
seq	Create sequence
add	Compute numeric sum
mul	Compute numeric product
select	Return operands that satisfy a condition
remove	Return operands that do not satisfy a condition
selectremove	Return operands that satisfy a condition and separately return operands that do not satisfy a condition
map	Apply command to the operands of an expression
zip	Apply binary command to the operands of two lists or vectors

Creating a Sequence

The **seq** command creates a sequence of values by evaluating a specified expression over a range of index values or the operands of an expression. See **Table 9.3**.

Table 9.3: The seq Command

Calling Sequence Syntax	Examples
seq(*expression*, *name* = *initial* .. *final***);**	`> seq(exp(x), x=-2..0);` $e^{-2}, e^{-1}, 1$
seq(*expression*, *name* **in** *expression***);**	`> seq(u, u in [Pi/4, Pi^2/2, 1/Pi]);` $\frac{\pi}{4}, \frac{\pi^2}{2}, \frac{1}{\pi}$

Adding and Multiplying Expressions

The **add** and **mul** commands add and multiply sequences of expressions over a range of index values or the operands of an expression. See **Table 9.4**.

Table 9.4: The add and mul Commands

Calling Sequence Syntax	Examples

add(*expression*, *name* = *initial* .. *final*);
 	
mul(*expression*, *name* = *initial* .. *final*);
	`> add(exp(x), x = 2..4);`
	$e^2 + e^3 + e^4$
	`> mul(2*x, x = 1 .. 10);`
	3715891200

add(*expression*, *name* **in** *expression*);
 	
mul(*expression*, *name* **in** *expression*);
	`> add(u, u in [Pi/4, Pi/2, Pi]);`
	$\frac{7}{4}\pi$
	`> mul(u, u in [Pi/4, Pi/2, Pi]);`
	$\frac{1}{8}\pi^3$

The endpoints of the index range (**initial** and **final**) in the **add** and **mul** calling sequence must evaluate to numeric constants. For information on symbolic sums and products, refer to the **sum** and **product** help pages.

Selecting Expression Operands

The **select**, **remove**, and **selectremove** commands apply a boolean-valued procedure or command to the operands of an expression. For information on operands, refer to the **op** help page.

- The **select** command returns the operands for which the procedure or command returns **true**.
- The **remove** command returns the operands for which the procedure or command returns **false** or **FAIL**.
- The selectremove command returns two expressions of the same type as the input expression.
  - The first consists of the operands for which the procedure or command returns **true**.
  - The second consists of the operands for which the procedure or command returns **false** or **FAIL**.

The structure of the output is the same as the structure of the input. See **Table 9.5**.

For information on Maple procedures, see *Procedures (page 305)*.

Table 9.5: The select, remove, and selectremove Commands

Calling Sequence Syntax	Examples
select(*proc_cmd*, *expression*);	`> select(issqr, {198331, 889249, 11751184, 9857934});` $\{889249, 11751184\}$
remove(*proc_cmd*, *expression*);	`> remove(var -> degree(var) > 3, 2*x^3*y - y^3*x + z );` z
selectremove(*proc_cmd*, *expression*);	`> selectremove(x -> evalb(x > round(x)), [sin(0.), sin(1.), sin(3.)]);` $[0.1411200081], [0., 0.8414709848]$

For information on optional arguments to the selection commands, refer to the **select** help page.

Mapping a Command over a Set or List

The **map** command applies a name, procedure, or command to each element in a set or list. See **Table 9.6**.

Table 9.6: The map Command

Calling Sequence Syntax	Examples

map(*name_proc_cmd*, *expression*);	`> map(f, {a, b, c});`
	 $\{f(a), f(b), f(c)\}$
	 `> map(u -> int(cos(x), x = 0 .. u), [Pi/4, Pi/7, Pi/3.0]);`
	 $\left[\frac{\sqrt{2}}{2}, \cos\left(\frac{5\pi}{14}\right), 0.8660254037\right]$

An alternative to the map command is to apply a function elementwise, using ~. For more information ~, see *Applying an Operation or Function to All Elements in a List, Set, Table, Array, Matrix, or Vector (page 289)*. For information on mapping over the operands of other expressions, optional arguments to the **map** command, and other mapping commands, refer to the **map** help page.

Mapping a Binary Command over Two Lists or Vectors

The **zip** command applies a name or binary procedure or command component-wise to two lists or vectors.

By default, the length of the returned object is that of the shorter list or vector. If you specify a value as the (optional) fourth argument, it is used as the value of the missing elements of the shorter list or vector. In this case, the length of the return value is that of the longer list or vector. See **Table 9.7**.

Table 9.7: The zip Command

Calling Sequence Syntax	Examples

zip(*proc_cmd*, *a*, *b*);
 zip(*proc_cmd*, *a*, *b*, *fill*);
	`> zip(f, [i, j], [k, l]);`
	 $[f(i, k), f(j, l)]$
	 `> zip(AiryAi, [1, 2], [0], 1);`
	 $\left[-\frac{3^{1/6}\,\Gamma\left(\frac{2}{3}\right)}{2\pi}, Ai''(1)\right]$
	 This is equivalent to $[\mathrm{AiryAi}(1, 0), \mathrm{AiryAi}(2, 1)]$.

For more information on the **zip** command, refer to the **zip** help page.

Additional Information

For more information on looping commands, refer to the corresponding command help page.

9.4 Procedures

A Maple procedure is a program consisting of Maple statements. Using procedures, you can quickly execute the contained sequence of statements.

Defining and Running Simple Procedures

To define a procedure, enclose a sequence of statements between **proc(...)** and **end proc** statements. In general, you assign a procedure definition to a name.

The following procedure returns the square root of 2.

```
> p := proc() sqrt(2); end proc;
```

$$p := \textbf{proc}(\,)\ \text{sqrt}(2)\ \textbf{end proc}$$

Note: Maple returns the procedure definition.

To improve readability of procedures, it is recommended that you define a procedure using multiple lines, and indent the lines using space characters. To begin a new line (without evaluating the incomplete procedure definition), press **Shift+Enter**. When you have finished entering the procedure, press **Enter** to create the procedure.

For example:

```
> p := proc()
      sqrt(2);
  end proc:
```

To run the procedure **p**, enter its name followed by parentheses (**( )**).

```
> p();
```

$$\sqrt{2}$$

Procedures with Inputs

You can define a procedure that accepts user input. In the parentheses of the **proc** statement, specify the parameter names. For multiple parameters, separate the names with commas.

```
> geometric_mean := proc(x, y)
      sqrt(x*y);
  end proc:
```

When the user runs the procedure, the parameter names are replaced by the argument values.

```
> geometric_mean(13, 17);
```

$$\sqrt{221}$$

```
> geometric_mean(13.5, 17.1);
```

$$15.19374871$$

For more information on writing procedures, including options and local and global variables, refer to the **procedure** help page.

Procedure Return Values

When you run a procedure, Maple returns **only** the last statement result value computed. Maple does not return the output for each statement in the procedure. It is irrelevant whether you use semicolons or colons as statement separators.

```
> p := proc(a, b)
      a + b;
      a - b:
  end proc:
```

```
> p(1, 2);
```

$$-1$$

Displaying Procedure Definitions

Unlike simple Maple objects, you cannot display the value of a procedure by entering its name.

```
> geometric_mean;
```

$$geometric\_mean$$

You must evaluate the name of the procedure using the **print** (or **eval**) command.

```
> print(geometric_mean);
```

$$\textbf{proc}(x, y)\ \ \mathrm{sqrt}(y*x)\ \ \textbf{end proc}$$

Displaying Maple Library Procedure Definitions

Maple procedure definitions are a valuable learning tool. To learn how to program in Maple, it is recommended that you examine the procedures available in the Maple library.

By default, the **print** command returns only the **proc** and **end proc** statements and (if present) the description fields of a Maple procedure.

```
> print(lcm);
```

$$\textbf{proc}(a, b)\ \ \ldots\ \ \textbf{end proc}$$

To display a Maple library procedure definition, first set the value of the **interface verboseproc** option to **2**. Then re-execute the **print** calling sequence.

```
> interface('verboseproc' = 2):
> print(lcm);
```

```
proc(a, b)
    option remember, Copyright (c) 1990 by the University of Waterloo. All rights reserved.;
    local q, t;
    if nargs = 0 then
        1
    elif nargs = 1 then
        t := expand(a); sign(t)*t
    elif 2 < nargs then
        foldl(procname, args)
    elif type(a, 'integer') and type(b, 'integer') then
        ilcm(a, b)
    else
        gcd(a, b, 'q'); q*b
    end if
end proc
```

Modules

Maple procedures associate a sequence of commands with a single command. The module, a more complex programming structure, allows you to associate related procedures and data.

A key feature of modules is that they *export* variables. This means that the variables are available outside the module in which they are created. Most Maple packages are implemented as modules. The package commands are exports of the module.

For more information on modules, refer to the **module** help page.

Objects

Objects take the idea of associating data and procedures beyond what modules provide. With objects, multiple instances of a class of objects can be created. Each individual object can have its own data, yet share other values and procedures with the entire class objects. A well implemented class of objects can be used in Maple as naturally as a built-in Maple type.

For more information on objects, refer to the **object** help page.

9.5 Programming in Documents

To write Maple code, you could simply open a Maple worksheet and start typing. However, if you want to create a readable document with the code interspersed or hidden, there are several options available: code edit regions and start up code. Both these features use a code editor which has features such as syntax highlighting and line numbers.

Code Edit Region

The code edit region allows you to program in one contained region, in a natural way. Features include the ability to press **Enter** for line breaking and indentation preservation. **Figure 9.1** shows the expanded code edit region.

To insert a new code edit region into your worksheet:

- From the **Insert** menu, select **Code Edit Region**.

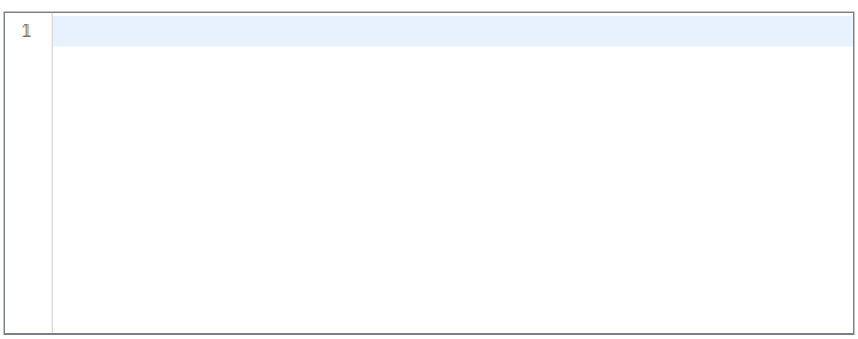


Figure 9.1: Code Edit Region

To execute the code within this region, click anywhere in this region, then click the **Execute Selection** icon (!) on the toolbar. (Alternatively, from the **Evaluate** menu, select **Execute Code**.)

You can hide the code in a code edit region by minimizing the region. To minimize, select **View → Collapse Code Edit Region**. When the region is minimized, an icon appears with the first line of the code written next to it. It is recommended that you make the first line a comment describing the program or programs contained in the region. See **Figure 9.2**.

Figure 9.2: Collapsed Code Edit Region

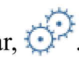
To re-execute the code in the region while it is collapsed, click this icon.

For more information, refer to the **CodeEditRegion** help page.

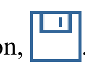
Startup Code

Startup code allows you to define commands and procedures that are executed each time the document is opened and after restart is called. This code is completely hidden to others reading the document. For example, use this region to define procedures that will be used throughout the document code but that would take up space and distract readers from the message of the document.

To enter startup code for a document:

1. From the **Edit** menu, select **Startup Code**. Alternatively, click the startup code icon in the toolbar, .
2. Enter commands to be run each time the worksheet is opened or **restart** is called.
3. To check the syntax of the entered code while entering your Maple commands or before closing the editor, from the **Edit** menu. select **Check Syntax Now**.

Note: You can also check the **Check Syntax Automatically** option to enable continuous syntax checking. It is recommended that you check the syntax before saving so that your startup code does not prevent Maple from opening successfully.

4. To save the contents, from the **File** menu, select **Save Code**. Alternatively, click the save icon, .
5. Close **Startup Code**.

