

Digits of Powers of 2 in Ternary Numeral System

Yagub Aliyev

yaliyev@ada.edu.az

ADA University

Baku, Azerbaijan

Let us write first powers of two ($1, 2, 4, \dots, 2^{18}$) in ternary numeral system so that their digits in the corresponding place values are aligned along vertical columns.

1	1	1	0	2	2	1	2	1	0	0	1									
	2	0	1	2	2	2	1	0	1	1	2									
		1	0	0	2	2	2	0	0	2	1									
			1	1	2	2	2	1	1	2	2									
				2	1	1	1	0	2	1	1									
					1	0	2	0	2	0	2									
						1	2	1	2	0	1									
							2	2	1	0	2									
								1	1	0	1									
									2	0	2									
										1	1									
											1									
												1								
													2							
														1						
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																1				
																	1			
																		2		
																			1	
																				1

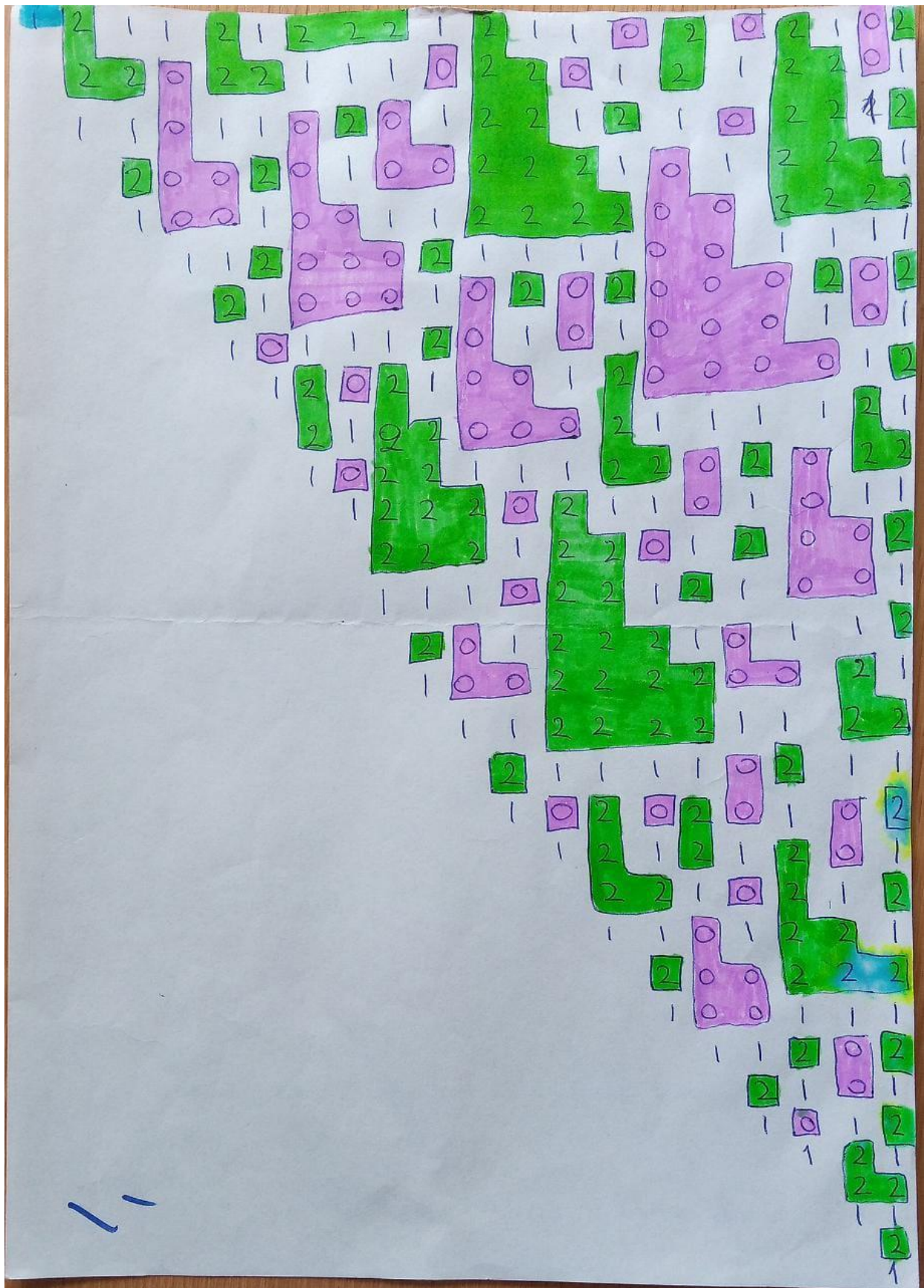
We can easily observe some interesting patterns in this short table. There are blocks of 0's and 2's in the shape of stairs, larger ones of which seem to have sizes which increase unboundedly. This means that there are arbitrarily large such stairs. Each stair of the stairs is of height either 1 or 2 digits. The base 3 representation of the above numbers were calculated using Maple. Example:

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> convert(2^31, base, 3);
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[2, 0, 1, 2, 0, 2, 0, 1, 1, 2, 1, 2, 2, 2, 1, 2, 1, 1, 2, 1]
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The topic has many interesting connections with The Euler Phi-Function, primitive roots, Benford's law, Erdős's conjecture about the powers of 2 which have ternary expansions that omit the digit 2, etc.

The following picture was created by me (numbers and stairs) and my daughter Khadija (11 yrs old) (coloring) for Maple Art and Creative Works Exhibit.



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