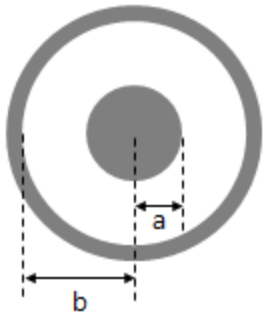
Coaxial Cable Transmission Line Design

An electrical engineer is asked to design a coaxial transmission line with a characteristic impedenace of 50 and a phase velocity of at least 1.8×10 m s^{-1}.

This application will calculate the outer radius of the cable.



Theory

Capacitance and inductance per unit length

$$c := \frac{2 \cdot \pi \cdot \epsilon_r \cdot \epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

$$l := \frac{\mu_0}{8 \cdot \pi} + \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right)$$

Phase velocity

$$v_p := \text{simplify}\left(\frac{1}{\sqrt{l \cdot c}}\right) = \frac{2}{\sqrt{\frac{\mu_0 \cdot \left(1 + 4 \cdot \ln\left(\frac{b}{a}\right)\right) \cdot \epsilon_r \cdot \epsilon_0}{\ln\left(\frac{b}{a}\right)}}}$$

Impedance of the coaxial cable transmission line

$$Z_0 := \text{simplify}\left(\sqrt{\frac{l}{c}}\right) = \frac{\sqrt{\frac{\mu_0 \cdot \left(1 + 4 \cdot \ln\left(\frac{b}{a}\right)\right) \cdot \ln\left(\frac{b}{a}\right)}{\epsilon_r \cdot \epsilon_0}}}{4 \cdot \pi}$$

Parameters

Permittivity and permeability of free space

$\epsilon_0 := 8.854187817 \times 10^{-12}\ \mathbf{F \cdot m^{-1}}$ $\mu_0 := 4 \cdot \pi \times 10^{-7}\ \mathbf{N \cdot A^{-2}}$

Dielectric constant for Teflon

$\epsilon_r := 2.1$

Radius of the center conductor is 22 gauge wire

$a := 0.5 \cdot 0.0253\ \mathbf{inch}$

Characteristic impedence

$eq := 50\ \mathbf{ohm} = Z_0$

Solution

Numerical solution

$res := \text{fsolve}(eq, \{b = 2 \cdot a\}) = \{b = 0.038\ \mathbf{in}\}$

Hence the phase velocity is

$\text{eval}(v_p, res) = 1.866 \times 10^8\ \frac{\mathbf{m}}{\mathbf{s}}$