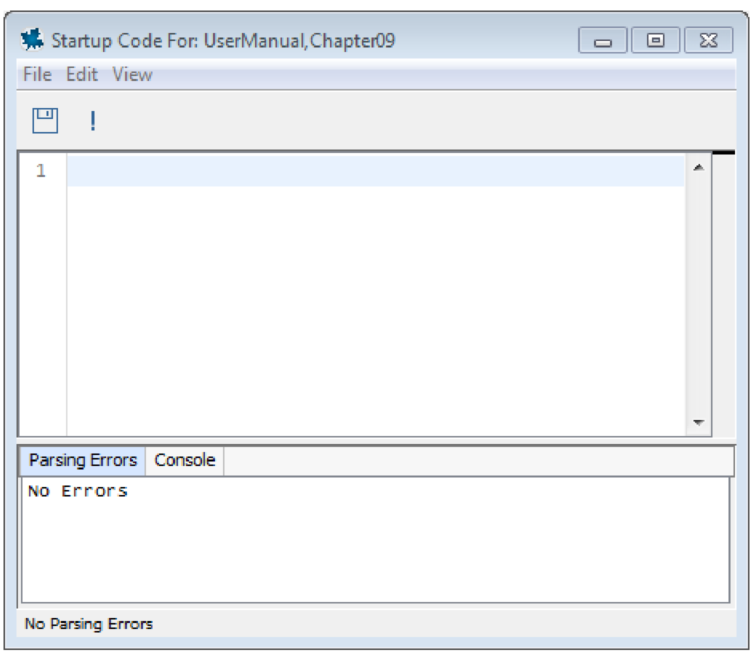


Figure 9.3: Startup Code Editor

For more information, refer to the **startupcode** help page.

9.6 Additional Information

The *Maple Programming Guide* provides an in-depth reference for programming in Maple. Topics include statements, data structures, procedures, packages, and debugging your code.

- Access via the help system. From the Table of Contents, select **Manuals>Programming Guide**.

The Programming Guide is also available as a PDF on the Maplesoft website.

http://www.maplesoft.com/documentation_center

10 Embedded Components and Maplets

These graphical components help you to create documents to use and share with colleagues or students, that interact with Maple code within the document without needing the reader to understand that Maple code. Other methods of interaction with Maple are described throughout this guide.

10.1 In This Chapter

Section	Topics
Using Embedded Components (page 311) - Basic interacting with Maple documents containing embedded components	• Interacting with Components • Printing and Exporting
Creating Embedded Components (page 314) - Methods for creating embedded components that work together and with your document	• Inserting Components • Editing Components • Removing Components • Integrating into a Document
Using Maplets (page 320) - Methods for launching a Maplet	• Maplet File • Maple Document
Authoring Maplets (page 321) - Methods for authoring and saving a Maplet	• Simple Maplet • **Maplet Builder** • **Maplets** Package • Saving

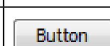
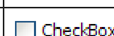
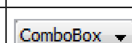
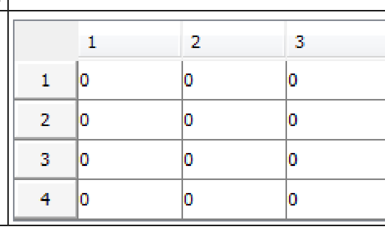
10.2 Using Embedded Components

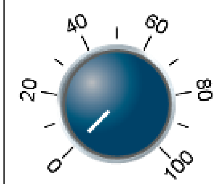
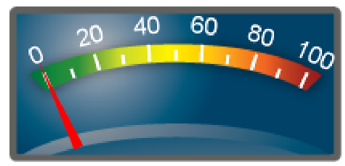
Interacting

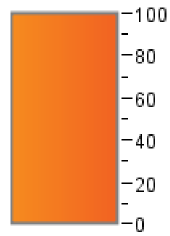
Embedded components allow readers to interact with Maple code through graphical components, rather than commands. They can be used alone, as with a button that you click to execute code, or together, such as a drop-down menu where you select an item, and a change takes place in a plot component.

Component Descriptions

Table 10.1: Embedded Component Descriptions

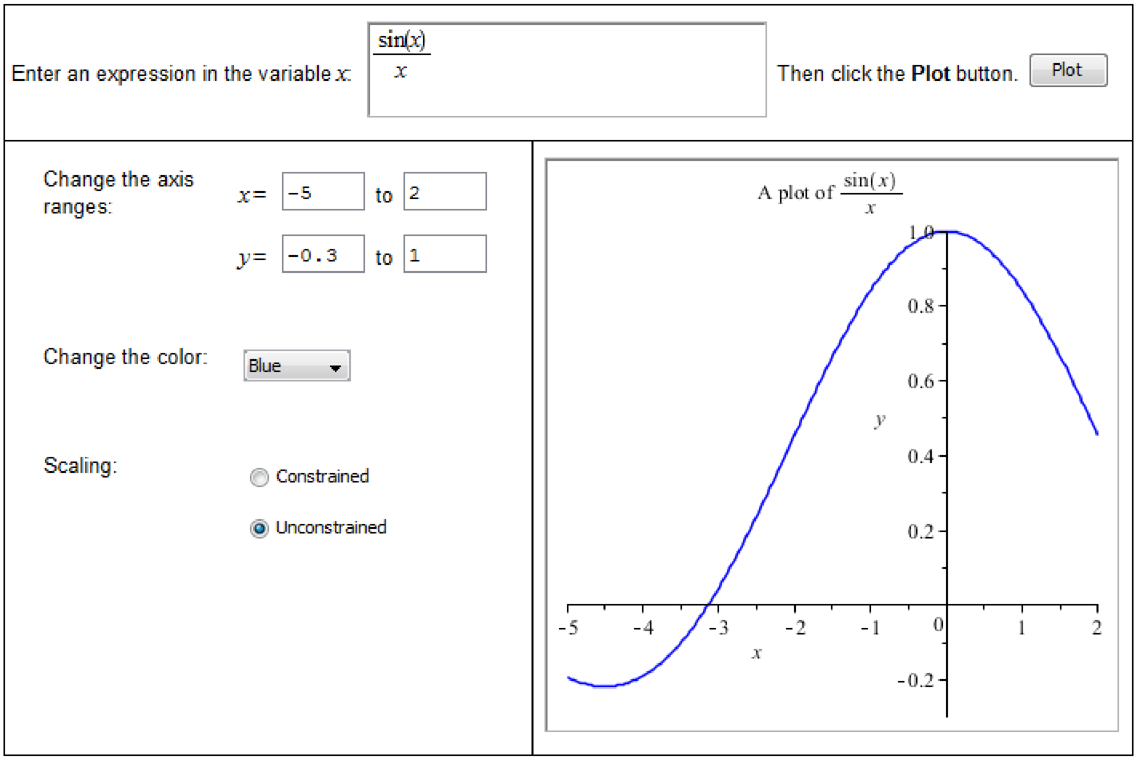
Component Name and Description	Inserted Image
Button - Click to perform an action; that is, execute code.	
Check Box - Select or de-select. Change the caption, and enter code to execute when the value changes.	
Combo Box - Select one of the listed options from the drop-down menu. Change the items listed, and enter code to execute when the value changes.	
Data Table - Link this embedded component to a Matrix, Vector, or Array in your worksheet.	

Component Name and Description	Inserted Image
Dial - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Label - Display a label. The value can be updated based on code in the document or another embedded component.	Label
List Box - Display a list of items. Change the items listed, and enter code to execute when an item is selected.	ListBox
Math Expression - Enter or display a mathematical expression. The value can be updated based on code in the document or another embedded component.	
Meter - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Microphone Component - Capture sound from a recording device. Change setting options from properties, and enter code for start and stop recording actions.	
Plot - Display a 2-D or 3-D plot or animation. This plot or animation can be interacted with in the same way as other plots (see *Plots and Animations (page 183)*). The value can be updated based on code in the document or another embedded component. You can also enter code to be executed when the **Click and Drag** pointer is used to click or drag in the plot region.	Embedded Plot Window
Radio Button - Use with other radio buttons to select one in a group. Enter code to execute when the value changes.	RadioButton
Rotary Gauge - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Slider - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Text Area - Enter or display plain text. The value can be updated based on code in the document or another embedded component, and you can enter code to execute when the value changes.	
Toggle Button - Select or display one of two options. Change the images displayed, and enter to code to execute when the value changes.	

Component Name and Description	Inserted Image
Video Player - Play a video. Enter code that specifies an action to perform when the video player reaches a marker during playback.	
Volume Gauge - Select or display an integer or floating-point value. Change the display, and enter code to execute when the value changes.	
Shortcut Component - Use to hyperlink various types of content, including help pages, MapleCloud documents, and URLs.	
Speaker Component - Play a sound. Customize component by modifying setting options under properties.	

Example 1 - Using Embedded Components

This example demonstrates several components working together to perform a task. The user inputs an expression, which is plotted when the button is clicked. Plot options are controlled by text areas, a combo box, a math expression, and radio buttons.



Printing and Exporting a Document with Embedded Components

Printing: When printing a document, embedded components are rendered as they appear on screen.

Exporting: Exporting a document with embedded components to other formats produces the following results.

- HTML format - components are exported as **.gif** files.
- RTF format - components are rendered as **bitmap** images in the **.rtf** document.
- LaTeX - components are exported as **.eps** files.
- PDF - components are rendered as static images.

10.3 Creating Embedded Components

Embedded Components are graphical components that you can add to your document. They provide interactive access to Maple code without requiring the user to know Maple commands, and include buttons, sliders, math and text input areas, plot display, and shortcut components.

Inserting Components

The graphical interface components can be inserted by using the **Components** palette (**Figure 10.1**) or by cutting/copying and pasting existing components to another area of the document. Although copied components have most of the same characteristics, they are distinct.

If the **Components** palette is not visible, see *Palettes (page 15)* for instructions on viewing palettes.

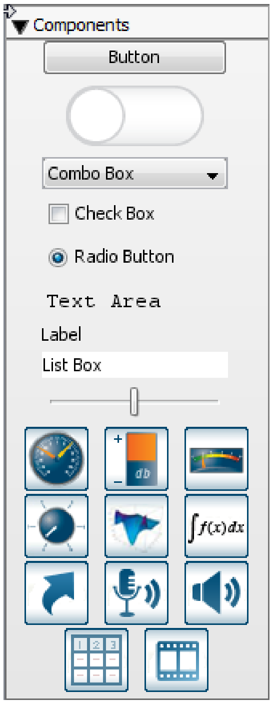


Figure 10.1: Components Palette

Editing Component Properties: General Process

To edit properties of components embedded in the document:

1. Click the component to display the context panel.
2. In the context panel, enter values and contents in the fields as necessary.
3. To define an action, such as an action to perform when the slider is moved, click the component, and then select **Edit Value Changed Code...** from the Context Panel. A **Code Edit Region** opens allowing you to enter Maple code that is executed when the event occurs. For details, refer to the **DocumentTools** help page.

Note: You can also edit embedded component properties and actions that are performed when a value changes by using Maple worksheet **Edit** menu options, **Edit > Component Properties**, and **Edit > Component Code**, respectively. For more information on options available in the **Edit** menu, see **Edit menu options**.

Removing Graphical Interface Components

You can remove an embedded component by:

- Using the **Delete** key
- Using the **Backspace** key
- Placing the cursor at the component and selecting from the document menu, **Edit→Delete Element**

Integrating Components into a Document

Use embedded components to display information from calculations, obtain input from a reader, or perform calculations at the click of a button, all without your readers having an understanding of Maple commands. They can be entered in any part of a Maple document, including a document block or table. For details on each component, see its help page.

This simple example inserts a slider with a label that indicates the current value of the slider.

1. Place the cursor in the location where the embedded component is to be inserted.
2. In the **Components** palette, click the **Slider** item. A slider is inserted into the document.
3. In the **Components** palette, click the **Label** item. A label is inserted next to the slider.

4. Click the label component. The **Label** context panel opens. See **Figure 10.2**.

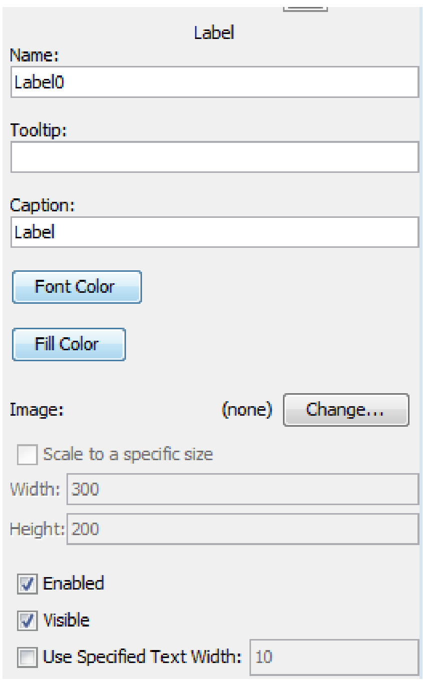


Figure 10.2: Label Properties Dialog

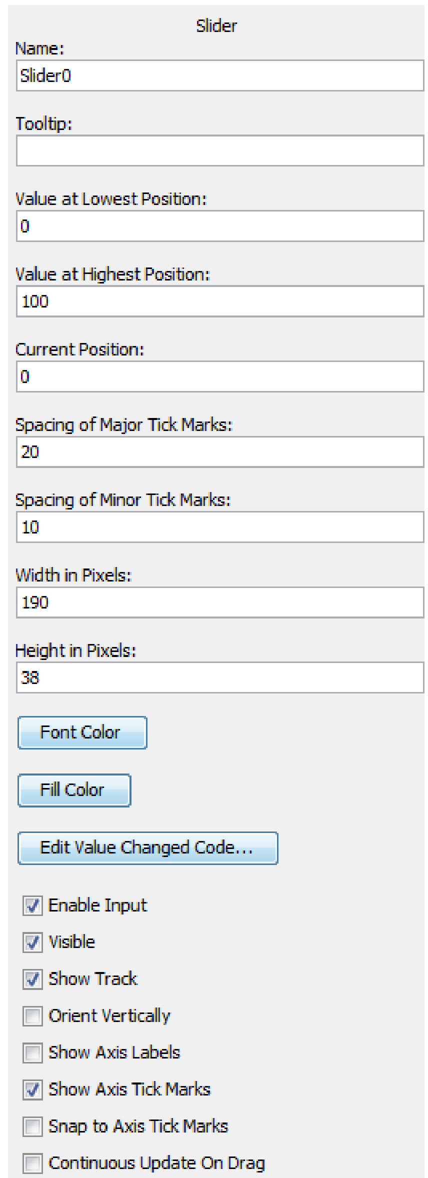


Figure 10.3: Slider Properties Dialog

5. Name the component **SliderLabel** and click **OK**.
6. Click the slider component. The **Slider** context panel opens. See **Figure 10.3**.

