Pyramidal Horn Design
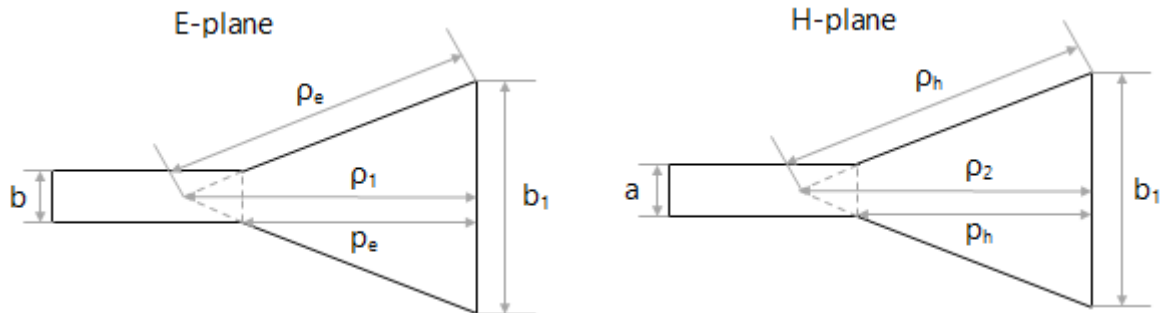
This application calculates the optimum design parameters for an X-band pyramidal antenna.

Parameters

Gain in dB at design frequency	$G_{odb} := 22.6$	$G_o := 10^{G_{odb}/10} = 181.970$
Speed of light	$c := 3 \times 10^{10} \, \text{cm} \cdot \text{s}^{-1}$	
Frequency	$f := 11 \cdot 10^9 \, \text{Hz}$	
Wavelength	$\lambda := \dfrac{c}{f} = 0.027 \, \text{m}$	
Geometrical constraints	$a := 2.286 \, \text{cm}$	$b := 1.016 \, \text{cm}$

Governing Equations

These are needed for optimum directivity

$$\text{cons1} := G_o = \frac{2 \cdot \pi}{\lambda^2} \cdot a_1 \cdot b_1 \qquad \text{cons2} := a_1 = \sqrt{3 \cdot \lambda \cdot \rho_h} \qquad \text{cons3} := b_1 = \sqrt{2 \cdot \lambda \cdot \rho_e}$$

$$\text{cons4} := \rho_e = (b_1 - b) \cdot \sqrt{\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}} \qquad \text{cons5} := \rho_h = (a_1 - a) \cdot \sqrt{\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}}$$

$$\text{cons6} := \rho_e = \rho_h$$

Numerical Solution

$res := fsolve(\{cons1, cons2, cons3, cons4, cons5, cons6\})$

$res = \{a_1 = 0.166\ \mathbf{m},\ b_1 = 0.130\ \mathbf{m},\ p_e = 0.280\ \mathbf{m},\ p_h = 0.280\ \mathbf{m},\ \rho_e = 0.310\ \mathbf{m},\ \rho_h = 0.335\ \mathbf{m}\}$

Plot the E-Plane Radiation Pattern

$$\rho_1 := \sqrt{\rho_e^2 - \left(\frac{b_1}{2}\right)^2}$$

$$t_1 := \theta\ \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(-\frac{b_1}{2} - \rho_1 \cdot \sin(\theta)\right) \qquad t_2 := \theta\ \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(\frac{b_1}{2} - \rho_1 \cdot \sin(\theta)\right)$$

$$F := \theta\ \text{FresnelC}(t_2(\theta)) - \text{FresnelC}(t_1(\theta)) - (1\,i) \cdot (\text{FresnelS}(t_2(\theta)) - \text{FresnelS}(t_1(\theta)))$$

$$E_\theta := \theta\ 20 \cdot \log10\left(|1 + \cos(\theta)| \cdot \frac{|F(\theta)|}{|F(0)|}\right)$$

$\text{plots:-polarplot}\left(\text{eval}(E_\theta(\theta), res) + 70,\ \theta = 0 .. 6.28,\ \text{filled} = \text{true}\right) =$