Automata Socks and Margolis Cowl

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The mathematical study of braids involves with mathematical representations of one-dimensional strands in three-dimensional space. These strands are also sometimes viewed as representing the movement through a time dimension of points in two-dimensional space. On the other hand, the study of cellular automata usually involves a one- or two-dimensional grid of cells which evolve through a time dimension according to specified rules. This time dimension is often represented as an extra spacial dimension. The ideas of representing both strands in space and cellular automata have also been explored in many artistic media, including drawing, sculpture, knitting, crochet, and weaving. Mathematics professor and amateur crafter Joshua Holden, and professional crafter and amateur mathematician Lana Holden, created a system of Stranded Cellular Automata which realistically captures the behavior of strands in certain media, such as knitting and crochet.

A cellular automaton is a mathematical construct which models a system evolving in time. It is characterized by a discrete set of cells, finite or infinite, in a regular grid, with a finite number of states that a cell can be in. Each cell has a well-defined finite neighborhood which determines how the cell will evolve through the different states. Time moves in discrete steps, and the state of each cell at time t is determined by the states of its neighbors at time $t-1$. Finally, each cell uses the same rule to determine its state. Examples include the "Game of Life", invented by John Conway [1], which has been implemented multiple times in Maple. (See [4], for example.)

In the "Game of Life", each cell can only be in one of two states. Since we want to represent "stranded" designs, we will need more states. Each cell can hold no strands, only a strand starting on the left, only a strand starting on the right, or strands starting on both sides. The strands can be upright or slant from one side of the cell to the other. If there are two slanting strands they will cross, and we need to specify which strand is on "top".

The neighborhood we will use will be a one-dimensional version of the so-called Margolus neighborhood. Unlike the standard for Elementary Cellular Automata, we will represent time as moving from the bottom of our pictures to the top, in order to make it resemble a knitting or crochet pattern. A cellular automata simulator in Maple was found to be a useful way to explore the space of rules and starting states. Two of these are shown in Figures 1 and 2. (The first of these also appeared as a knitting pattern in [3]. For more details on both patterns, see [2].)

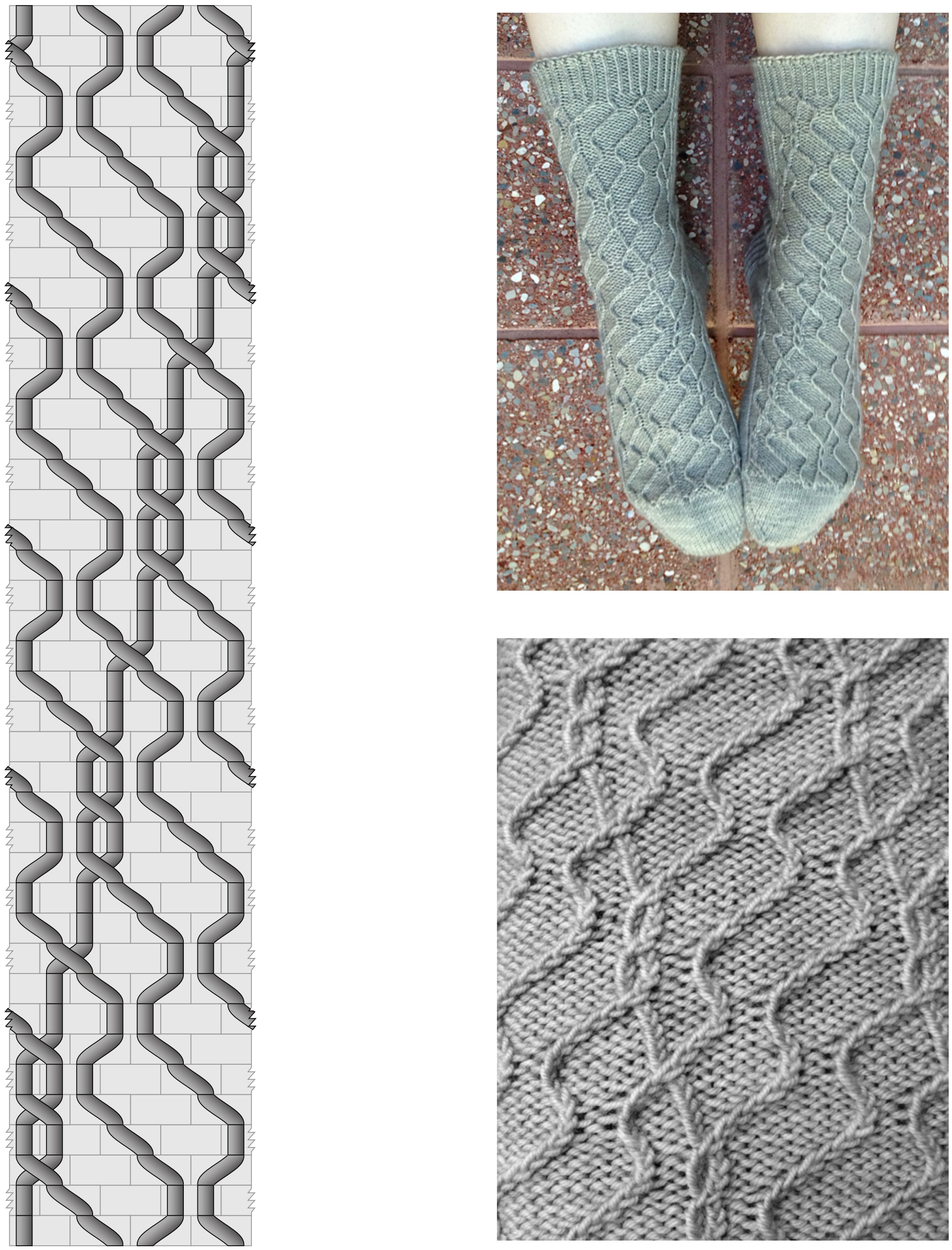

Figure 1 : *Left: Rules 47 and 0. Right: The pattern depicted as cables on a sock, overview and close-up.*

Figure 2 : *Left: Rules 201 and 39. Right: The pattern depicted as cables on a cowl, overview and close-up.*

References

[1] Martin Gardner, *Mathematical Games: The fantastic combinations of John Conway's new solitaire game "life"*, Scientific American **223** (October 1970), 120–123.

[2] Joshua Holden and Lana Holden, *Modeling Braids, Cables, and Weaves with Stranded Cellular Automata*, Proceedings of Bridges 2016, Tessellations Publishing, 2016, pp. 127–134.

[3] Lana Holden, *Knit Stranded Cellular Automata*, Sockupied (Spring 2014), 10–11.

[4] Miles Simmons, *Conway's Game of Life Simulator* (2023), `https://learn.maplesoft.com/doc/m8qbjpgw7r`.