7. Name the component **Slider1.**
8. Enter the value at the lowest position as **0** and the highest as **100.**
9. Enter major tick marks at **20** and minor tick marks at **10**.
10. Make sure that the **Continuous Update On Drag** check box is selected.
11. Click **OK**.
12. To define an action, click the slider component, and then select **Edit Value Changed Code...** from the context panel. This launches a dialog that allows you to program the action of displaying the slider value in the label component. The dialog includes information on how to program actions between embedded components. The **use...in/end use;** statement allows you to specify routines using the short form of accessing a package command without invoking the package. For details on this command, refer to the **use** help page.
13. Before the **end use;** statement at the bottom of the dialog, enter the following command.

Do(%SliderLabel(caption)=%Slider0(value));

14. Save the code, and exit the code editor.

As you move the arrow indicator, the value from the slider populates the **Label** caption field.

For details on this command, refer to the **DocumentTools[Do]** help page.

Example 2 - Creating Embedded Components

In chapter 7 (see *Embedded Components (page 263)*), you created a document that included embedded components, imported from a task template. Here, we re-create that configuration of components. This example takes two parameters, a and b, as inputs, then plots the function $y = b\,x + a$ and calculates $\frac{a}{b}$.

1. **Create the components.**

The table layout is best done after the components are finished, in case the configuration of the components changes as you are working.

Create two **DialComponent**s to set the parameters, a and b, one **RotaryGaugeComponent** to display the result, $\frac{a}{b}$, one **PlotComponent** to display the plot, and one **MathContainerComponent** to display the function. Note that you do not have to use the dial and rotary gauge components here, you can also use the slider instead.

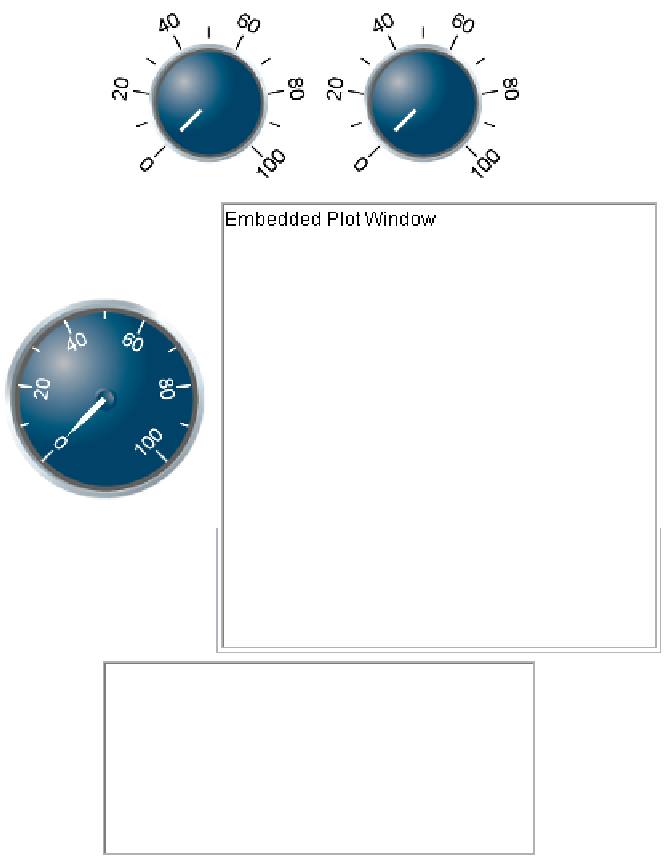


Figure 10.4: The Inserted Components

2. **Edit the display of the components.**

Open the context panel for the first **DialComponent**, and notice that it already has a name. This name is used to reference the component from other components, and is unique. Change the display of each of the components as follows:

- **Dial0**: no changes.
- **Dial1**: change the **Value at Lowest Position** to 1, the **Value at Highest Position** to 10, the **Spacing of Major Tick Marks** to 1, and the **Spacing of Minor Tick Marks** to 1.
- **RotaryGauge0**: change the **Value at Highest Position** to 40, the **Spacing of Major Tick Marks** to 5, and the **Spacing of Minor Tick Marks** to 1.
- **Plot0**: no changes.
- **MathContainer0**: change the **Width in Pixels** to 200, and the **Height in Pixels** to 45.

Note the names of all of the components, and close each dialog before moving on.

3. **Create actions for the components.**

Components can perform actions when their values are changed, so the code to execute needs to be in the dials. That way, whenever one of them is changed, the other components are updated to reflect that change.

The following Maple commands retrieve the values of the parameters and display them in the other three components:

```
> parameter1:=Do(%Dial0):
```

```
> parameter2:=Do(%Dial1):
```

```
> Do(%RotaryGauge0=parameter1/parameter2);
> Do(%Plot0=plot((parameter2*x+parameter1), x=-50..50, y=-50..50));
> Do(%MathContainer0=(y=parameter2*x+parameter1));
```

4. **Test the actions.**

To test these commands, first load the **DocumentTools** package with the following command.

```
> with(DocumentTools):
```

After loading the package, execute the commands in the document, and verify that the components you inserted are updated: the gauge should change to the computed value, a plot should appear in the plot component, and the function should display in the math container.

5. **Troubleshooting.**

The first **Do** command gives an error, because the second parameter is 0. One way to avoid this problem is to change the range of the second dial. In the **Component Properties** dialog for the second **DialComponent**, change the **Value at Lowest Position** from 0 to 1. Alternatively, you could change the code to compensate, with an **if** statement.

6. **Copy the actions to the components.**

After the commands work as expected, you can copy them into the components.

- Click the first **DialComponent** and select **Edit Value Changed Code** from the context panel. Copy and paste the commands into the space between the **use** statements.

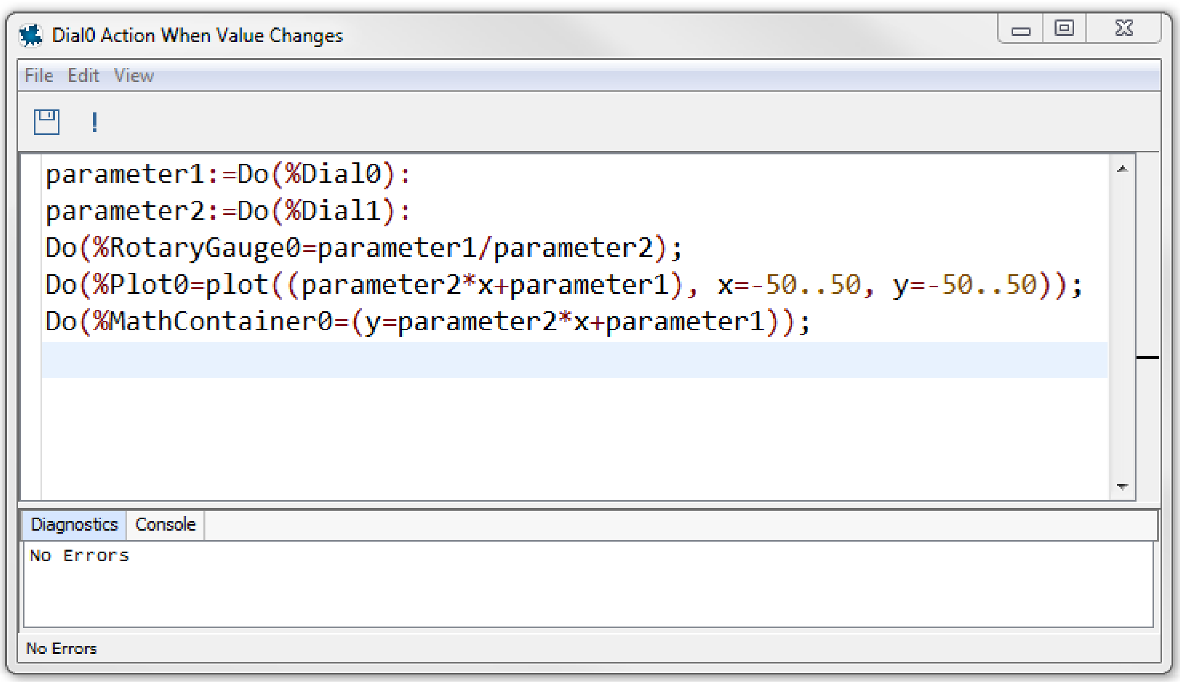
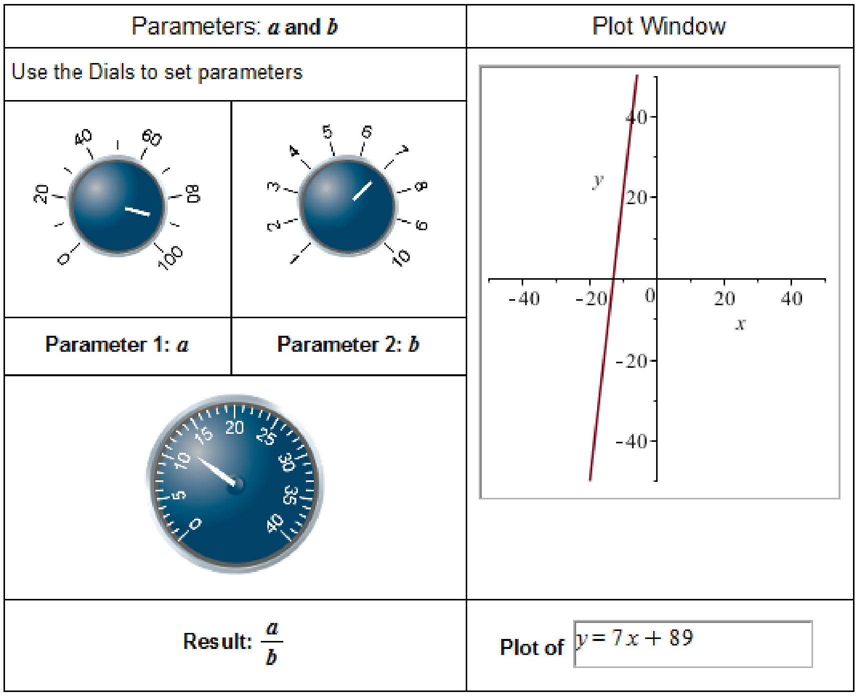


Figure 10.5: DialComponent Action Code Editor

- Save the code, and exit the code editor.
- Do the same for the second **DialComponent**. Now, moving either dial will update the rotary gauge, plot, and formula displayed in the math component.

7. **Create the layout for the components.**

Create a table, and then cut and paste the components into it, along with explanatory text. **Important:** you must cut, not copy, the components, or their names will be changed to avoid duplication. For information on creating and modifying tables, refer to *Tables (page 245)*.



10.4 Using Maplets

A Maplet is a popup graphical user interface that provides interactive access to the Maple engine through buttons, text regions, slider bars, and other visual interfaces. You can create your own Maplets, and you can take advantage of the built-in Maplets that cover numerous academic and specialized topics. Built-in Maplets include some assistants and tutors, such as the ODE Analyzer. For more information on this assistant, see *Ordinary Differential Equations (ODEs) (page 87)*.

Maplet applications are launched by executing Maplet code. Maplet code can be saved in a Maplet (**.maplet**) file or Maple document (**.mw**).

Maplet File

To launch a Maplet application saved as a Maplet file:

- In Windows, double-click the file from a Windows file browser.
- In UNIX and on Macintosh, use the command-line interface. At the command-line, enter **maple -q** <*maplet_filename*>.

To view and edit the Maplet code contained within the .maplet file:

1. Start Maple.
2. From the **File** menu, select **Open**. Maple displays the **Open** dialog.
3. In the **Files of Type** drop-down list, select **.maplet**.
4. Navigate to the location of the **.maplet** file and select the file.

5. Click **Open**.

Maple Document

To launch a Maplet application for which the Maple code is contained in a Maple document, you need to execute the Maplet code. To display the Maplet application, you must use the **Maplets[Display]** command. **Note:** The Maplet code may be quite large if the Maplet application is complex. In this case, execute the document to ensure user-defined procedures that are referenced in the Maplet application are also defined.

Typical procedure:

1. If present, evaluate user-defined procedures.

```
Myproc:=proc..
```

2. Load the **Maplets[Elements]** package.

```
with( Maplets[Elements] );
```

3. Evaluate the Maplet definition.

```
Maplet_name:=Maplet( Maplet_definition );
```

4. Display the Maplet application.

```
Maplets[Display]( Maplet_name );
```

Important: When a Maplet application is running, you cannot interact with the Maple document.

10.5 Authoring Maplets

To author Maplets, you can use the **Maplet Builder** (GUI-based) or the **Maplets** package (syntax-based). The **Maplet Builder** allows you to drag and drop buttons, sliders, text regions, and other elements to define the Maplet application and set the element properties to perform an action on selection or update of the element. The **Maplet Builder** is designed to create simple Maplets. The **Maplets** package offers more capabilities, control, and options when designing complicated Maplet applications.

Designing a Maplet application is similar to constructing a house. When building a house, you first construct the skeletal structure (that is, foundation, floors, and walls) and then proceed to add the windows and doors. Constructing a Maplet is no different. First define the rows and columns of the Maplet application and then proceed to add the body elements (such as buttons, text fields, and plot regions).

Simple Maplet

A Maplet application can be defined using the commands in the **Maplets[Elements]** package and then launched using the **Maplets[Display]** command. The following commands define and run a very simple Maplet application that contains the text string "Hello World".

```
> with(Maplets[Elements]):
> MySimpleMaplet:= Maplet([["Hello World"]]):
> Maplets[Display](MySimpleMaplet):
```

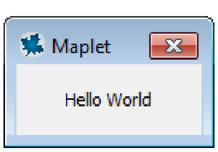


Figure 10.6: A Simple Maplet

Maplet Builder

To start the **Maplet Builder**:

- From the **Tools** menu, select **Assistants** → **Maplet Builder**.

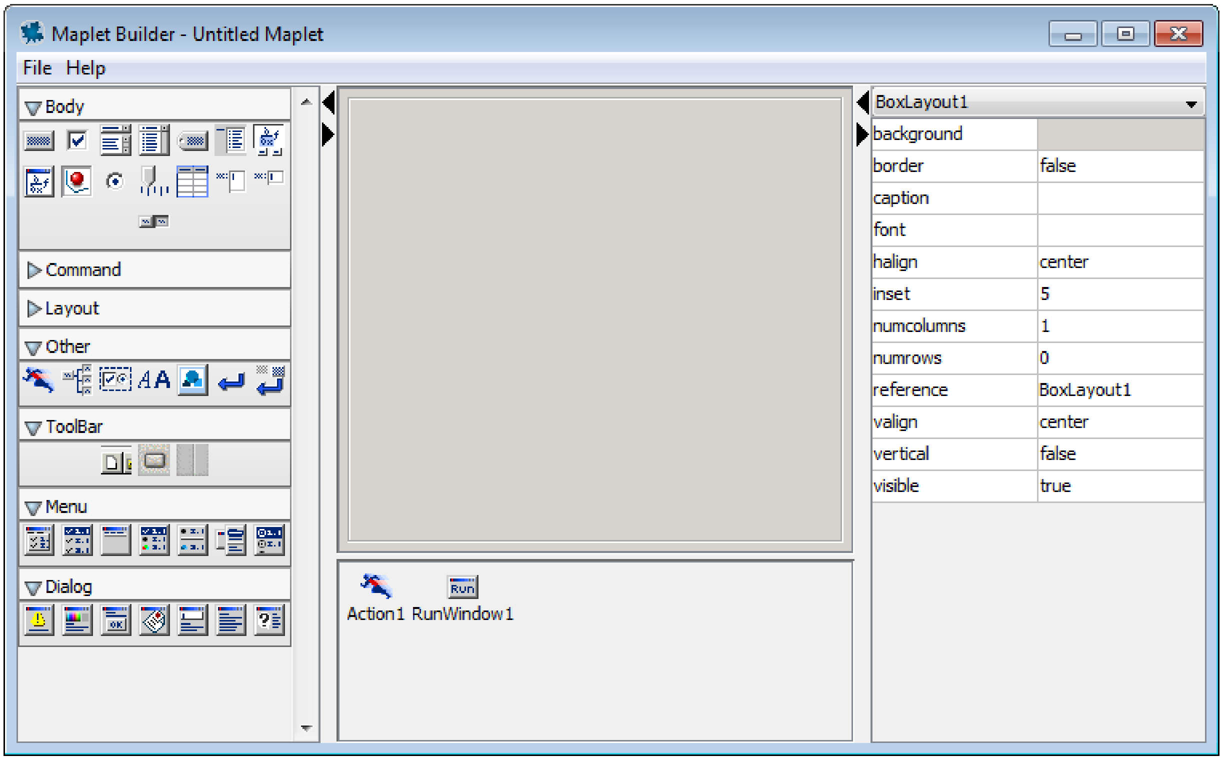


Figure 10.7: Maplet Builder Interface

The **Maplet Builder** is divided into four different panes.

- The **Palette** pane displays palettes, which contain Maplet elements, organized by category. For a description of the elements, see the **MapletBuilder/Palette** help page. The **Body** palette contains the most popular elements.
- The **Layout** pane displays the visual elements of the Maplet.
- The **Command** pane displays the commands and corresponding actions defined in the Maplet.
- The **Properties** pane displays the properties of an instance of a defined element in the Maplet.

Example 3 - Design a Maplet Using the Maplet Builder

In this example, shown in **Figure 10.8**, the Maplet user enters a function and plots the result.

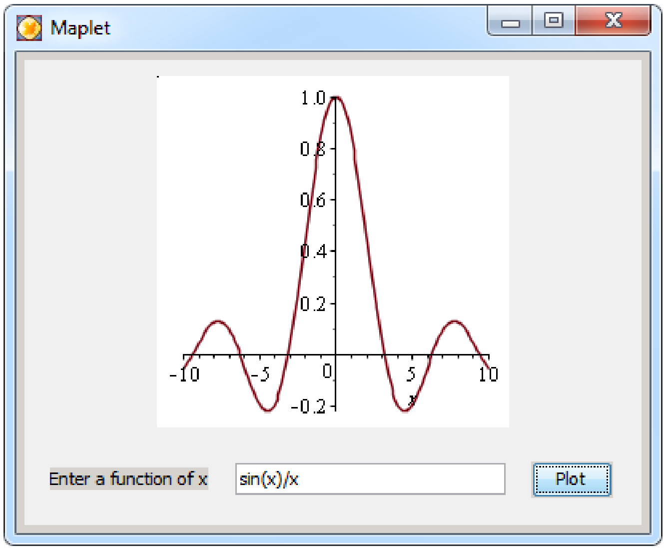


Figure 10.8: Image of the Maplet

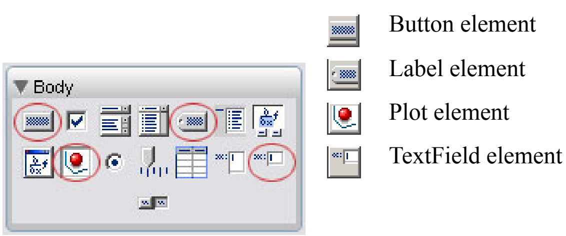
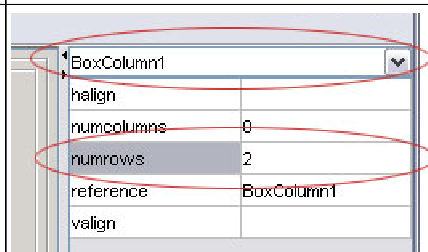
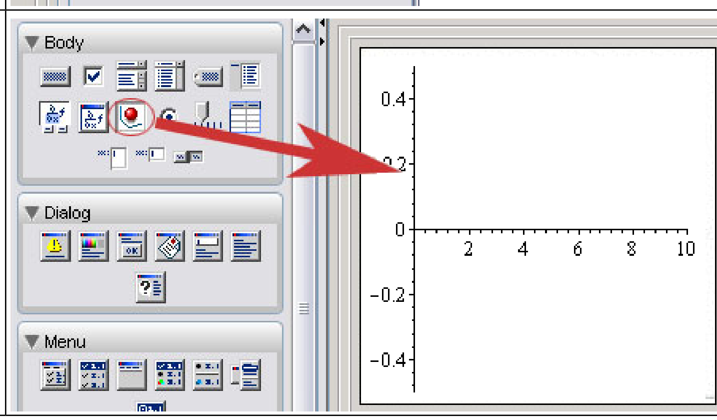
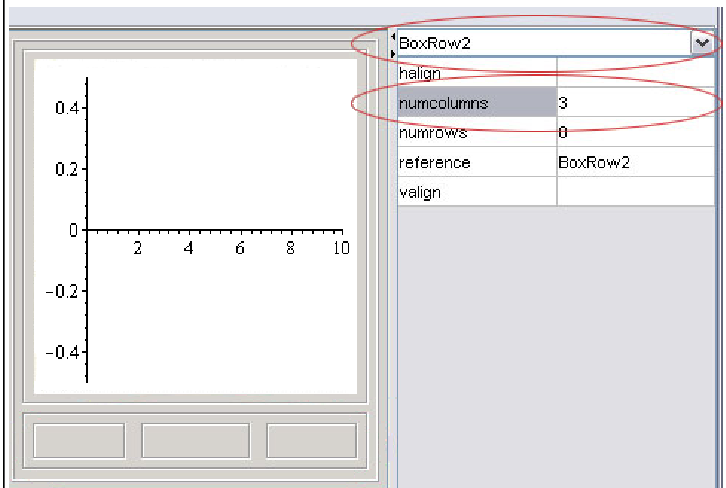
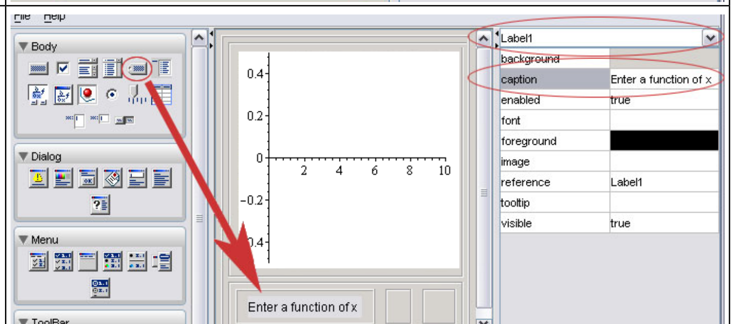
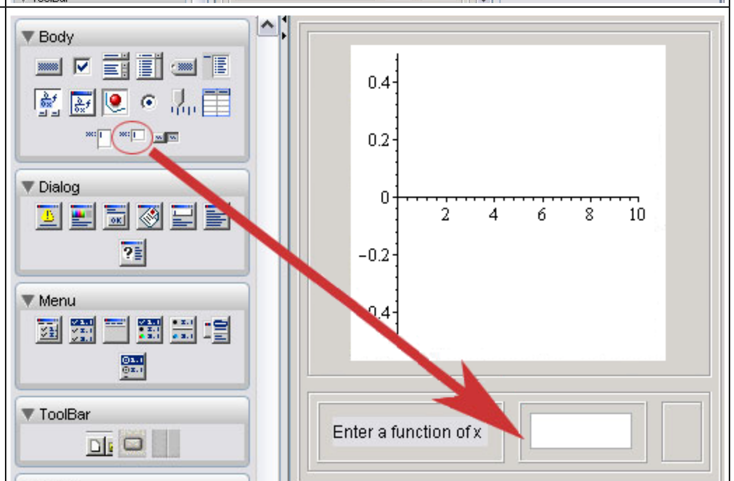
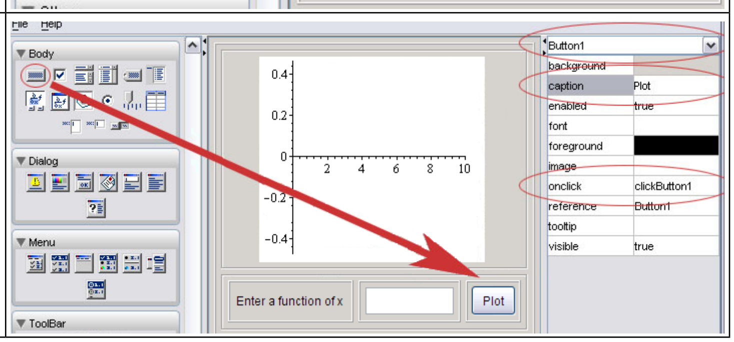
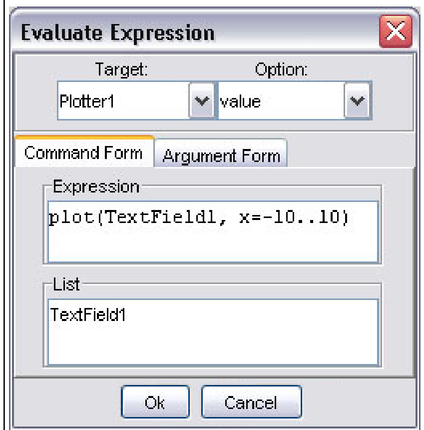


Figure 10.9: Body Elements Used to Define This Maplet

Action	Result in MapletBuilder
Define the number of rows in the Maplet: 1. In the **Properties** pane: a. In the drop-down list, select **BoxColumn1**. b. Change the **numrows** field to **2**.	
Add a plot region to row 1: 2. From the **Body** palette, drag the **Plotter** element to the first row in the **Layout** pane.	

Action	Result in MapletBuilder
Add columns to row 2: 3. In the **Properties** pane: a. In the drop-down list, select **BoxRow2**. b. Change the **numcolumns** field to **3**.	
Add a label to row 2: 4. From the **Body** palette, drag the **Label** element to the left column in the **Layout** pane. 5. In the **Properties** pane: a. In the drop-down list, select **Label1**. b. Change the caption field to **Enter a function of x**.	
Add a text region to row 2: 6. From the **Body** palette, drag the **TextField** element to the middle column. The **TextField** element allows the Maplet user to enter input that can be retrieved in an action. 7. If necessary, resize the Maplet Builder to display the entire **Layout** pane.	
Add a button to row 2: 8. From the **Body** palette, drag the **Button** element to the right column in the **Layout** pane. 9. In the **Properties** pane: a. In the drop-down list, select **Button1**. b. Change the **caption** field to **Plot**. c. In the **onclick** property drop-down list, select **<Evaluate>**.	

<table>
<tr><th>Action</th><th>Result in MapletBuilder</th></tr>
<tr><td>10. In the Evaluate Expression dialog that displays, the Target drop-down list contains the defined elements to which you can send information, in this case, Plotter1 and TextField1. The List group box, located below the Expression group box, displays the defined elements to which you can retrieve information, in this case, TextField1.
a. In the Target drop-down list, select Plotter1.
b. In the Command Form tab, enter plot(TextField1, x=-10..10) in the Expression group box. (Note: Do not include a semicolon (;) at the end of the plot command.) You can also double-click TextField1 in the List group box to insert this element in the command syntax.
c. Click Ok.</td><td>
</td></tr>
<tr><td colspan="2">Run the Maplet:
11. From the File menu, select Run. You are prompted to save the Maplet. Maplets created with the Maplet Builder are saved as .maplet files.
12. Click Yes and navigate to a location to save this Maplet.</td></tr>
</table>

For further information on the **Maplet Builder**, see the **MapletBuilder** help page. For more examples of designing Maplets using the **Maplet Builder**, see **examples/MapletBuilder**.

Maplets Package

When designing a complicated Maplet, the **Maplets** package offers greater control. The **Maplets[Elements]** subpackage contains the elements available when designing a Maplet application. After you define the Maplet, use the **Maplets[Display]** command to launch the Maplet.

For more information on the **Maplets** package, refer to the **MapletsPackage** help page. For more examples of designing Maplets using the **Maplets** package, see the **Maplets/Roadmap** help page.

Example 4 - Design a Maplet Using the Maplets Package

To introduce the structure of designing Maplets using the **Maplets** package, this example illustrates the equivalent syntax for the *Example 3 - Design a Maplet Using the Maplet Builder (page 322)*.

Load the **Maplets[Elements]** package.

```
> with(Maplets[Elements]):
```

Define the Maplet application. To suppress the display of the data structure associated with the Maplet application, end the definition with a colon.

```
> PlottingMaplet:=Maplet(
    BoxLayout(
     BoxColumn(
       # First Box Row
           BoxRow(
         # Define a Plot region
             Plotter('reference' = Plotter1)
       # End of first Box Row
```

```
                ),
        # Second Box Row
            BoxRow(
          # Define a Label
              Label("Enter a function of x "),
           # Define a Text Field
              TextField('reference' = TextField1),
           # Define a Button
              Button(caption="Plot", Evaluate(value = 'plot(TextField1,
                     x = -10..10)', 'target' = Plotter1))
        # End of second Box Row
              )
       # End of BoxColumn
         )
     # End of BoxLayout
     )
   # End of Maplet
   ):
```

Launch the Maplet.

```
> Maplets[Display](PlottingMaplet);
```

For further examples using both the **MapletBuilder** and **Maplets** package commands, see the Maplets example worksheets. For a listing, refer to the **examples/index** help page.

Saving

When saving a Maplet, you can save the document as an **.mw** file or you can export the document as a **.maplet** file.

Maple Document

To save the Maplet code as an **.mw** file:

1. From the **File** menu, select **Save**.
2. Navigate to the save location.
3. Enter a filename.
4. Click **Save**.

If the document contains only Maplet code, it is recommended that you export the document as a **.maplet** file.

Maplet File

To export the Maplet code as a **.maplet** file:

1. From the **File** menu, select **Export As**.
2. In the **Files of Type** drop-down list, select **Maplet**.
3. Navigate to the export location.
4. Enter the filename.
5. Click **Save**.

11 Input, Output, and Interacting with Other Products

11.1 In This Chapter

Section	Topics
Writing to Files (page 327) - Saving to Maple file formats	• Saving Data to a File • Saving Expressions to a File
Reading from Files (page 329) -Opening Maple files	• Reading Data from a File • Reading Expressions from a File
Exporting to Other Formats (page 331) - Exporting documents in file formats supported by other software	• Exporting Documents • MapleNet
Connectivity (page 333) - Using Maple with other programming languages and software	• Translating Maple Code to Other Programming Languages • Accessing External Products from Maple • Accessing Maple from External Products • Sharing and Storing Maple Worksheet Content with the MapleCloud™

11.2 Writing to Files

Maple supports file formats in addition to the standard **.mw** file format.

After using Maple to perform a computation, you can save the results to a file for later processing with Maple or another program.

Note: Make sure you have right access to the **directory** in order to execute the example in the following subsections.

Saving Data to a File

If the result of a Maple calculation is a long list or a large array of numbers, you can convert it to Matrix form and write the numbers to a file using the **ExportMatrix** command. This command writes columns of numerical data to a file, allowing you to import the numbers into another program. To convert a list or a list of lists to a Matrix, use the **Matrix** constructor. For more information, refer to the **Matrix** help page.

$$> L := \begin{bmatrix} -81 & -98 & -76 & -4 & 29 \\ -38 & -77 & -72 & 27 & 44 \\ -18 & 57 & -2 & 8 & 92 \\ 87 & 27 & -32 & 69 & -31 \\ 33 & -93 & -74 & 99 & 67 \end{bmatrix}:$$

$> \mathit{ExportMatrix}(\text{"matrixdata.txt"}, L):$

If the data is a Vector or any object that can be converted to type Vector, use the **ExportVector** command. To convert lists to Vectors, use the **Vector** constructor. For more information, refer to the **Vector** help page.

> $R := [3, 3.1415, -65, 0]$

$$R := [3, 3.1415, -65, 0] \tag{11.1}$$

> $V := Vector(R)$

$$V := \begin{bmatrix} 3 \\ 3.1415 \\ -65 \\ 0 \end{bmatrix} \tag{11.2}$$

> $ExportVector(\text{"vectordata.txt"}, V)$:

You can extend these routines to write more complicated data, such as complex numbers or symbolic expressions. For more information, refer to the **ExportMatrix** and **ExportVector** help pages.

For more information on matrices and vectors, see *Linear Algebra (page 116)*.

Saving Expressions to a File

If you construct a complicated expression or procedure, you can save them for future use in Maple. If you save the expression or procedure in the Maple internal format, Maple can retrieve it more efficiently than from a document. Use the **save** command to write the expression to a **.m** file. For more information on Maple internal file formats, refer to the **file** help page.

> $qbinomial := (n, k) \rightarrow \frac{\prod_{i=n-k+1}^{n} (1 - q^i)}{\prod_{i=1}^{k} (1 - q^i)}$:

In this example, small expressions are used. In practice, Maple supports expressions with thousands of terms.

> $expr := qbinomial(10, 4)$

$$expr := \frac{(1 - q^7)(1 - q^8)(1 - q^9)(1 - q^{10})}{(1 - q)(1 - q^2)(1 - q^3)(1 - q^4)} \tag{11.3}$$

> $nexpr := normal(expr)$

$$nexpr := (q^6 + q^5 + q^4 + q^3 + q^2 + q + 1)(q^4 + 1)(q^6 + q^3 + 1)(q^8 + q^6 + q^4 + q^2 + 1) \tag{11.4}$$

You can save these expressions to the file **qbinom.m**.

> **save** *qbinomial, expr, nexpr,* "qbinom.m"

Clear the memory using the **restart** command and retrieve the expressions using the **read** command.

> *restart*

> **read** "qbinom.m"

> *expr*

$$\frac{(1-q^7)(1-q^8)(1-q^9)(1-q^{10})}{(1-q)(1-q^2)(1-q^3)(1-q^4)} \tag{11.5}$$

For more information on writing to files, refer to the **save** help page.

Saving Data as Part of a Workbook

You can save all files related to a common Maple project as a workbook (.maple) file. Saving your data files and worksheets (or documents) as a workbook allows you to use this saved data across all .mw file inside your workbook.

11.3 Reading from Files

The most common reason for reading files is to load data, for example, data generated in an experiment. You can store data in a text file, and then read it into Maple.

Reading Data from a File

Import Data Assistant

If you generate data outside Maple, you can read it into Maple for further manipulation. This data can be an image, a sound file, or columns of numbers in a text file. You can easily import this external data into Maple using the **Import Data Assistant**, where the supported file formats include files of type Excel®, MATLAB®, Image, Audio, Matrix Market, and Delimited.

To launch the Import Data Assistant:

1. From the **Tools** menu, select **Assistants**, and then **Import Data**.
2. A dialog window appears where you can navigate to your data file. Select the file that you want to import data from, and then select the file type before clicking **Next**.
3. From the main window, you can preview the selected file and choose from the applicable options based on the format of the file read in before importing the data into Maple. See **Figure 11.1** for an example.

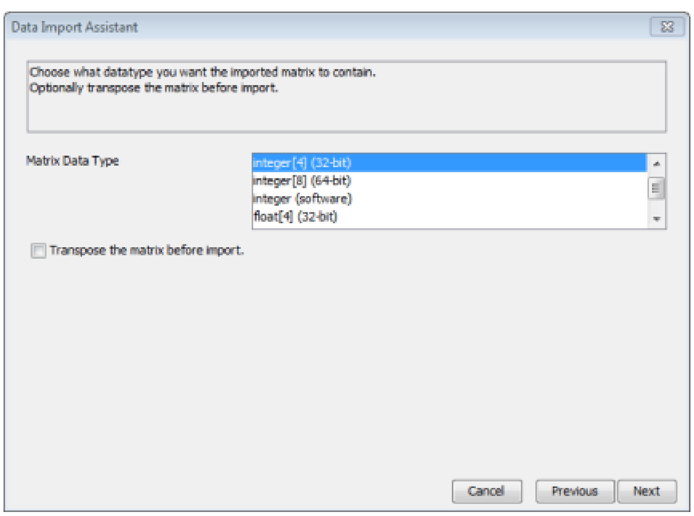


Figure 11.1: Import Data Assistant

ImportMatrix Command

The **Import Data Assistant** provides a graphical interface to the **ImportMatrix** command. For more information, including options not available in the assistant, refer to the **ImportMatrix** help page.

Reading Expressions from a File

You can write Maple programs in a text file using a text editor, and then import the file into Maple. You can paste the commands from the text file into your document or you can use the **read** command.

When you read a file with the **read** command, Maple treats each line in the file as a command. Maple executes the commands and displays the results in your document but it does *not,* by default, insert the commands from the file in your document.

For example, the file **ks.txt** contains the following Maple commands.

S:= n -> sum(binomial(n, beta) * ((2*beta)! / 2^beta - beta!*beta), beta=1..n);

S(19);

Note that the file should not contain prompts (>) at the start of lines.

When you read the file, Maple displays the results but not the commands.

$$S := n \to \sum_{\beta=1}^{n} \text{binomial}(n, \beta) \left(\frac{(2\,\beta)!}{2^{\beta}} - \beta!\,\beta \right)$$

$$1024937361666644598071114328769317982974 \tag{11.6}$$

> *filename* := *cat*(*kernelopts*(*datadir*), *kernelopts*(*dirsep*), "ks", *kernelopts*(*dirsep*), "ks.txt") :

> **read** *filename*

$$S := n \to \sum_{\beta=1}^{n} \text{binomial}(n, \beta) \left(\frac{(2\,\beta)!}{2^{\beta}} - \beta!\,\beta \right)$$

$$1024937361666644598071114328769317982974 \tag{11.7}$$

If you set the **interface echo** option to 2, Maple inserts the commands from the file into your document.

> *interface*(*echo* = 2) :
read *filename*

```
> S:= n -> sum( binomial( n, beta ) * ( ( 2*beta )! / 2^beta - beta!*beta ), beta=1..n );
```

$$S := n \to \sum_{\beta=1}^{n} \text{binomial}(n, \beta) \left(\frac{(2\,\beta)!}{2^{\beta}} - \beta!\,\beta \right)$$

```
> S(19);
```

$$1024937361666644598071114328769317982974 \tag{11.8}$$

For more information, refer to the **read** and **interface** help pages.

Reading Data From Workbook Attachments

Data stored in a workbook in the form of an attachment, can be accessed easily using the workbook URI. For information on workbook attachments, see **worksheet,workbook,attachFiles**. For information on the workbook URI format, see **worksheet,workbook,uri**.

11.4 Exporting to Other Formats

Exporting Documents

You can save your documents by selecting **Save** or **Save As** from the **File** menu. By selecting **Export As** from the **File** menu, you can also export a document in the following formats: HTML, LaTeX, Maple input, Maplet application, Maple text, plain text, PDF, and Rich Text Format. This allows you to access your work outside Maple.

HTML

The **.html** file that Maple generates can be loaded into any HTML browser. Exported mathematical content can be displayed in one of the following formats: GIF, MathML 2.0 Presentation, MathML 2.0 Content, or Maple Viewer, and is saved in a separate folder. MathML is the Internet standard, sanctioned by the World Wide Web Consortium (W3C), for the communication of structured mathematical formulae between applications. For more information about MathML, refer to the **MathML** help page.

Maple documents that are exported to HTML translate into multiple documents when using frames. If the frames feature is not selected, Maple creates only one page that contains the document contents.

LaTeX

The **.tex** file generated by Maple is ready for processing by LaTeX. All distributions of Maple include the necessary style files. By default, the LaTeX style files are set for printing the **.tex** file using the **dvips** printer driver. You can change this behavior by specifying an option to the **\usepackage** LaTeX command in the preamble of your **.tex** file. For more information, refer to the **exporttoLaTeX** help page.

Maple Input

You can export a Maple document as Maple input so that it can be loaded using the Maple Command-line version.

Important: When exporting a document as Maple input for use in Command-line Maple, your document must contain explicit semicolons in 1-D Math input. If not, the exported **.mpl** file does not contain semicolons, and Command-line Maple generates errors.

Maplet Application

The **Export as Maplet** facility saves a Maple document as a **.maplet** file, so that you can run it using the command-line interface or the **MapletViewer**. The MapletViewer is an executable program that can launch saved Maplet applications. It displays and runs Maplet applications independently of the Maple Worksheet interface.

Important: When exporting a document as a Maplet Application for use in Command-line Maple or the MapletViewer, your document must contain explicit semicolons. If not, the exported **.maplet** file does not contain semicolons, and Command-line Maple and the MapletViewer generates errors.

Maple Text

Maple text is marked text that retains the distinction between text, Maple input, and Maple output. Thus, you can export a document as Maple text, send the text file by email, and the recipient can import the Maple text into a Maple session and regenerate the computations in the original document.

PDF

Export a Maple document to a Portable Document Format (PDF) file so that you can open the file in a reader such as Adobe® Acrobat®. The PDF document is formatted as it would appear when the Maple worksheet is printed using the active printer settings.

Note: Images, plots, and embedded components may be resized in the PDF file.

Plain Text

Export a Maple document as plain text so that you can open the text file in a word processor.

Rich Text Format (RTF)

Export a Maple document to a rich text format file so that you can open and edit the file in a word processor.

Note: The generated **.rtf** format is compatible with Microsoft Word and Microsoft WordPad only.

Summary of Translation

Table 11.1: Summary of Content Translation When Exporting to Different Formats

Content	HTML	LaTeX	Maple Input	Maplet Application	Maple Text	Plain Text	Rich Text Format	PDF Format
Text	Maintained	Maintained	Preceded by #	Preceded by #	Preceded by #	Maintained	Maintained	Maintained
1-D Math	Maintained	Maintained	Maintained	Maintained	Preceded by >	Preceded by >	Static image	Static image
2-D Math	GIF or MathML	LaTeX	1-D Math (if possible)	1-D Math (if possible)	1-D Math or character-based typesetting	1-D Math or character-based typesetting	Static image	Either text or shapes depending on option selected
Plot	GIF	Postscript file	Not exported	Not exported	Not exported	Not exported	Static image	Static image
Animation	Animated GIF	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Static image
Hidden content	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported	Not exported
Manually inserted page break	Not supported	Not supported	Not supported	Not supported	Not supported	Not supported	RTF page break object	Maintained
Hyperlink	Links to help pages become plain text. Links to documents are renamed and converted to HTML links	Plain text	Plain text	Plain text	Plain text	Plain text	Plain text	Plain text
Embedded image or sketch output	GIF	Not exported	Not exported	Not exported	Not exported	Not exported	Static image	Static image
Spreadsheet	HTML table	LaTeX tables	Not exported	Not exported	Not exported	Not exported	RTF table	Static image

Content	HTML	LaTeX	Maple Input	Maplet Application	Maple Text	Plain Text	Rich Text Format	PDF Format
Document style	Approximated by HTML style attributes	LaTeX environments and sections, LaTeX macro calls	Not exported	Not exported	Not exported	Not exported	RTF style	Maintained

MapleNet

Overview of MapleNet

Using MapleNet, you can deploy Maple content on the web. Powered by the Maple computation engine, MapleNet allows you to embed dynamic formulas, models, and diagrams as **live** content in webpages. The MapleNet software is not included with the Maple software. For more information on MapleNet, visit **http://www.maplesoft.com/maplenet**.

MapleNet Documents and Maplets

After you upload your Maple document to the MapleNet server, it can be accessed by anyone in the world using a web browser. Even if viewers do not have a copy of Maple installed, they can view documents and Maplets, manipulate 3-D plots, and execute code at the click of a button.

Custom Java Applets and JavaServer Pages™ Technology

MapleNet provides a programming interface to the Maple math engine so commands can be executed from a Java applet or using JavaServer Pages™ technology. Embed MapleNet into your web application, and let Maple handle the math and visualization.

11.5 Connectivity

Translating Maple Code To Other Programming Languages

Code Generation

The CodeGeneration package is a collection of commands and subpackages that enable the translation of Maple code to other programming languages. Languages currently supported include: C, C#, Fortran 77, Java, MATLAB®, Visual Basic, Perl, and Python.

For details on Code Generation, refer to the **CodeGeneration** help page.

Accessing External Products from Maple

External Calling

External calling allows you to use compiled C, C#, Fortran 77, or Java code in Maple. Functions written in these languages can be linked and used as if they were Maple procedures. With external calling you can use pre-written optimized algorithms without the need to translate them into Maple commands. Access to the NAG library routines and other numerical algorithms is built into Maple using the external calling mechanism.

External calling can also be applied to functions other than numerical algorithms. Routines exist that accomplish a variety of non-mathematical tasks. You can use these routines in Maple to extend its functionality. For example, you can link to controlled hardware via a serial port or interface with another program. The **Database** package uses external calling to allow you to query, create, and update databases in Maple. For more information, refer to the **Database** help page.

For more information on using external calling, refer to the **ExternalCalling** help page.

Mathematica Translator

The **MmaTranslator** package provides translation tools for converting Mathematica® expressions, command operations, and notebooks to Maple. The package can translate Mathematica input to Maple input and Mathematica notebooks to Maple documents. The **Mma** subpackage contains commands that provide translation for Mathematica commands when no equivalent Maple command exists. In most cases, the command achieves the translation through minor manipulations of the input and output of similar Maple commands.

Note: The **MmaTranslator** package does not convert Mathematica programs.

There is a Maplet interface to the **MmaTranslator** package. For more information, refer to the **MmaToMaple** help page.

Matlab Package

The **Matlab** package enables you to translate MATLAB® code to Maple, as well as call selected MATLAB® functions from a Maple session, provided you have MATLAB® installed on your system.

For more information, refer to the **Matlab** help page.

Accessing Maple from External Products

Microsoft Excel Add-In

Maple is available as an add-in to Microsoft Excel. This add-in is supported for Excel 2019 and Excel 2016 for Windows, and provides the following features.

- Access to Maple commands from Excel
- Ability to copy and paste between Maple and Excel
- Access to a subset of the Maple help pages
- Maple Function Wizard to step you through the creation of a Maple function call

To enable the Maple Excel Add-in:

1. In Excel, click the **File** menu and select **Options**.
2. Click **Add-ins**.
3. In the **Manage** box select **Excel Add-ins**, and then **Go**.
4. Navigate to the Excel subdirectory of your Maple installation and select the file:
    - **WMIMPLEX64.xla** (that is, select $MAPLE/Excel/WMIMPLEX64.xla), and click **OK**.
5. Select the **Maple Excel Add-in** check box.
6. Click **OK**.

For further details on enabling the Maple Excel Add-in, refer to the **Excel** help page.

For information on using this add-in, refer to the **Using Maple in Excel** help file within Excel.

To view this help file:

1. Enable the add-in.
2. From the **Add-ins** tab, view the **Maple** toolbar.
3. On the Maple toolbar, click the Maple help icon .

OpenMaple

OpenMaple is a suite of functions that allows you to access Maple algorithms and data structures in your compiled C, C#, Java, or Visual Basic programs. (This is the reverse of external calling, which allows access to compiled C, C#, Fortran 77, and Java code from Maple.)

To run your application, Maple must be installed. You can distribute your application to any licensed Maple user. For additional terms and conditions on the use of OpenMaple, refer to the **extern/OpenMapleLicensing.txt** file in your Maple installation.

For more details on using OpenMaple functions, refer to the **OpenMaple** help page.

MapleSim

MapleSim™ is a complete environment for modeling and simulating multidomain engineering systems. During a simulation, MapleSim uses the symbolic Maple computation engine to generate the mathematical models that represent the system behavior.

Because both products are tightly integrated, you can use Maple commands and technical document features to edit, manipulate, and analyze a MapleSim model. For example, you can use Maple commands and tools to manipulate your model equations, develop custom components based on a mathematical model, and visualize simulation results.

MapleSim software is not included with the Maple software. For more information on MapleSim, visit **http://www.maplesoft.com/maplesim**.

MaplePlayer for iPad

The Maple Player is a free application for the iPad that uses the Maple computation engine to enable you to view and interact with documents created in desktop Maple.

For more information on the Maple Player for iPad, visit **http://www.maplesoft.com/products/MaplePlayer**.

Sharing and Storing Maple Content

The MapleCloud

You can use the MapleCloud to share or store your Maple documents and workbooks. Upload Maple documents or workbooks. Package workbooks offer a way to share a Maple package with other users, including source code, documentation and examples.

The MapleCloud has private and public sharing. You can share with all Maple users, share with a private group, or upload and store content in a user-specific area that only you can access.

Users need an internet connection to use the MapleCloud. Anyone can access publicly shared documents. To share content, create, manage and join user groups; and view group-specific content, you must log in to the MapleCloud using a Maplesoft.com, Gmail™, or Google Mail™ account name and password.

A Maplesoft.com membership account gives you access to thousands of free Maple resources and MaplePrimes, which is an active web community for sharing techniques and experiences with Maple and related products. To sign up for a free Maplesoft.com membership account, visit **http://www.maplesoft.com/members/sign_up_form.aspx**. The MapleCloud is integrated with several of these online features, so it is strongly recommended that you use a Maplesoft.com membership account.

For more information on the MapleCloud, refer to the **MapleCloud** help page.

Index

Symbols

! toolbar icon, 47
!!! toolbar icon, 48
"", 277
$, 131
%H, 125
%T, 125
&x, 125
', 67, 291
(), 306
->, 67
., 124
1-D Math, 56
2-D Math, 56
    converting to 1-D, 57
    entering, 4
    shortcuts, 6
    switching to 1-D, 57
:, 56, 57
::, 104
:=, 66
;, 56, 57
<>, 116, 118
>, 56
?
    help topic, 37
[], 123, 269, 270
^, 5, 79
    entering, 79
_, 68
_ZN~, 83
`, 68
{}, 270
|, 118
~, 83, 105
    element-wise operations, 289

A

about command, 105
abs command, 77
absolute value, 77
add
    word to your dictionary, 267
add command, 303
additionally command, 105
algebra, 115
    linear, 127
    polynomial, 110
algsubs command, 287
alignment format, 231
American spelling
    spellcheck, 265
and operator, 296
angle brackets, 116, 118, 152
angles, 284
animations
    creating, 217
    customizing, 225
Application Center, 41
applications
    sample documents, 40
apply
    character styles, 232
    paragraph styles, 235
approximation, 75
    least-squares, 127
    numeric, 287
arguments, 306
arithmetic, 47
    finite-precision, 74
    interval, 101
    matrix and vector, 124
    modular, 77, 78
    polynomial, 110
Arrays, 271
    indexing, 272
    large, 273
arrow operator, 67
assign command, 86
assigned command, 291
assignment operator (:=), 66
Assistants
    CAD Link, 24
    Code Generation, 24
    Curve Fitting, 115
    Data Set Search, 24
    eBook Publisher, 24
    Import Data, 24, 329
    Installer Builder, 24
    Library Browser, 24
    Maplet Builder, 24
    ODE Analyzer, 87
    Optimization, 138
    overview, 24
    Plot Builder, 24, 184
    Scientific Constants, 24
    Special Functions, 25
    Student Practice Sheets, 26
    Thermophysical Properties Calculator, 25
    Tools menu, 24
    Unit Converter, 284
    Units Converter, 25, 94
    Worksheet Migration, 25

assume command, 104
  adding assumptions, 105
  and procedure variables, 106
  imposing multiple assumptions, 105
  removing assumptions, 105
  setting relationships between variables, 104
  setting variable properties, 104
  testing property, 105
  using with assuming command, 106
  viewing assumptions, 105
assuming command, 104, 106, 135, 283
  additionally option, 106
  and procedure variables, 106
  applying to all names, 106
  using with assume command, 106
Attributes submenu
  character, 230
  paragraph, 231
auto-execute, 243
  repeating, 244
  security levels, 244
Avogadro constant, 98

B

bar chart, 143
basis
  vector space, 127
binary numbers, 78
Bohr radius, 98
bold format, 229
bookmarks
  using, 261
boolean expressions, 288, 296, 300
brackets
  angle, 116, 118
break statement, 303
browser
  Matrix, 118, 273
  Task, 64
bullets
  format, 231
button
  embedding, 263
Button component, 311
by clause, 298
  excluding, 298
  negative, 299

C

CAD Link Assistant, 24
calculus, 137
  clickable problem solving, 181
  multivariate, 136
    Student package, 137
  of variations, 137
  packages, 136
  study guides, 146
  teaching, 137, 147
  vector, 136
    Student package, 137
calling sequence, 58
canvas
  inserting, 255
canvas style
  sketch pad, 256
caret
  entering, 79
central tendency, 101
character styles
  creating, 233
  description, 232
Check Box component, 311
Cholesky decomposition, 126
clickable math
  expressions, 27
Clickable Math, 181
  Drag-to-Solve, 157
  Smart Popups, 157
Code Edit Region, 308
Code Generation, 24
CodeGeneration
  package description, 60
coeff command, 114
coefficients
  polynomials, 114
coeffs command, 114
collect command, 114
colon, 56, 57
color
  of plots, 211
combine command, 283
  errors option, 103
Combo Box component, 311
command completion, 6, 33
Command-line Interface, xiii
commands, 59
  and task templates, 64
  displaying procedures, 307
  entering, 32
  help, 37
  hiding, 308, 309
  iterative, 303
  mapping over set or list, 305
  package, 59
  top, 59
  top-level, 58
complex expressions, 288

complex numbers, 21
compoly command, 115
components
  adding GUI elements, 263
  palette, 263
computations
  assistants, 64
  commands, 59
  context panel, 63
  errors, 76
    avoiding, 76
  integers, 79
  interrupting, 303
  linear algebra, 124
  mathematics, 109
  numeric, 75
  palettes, 61
  performing, 73, 109
  Real number system, 103
  symbolic, 75
  syntax-free, 53
  task templates, 64
  tutors, 64
  under assumptions, 104
    single evaluation, 106
  updating, 47
  with uncertainty, 102
  with units, 96
conditional execution, 295
constants, 44
content command, 115
context
  of unit, 94
context panel, 63, 126
  access tutors, 53
  computing with, 48
  customizing animations, 221
  integer, 63, 88
  overview, 27
  Plot Builder access, 185
  using, 27
context-sensitive operation, 27
  customize text on arrow, 51
  equation, 80
  integer, 76
  using, 10
convert command, 284
  base option, 78, 302
  degrees option, 284
  mathematical functions, 284
  polynom option, 133
  set option, 284
  temperature option, 95
  units option, 94, 284
copy, 228
  and drag, 10
  examples, 39
copy expressions, 10
correlation, 102
coulditbe command, 105
covariance, 102
cross product, 125
Curl command, 136
Curve Fitting
  package
    PolynomialInterpolation command, 115
Curve Fitting Assistant, 115
cut and paste
  in tables, 247

D

D operator, 131
data structures, 44, 269
  creating, 277
Data Table component, 311
Database Integration, 333
datatype option, 121
degree
  command, 114
  polynomials, 114
denom command, 280
derivatives, 130
  directional, 132
  partial, 45, 130
  prime notation, 243
  Tutor, 147
Dial component, 312
dictionary, 40, 146
dictionary topic
  adding hyperlink to, 260
diff command, 88, 130
differential equations
  ordinary, 87
  partial, 91
  Student package, 137
differentiation, 130
  with uncertainty, 102
  with units, 96
Differentiation Methods Tutor, 147
Digits environment variable, 75
dimension, 93, 126
  base, 93
Directional Derivative Tutor, 132
discrim command, 115
display
  bookmark, 261
distribution

probability, 141
divide command, 111
divisors, 77
document blocks, 35, 241
Document mode, 43
documents
running, 8
DocumentTools, 318
double colon operator, 104
dsolve command, 90

E

e (exponential)
entering, 45
e-notation, 75
eBook Publisher Assistant, 24
Edit menu
in help system, 39
eigenvalues, 126
eigenvectors, 126
element-wise operators, 289
elementary charge, 98
elements, 97
definition, 99
isotopes, 99
definition, 99
properties, 99
list, 99
properties
list, 99
uncertainty, 100
units, 100
using, 98
value, 100
value and units, 100
elif clauses, 297
order, 297
else clause, 296
email
adding hyperlink to, 259
embedded components, 263, 311
example, 315, 317
inserting, 314
properties, 315
end do keywords, 298, 300
end if keywords, 295
end proc keywords, 306
engineers
portal for, 40
Enter key, 46
environment variables
Digits, 75
Order, 133
equation
solving step-by-step, 163
equation labels, 71
displaying, 69
features, 70
formatting, 35
inserting, 34
numbering schemes, 70
overview, 34
references to, 69
versus names, 70
with multiple outputs, 70
Equation Manipulator, 163
equations
solving, 80
for real solutions, 103
numerically, 84
symbolically, 82
transcendental, 83
errors
quantities with, 101
Euclidean algorithm, 115
eval command, 286, 307
evalb command, 288
evalc command, 288
evalf command, 75, 83, 100, 102, 287
with Int command, 136
with Limit command, 129
evaln command, 291
evaluate
and display inline, 46
evaluation
boolean expressions, 288
complex expressions, 288
delaying, 291
levels of, 290
Maple expressions, 285
of expression at a point, 285
output below, 47
output inline, 46, 49
updated computations, 47
exact
computation, 74
numbers, 74
quantities
converting to floating-point, 75
example worksheets
copy, 39
execute, 46
execution group, 56
execution groups, 14
expand
command, 283
document block, 242

execution group, 242
series, 133
Exploration Assistant, 29
exponents
entering, 5
export, 308
to HTML, 331
to LaTeX, 331
to Maple input, 331
to Maple text, 331
to Maplet application, 331
to other formats, 331
to PDF, 332
to plain text, 332
to Rich Text Format, 332
worksheets, 331
exporting
embedded components, 314
expression sequences, 82, 269
creating, 303
expressions, 44, 269
adding, 303
clickable math, 27
evaluating, 285
manipulating, 282
multiplying, 303
versus functional operators, 275

F

factor
integers, 76
polynomials, 114
QR factorization, 127
factor command, 114, 282
factored normal form, 284
factorial command, 77
FAIL, 296, 300
false, 296, 300
Faraday constant, 98
Favorites palette, 15, 20
files
image formats, 257
reading from, 329
writing to, 327
fill option, 121
finite fields, 78
solving equations, 92
finite rings, 78
floating-point
computation, 75
accuracy, 76
hardware, 76
significant digits, 75
numbers, 74
rational approximation, 63
Flux command, 137
font color, 229
foot-pound-second (FPS) system, 94
footers, 239
for/from loops, 298
for/in loops, 299
formal power series solutions, 91
format labels, 35
Format menu
bookmarks, 261
quick formatting, 229
frac command, 106
fractions
approximating, 50
entering, 4
frequency plot, 143
Frobenius form
matrix, 127
from clause, 298
excluding, 298
fsolve command, 84
full evaluation, 290, 292
function, 46
FunctionAdvisor command, 58
functional operators, 274
differentiating, 131
plotting, 276
versus expressions, 275
functions
converting between, 284
defining as functional operators, 274
how to define in Maple, 274

G

Gaussian elimination, 127
Gaussian integers, 79
GaussInt package, 79
gcd command, 115
gcdex command, 115
Global Optimization Toolbox, 138
global variables, 306
glossiness
of 3-D plots, 211
go to
bookmark, 262
gradient, 148
Gradient Tutor, 148
greatest common divisor, 77, 115

H

has command, 279

hastype command, 279
HazardRate command, 143
headers, 239
Help Navigator
  Using, 38
help page
  adding hyperlink to, 259
help system
  accessing, 37
  description, 40
  Edit menu, 39
  Help Navigator, 37
  manuals, 39
  search, 39
  table of contents, 39
  tasks, 39
  View menu, 39
Hermitian transpose
  matrix and vector, 125
Hessenberg form, 127
hexadecimal numbers, 78
hide
  worksheet content, 239
highlight color, 229
Hilbert Matrix, 127
histogram, 143
Homework practice sheet assistant, 26
How Do I ... topics, 40
hyperlinks
  in worksheet, 258

I

i
  entering, 21, 80
icons
  open as example worksheet, 39
if statement, 295
ifactor command, 76, 77, 283
igcd command, 77
images
  adding hyperlink to, 259
  file format, 257
  inserting, 257
imaginary unit
  entering, 21, 80
implied multiplication, 5
implies operator, 296
Import Data Assistant, 24, 329
indent
  format, 231
indeterminates, 281
indets command, 281
indices, 58, 123
inequations
  solving, 80
    for real solutions, 103
    symbolically, 82
infinite loops, 303
infolevel command, 91
input
  2-D Math, 56
  2-D math notation, 56
  Maple notation, 56
  prompt, 56
  separating, 57
  setting default mode, 57
insert
  bookmark, 261
  hyperlink, 258
  images, 257
  section, 238
  sketch pad, 255
  table, 245
Installer Builder Assistant, 24
instructor resources, 155
int command, 135
Int command, 136
integers
  commands, 77
  computations, 79
  context panel, 63, 88
  factoring, 76
  Gaussian, 79
  modulo m, 78
  solving equations, 92
  solving modular equations, 92
integration, 48, 62, 134
  definite, 135
  functional operators, 277
  indefinite, 134
  iterated, 135
  line, 135, 150
  numeric, 136
  surface, 135
  with units, 96
interactive commands
  Student, 26
interactive document
  building, 311
Interactive Linear System Solving tutor, 53
Interactive Plot Builder Assistant
  creating animations, 215
  creating plots, 184
  customizing animations, 221
  customizing plots, 206
interface command
  rtablesize option, 120

verboseproc option, 307
international system (SI), 94
InterquartileRange command, 142
interval arithmetic, 101
iquo command, 77
iroot command, 77
is command, 105
isprime command, 77
isqrt command, 77
italic format, 229

J

j
entering, 80
Jordan form, 126

K

keyboard keys
Command Completion, xiii
keystrokes, 5

L

Label component, 312
labels, 71
last name evaluation, 291
lcm command, 115
lcoeff command, 114
ldegree command, 114
least-squares, 127
left single quotes, 68
left-hand side, 280
levels of evaluation, 290
lexicographic order, 112
lhs command, 280
Library Browser
description, 24
limit command, 129
Limit command, 129
limits, 128
multidimensional, 129
line break, 231
line integrals, 150
linear algebra, 127
computations, 124
efficiency, 120, 128
LinearAlgebra package, 127
teaching, 128, 147
Linear System Solving tutor, 53
linear systems
solving, 92, 127
interactive, 53
LinearAlgebra
package description, 60
LinearAlgebra package, 125
commands, 127
numeric computations, 128
LinearSolve command, 92
Linux
command complete, 6
List Box component, 312
lists, 123, 270
returning solutions as, 82
local variables, 306
logical operators, 296
loops, 298
general, 302
infinite, 303

M

Mac
command complete, 6
manipulate
equation, 163
map command, 305
Maple Application Center, 146
Maple library, 31
Maple Portal, 40, 146
Maple Student Help Center, 146
Maple Workbook, 4
MapleCloud, 335
MaplePrimes, 41
Maplet Builder
description, 24
launching, 322
Maplet authoring, 322
Maplets
adding hyperlink to, 260
authoring, 325
Maplet Builder, 322
Maplets package, 325
launching
Maple worksheet, 321
Maplet file type, 320
Maplets package
Display command, 325
Elements subpackage, 325
Maplet authoring, 325
saving
Maple worksheet, 326
maplet file, 326
using, 320
markers
bookmarks, 261
displaying, 36
for document blocks, 241
Math Apps, 26

in help system, 39
math dictionary
description, 40
Math Expression component, 312
Math mode, 14
shortcuts, 6
mathematical functions
list, 58
mathematics
computations, 109
teaching and learning, 155
matrices, 273
arithmetic, 124
context panel, 126
data type, 121, 122
defining, 116
efficiency, 120
filling, 122
Hermitian transpose, 125
image, 120
large, 119
multiplication, 124
operations, 126
random, 119
scalar multiplication, 124
selecting submatrices, 123
shape, 120, 122
transpose, 125
type, 120
Matrix
Browser, 118, 273
constructor, 121
data structure, 116
palette, 92, 116, 120
Matrix command, 116
max command, 77
maximize, 138
maximum, 77
Mean command, 142
Meter component, 312
Microphone component, 312
min command, 77
minimize, 138
minimum, 77
mod command, 77
mod operator, 78
modes
Document, 43
Worksheet, 43
modify
table, 246
modp command, 79
mods command, 79
modular arithmetic, 77, 78
modules, 308
MPS(X) files, 141
msolve command, 92
mul command, 303
multiplication
implied, 5

N

names, 44, 67
adding assumptions, 104
and symbols, 21
assigned, 291
assigning values to, 66
logical, 296
previously assigned, 291
protected, 67
removing assumptions, 105
reserved, 67
unassigning, 67, 105, 292
valid, 68
versus equation labels, 70
with assumptions, 105
nops command, 281
norm command, 115, 126
normal command, 284
normal form, 284
not operator, 296
numbers, 44
exact, 74
floating-point, 74
non-base 10, 78
NumberTheory[Divisors] command, 77
numer command, 280
numeric
approximation, 287
computation, 74

O

objects, 308
ODE Analyzer Assistant, 87
ODEs
Student package, 137
online help, 41
operands, 281
selecting, 304
operators, 44
functional, 274
logical, 296
relational, 296
Optimization
package description, 61
optimization, 139
efficiency, 140

plotting, 139
point-and-click interface, 138
Optimization Assistant, 138
Plotter, 139
Options dialog, 15
or operator, 296
Order environment variable, 133
ordinary differential equations
plotting solution, 90
solving, 87
orthogonal matrix, 127
output
suppressing, 56

P

packages, 58
accessing commands, 33
definition, 31
help, 37
loading, 59
top, 61
unloading, 60
page break, 231
page headers and footers, 239
palettes, 45, 48, 61, 286
categories, 18
Components, 314
favorites, 15
Favorites, 20
managing, 18
Matrix, 116, 120
overview, 15
saving expressions for ease of reuse, 20
Units, 52, 95
paragraph styles
creating, 236
description, 232
parameters, 306
parametric solutions, 84
partial derivative
entering, 45
partial differential equations
solving, 91
paste, 228
examples, 40
PDEs, 91
pdsolve command, 91
pencil
sketch pad, 255
Physics
package description, 61
pie chart, 143
piecewise command, 142
Planck constant, 98
Plot Builder
description, 24
Plot component, 312
plot3d command, 276
plots
analyzing, 213
pan, 213
point probe, 213
rotate, 213
scale, 213
code for color plates, 226
creating, 205
context panel, 189
displaying multiple plots, 202
Interactive Plot Builder, 184
plot command, 191
plot3d command, 191
plots package, 198
creating animations
animate command, 215
Interactive Plot Builder, 215
plot3d[viewpoint] command, 217
customizing, 211
context panel, 207
Interactive Plot Builder, 206
plot options, 211
plot3d options, 211
customizing animations, 225
command-line options, 222
context panel, 221
Interactive Plot Builder, 221
data, 214
dragging expression to add to a plot, 202
exporting, 226
functional operators, 276
gradient, 150
line integral, 150
Live Data Plots palette, 214
ODEs
symbolic solution, 89
optimization problem, 139
playing animations, 219
plots package
animate command, 215
contourplot command, 201
display command, 203
matrixplot command, 199
pointplot command, 199
statistics, 143
viewing animations
animate context bar, 219
plotting
using the context panel, 27

point-and-click, 24
polynomial equations
  solving, 83
    numerically, 84
polynomials
  algebra, 110
  arithmetic, 110
  coefficients, 114
  collecting terms, 114
  degree, 114
  division, 110, 111
  expanding, 111
  factoring, 114
  implied multiplication, 111
  numeric
    algebraic manipulation, 115
  operations, 115
  sorting, 111
    pure lexicographic, 112
    total degree, 112
PolynomialTools package, 115
  IsSelfReciprocal command, 115
powers
  entering, 5
precalculus
  study guides, 146
  teaching, 147
precision, 75
prem command, 115
previously assigned, 291
primality testing, 77
primpart command, 115
print
  command, 307
  table, 251
printing
  embedded components, 314
probability distribution, 141
proc key word, 306
procedures, 307
  and assumptions, 106
  calling, 306
  defining, 306
  displaying, 307
  inputs, 306
  multiple lines, 306
  output, 306
  using, 306
product command, 304
products
  entering, 5
  implied, 5
programming, 295
  access to the Maple Programming Guide, 40
programs, 295
  modules, 308
  objects, 308
  procedures, 307
prompt
  input, 56
properties
  testing, 105
protected names, 67

Q

QPSolve command, 140
QR factorization, 127
quadratic programs, 140
quantities with uncertainty, 101
  accessing error, 102
  accessing value, 102
  computing with, 102
  constructing, 101
  element properties, 102
  rounding the error, 102
  scientific constants, 102
  with units, 102
quick
  character formatting, 229
  paragraph formatting, 231
quit statement, 303
quo command, 110
quotes
  double, 277
  left single, 68
  right single, 67, 291
  unevaluation, 291
quotient
  integer, 77

R

Radio Button component, 312
random
  matrices, 119
  variables, 141
randpoly command, 115
range
  in plots, 208
  operator, 123
rank, 126
rational expressions
  entering, 4
read
  from files, 329
RealDomain
  package description, 61
recurrence relation

solving, 92
reference
equation labels, 71
names, 67
relational operators, 296
rem command, 110
remainder
integer, 77
remove command, 304
repetition statements, 298
reserved names, 67
resources
in help system, 39
restart command, 60, 68
resultant command, 115
return
statement, 303
values, 306
rhs command, 280
right single quotes, 67, 291
right-hand side, 280
RootOf structure, 83
roots
command, 115
of equations, 83
Rotary Gauge component, 312
row vector
creating, 121
rsolve command, 92
running
documents, 8
worksheets, 8

S

saving a Maple Document, 13
as a Workbook, 13
scatter plot, 143
scientific constants, 97
list, 98
name, 98
symbol, 98
uncertainty, 100
units, 100
using, 98
value, 100
value and units, 100
Scientific Constants Assistant, 24
ScientificConstants
package description, 61
ScientificConstants package, 98
extensibility, 101
objects, 99
ScientificErrorAnalysis
package description, 61
ScientificErrorAnalysis package, 101
extensibility, 103
objects, 101
search
help system, 38
sections
in worksheet, 237
security levels
auto-execute, 244
security tab
options dialog, 244
select command, 304
selection
execute, 8
selectremove command, 304
semicolon, 56, 57
seq command, 303
series, 133
command, 133
Taylor, 133
type, 133
Set Search Assistant, 24
sets, 270
shape option, 121
Shortcut component, 313
show
worksheet content, 239
show contents dialog
using, 240
significant digits, 75
simplify command, 282, 287
slider
embedding, 263
Slider component, 312
solutions
assigning as expression, 86
assigning as function, 86
details, 91
formal, 91
formal power series, 91
integers, 92
real, 103
series, 91
verifying, 85
solve
equations, 80
for real solutions, 103
numerically, 84
symbolically, 82
inequations, 80
for real solutions, 103
symbolically, 82
integer equations, 92

linear system, 92, 127
modular integer equations, 92
ODEs, 87
PDEs, 91
recurrence relation, 92
transcendental equations, 83
solve command, 82, 271
finding all solutions, 83
finding parametric solutions, 84
real solutions, 103
solving procedures, 84
sort
lists, 285
polynomials, 111, 285
sort command, 111, 285
plex option, 112
source code
displaying, 307
spacing format, 231
Speaker component, 313
Special Functions Assistant, 25
spellcheck, 265
American spelling, 265
dictionary, 267
sqrfree command, 115
Standard Document Interface, xiii
starting, 2
Standard Worksheet Interface, xiii
startup code, 8
Startup Code, 309
statements
multiple lines, 306
Statistics
Student package, 145
Statistics package, 143
continuous distributions, 141
description, 61
discrete distributions, 141
plots, 143
strings, 277
StringTools package, 278
Student
package description, 61
Student Help Center, 41
Student package, 132, 146, 147
calculus subpackages, 137
LinearAlgebra subpackage, 128
Maplets, 146
statistics, 145
Tutors, 146
Student Practice Sheets
Assistant, 26
student resources, 155
students
portal for, 146
study guides, 146
style set management, 237
subscripts
entering, 5
format, 229
substitute, 285
sum command, 304
superscript format, 229
Sylvester matrix, 127
symbol completion, 6
symbolic
computation, 74
objects, 74
symbols
entering, 21
names, 21
system of units, 94
controlling, 97
systeme international (SI), 94

T

Tab
inserting, 62
key, 62
Tab key, 240
table of contents
help system, 39
tables, 273
alignment, 249
appearance, 248
borders, 248
contents, 245
editable, 252
execution order, 251
physical dimensions, 248
printing, 251
using, 245
visibility of cell content, 251
Task Browser, 64
task template, 27
task templates, 64, 76, 93, 116, 128
taylor command, 133
Taylor series, 133
tcoeff command, 114
Teacher Resource Center, 41
teaching with Maple, 155
Technical Support
access, 42
temperature conversion, 94
Text Area component, 312
text field
embedding, 263

Text mode, 14
text regions, 65
Thermophysical Properties Calculator Assistant, 25
tilde, 83, 105, 289
to clause, 298
  excluding, 298
Toggle Button component, 312
Tolerances package, 101
toolboxes
  Global Optimization, 138
Tools menu
  assistants, 24
  Assistants and Tutors, 64
  Tasks, 64
Torsion command, 137
total degree, 112
transparency
  of 3-D plots, 212
transpose
  matrices and vectors, 125
true, 296
Tutorials, 40
Tutors, 146, 147
  Derivatives, 147
  Differentiation Methods, 147
  Directional Derivative, 132
  Gradient, 148
  Linear System Solving, 53
  using, 25
tutors
  accessing, 25
type command, 279
types, 104, 278
  converting, 284
  series, 133
  testing, 279
    subexpressions, 279

U

unapply command, 86
unassign command, 67
unassigning names, 67, 292
uncertainty, 101
  quantities with, 101
underline format, 229
unevaluation quotes, 67, 291
union
  of sets, 270
Unit Converter Assistant, 284
units, 52, 93, 284
  adding to expressions, 52
  applying to expression, 95
  computing with, 96
  context, 94
  converting between, 94
  environment, 96
  evaluating with, 53
  in 1-D Math, 96
  inserting, 95
  overview, 93
  prefixes, 96
  system of
    controlling, 97
  systems of, 94
Units
  package description, 61
Units Converter, 94
Units Converter Assistant, 25
Units package, 93
  environments, 96
  extensibility, 97
  UseSystem command, 97
  UsingSystem command, 97
Units palette, 52, 95
universal gravitational constant, 98
until loop, 300
until loops, 300
unwith command, 60
URL
  adding hyperlink to, 259

V

variables, 44
variance, 102
VariationalCalculus package, 137
Vector
  constructor
    vectorfield attribute, 136
  data structure, 116
vector fields, 136
vector spaces
  basis, 127
VectorCalculus
  package description, 61
VectorCalculus package, 136
  Student version, 137
vectors, 273
  arithmetic, 124
  column, 118
  context panel, 126
  cross product, 125
  data type, 121
  defining, 118
  efficiency, 120
  filling, 121
  large, 119

multiplication, 124
row, 118, 121
scalar multiplication, 124
selecting entries, 122
shape, 121
transpose, 125
Video Player component, 313
View menu
in help system, 39
markers, 36
Volume Gauge component, 313

W

Webpage
adding hyperlink to, 259
Website
access to Maple help pages, 41
Application Center, 41, 146
MaplePrimes, 41
Student Center, 146
Student Help Center, 41
Teacher Resource Center, 41
Technical Support, 42
Training, 41
while loops, 300
Windows
command complete, 6
with command, 59
Workbook Attachment
adding hyperlink to, 260
Workbook File
adding hyperlink to, 261
worksheet
adding hyperlink to, 259
Worksheet Environment, 1
Worksheet Migration Assistant, 25
Worksheet mode, 43, 55
worksheets
running, 8
write
to files, 327

X

xor operator, 296

Z

zero recognition, 285
zip command, 305