# Worst Case Circuit Analysis with Monte Carlo Simulation

Electrical components (such as resistors and capacitors) are manufactured in large quantities. Inconsistencies in the raw materials and the manufacturing process means that component parameters have a statistical distribution. That is, the resistance of a batch of resistors might be described by a normal distribution.

Given the number of components in a circuit and the distribution of their parameters, the circuit may not perform as specified. This is a risk that must be identified, managed and mitigated early in the design process.

Worst Case Circuit Analysis (WCCA) is a set of techniques used to analyze how variations in parameters influence circuit performance. One approach is Monte Carlo analysis, in which parameters are randomly selected from a distribution, and the circuit simulated, anywhere from 1000 to 100000 times.

Given the results, circuits can be redesigned to minimize failures due to parameter variations (or an initially overdesigned circuit could be made cheaper to manufacture with less costly components that have a broader parameter distribution).

This application analyses the performance of this circuit using a Monte Carlo approach.
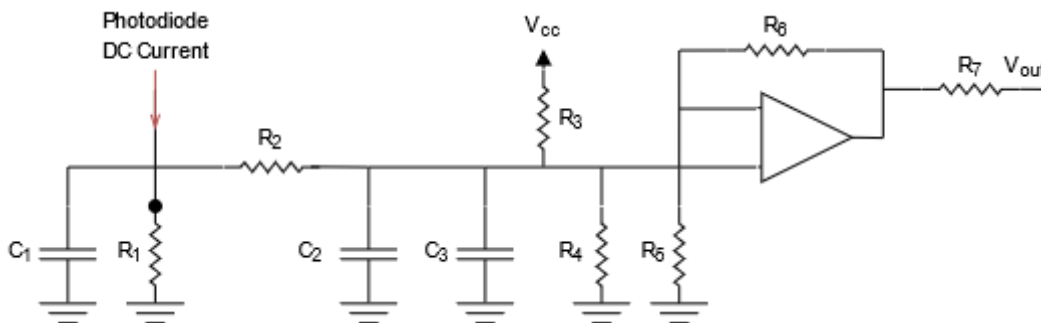
Output voltage

$$V_{out} := \left(R_1, R_2, R_3, R_4, R_5, R_6, R, V_{cc}, P\right) \; \frac{\left(\left(P \cdot R \cdot R_3 + V_{cc}\right) \cdot R_1 + V_{cc} \cdot R_2\right) \cdot R_4 \cdot \left(R_5 + R_6\right)}{\left(R_1 \cdot \left(R_3 + R_4\right) + \left(R_2 + R_4\right) \cdot R_3 + R_2 \cdot R_4\right) \cdot R_5}$$

Number of samples $N := 1000$

Component values $R_1 := $ Statistics:-Sample$($Normal$(9000, 180), N)$

$R_2 := $ Statistics:-Sample$($Normal$(67500, 1350), N)$

$R_3 := $ Statistics:-Sample$($Normal$(2050000, 20500), N)$

$R_4 := \text{Statistics:-Sample}(\text{Normal}(89200, 1338), N)$

$R_5 := \text{Statistics:-Sample}(\text{Normal}(90000, 1350), N)$

$R_6 := \text{Statistics:-Sample}(\text{Normal}(87000, 435), N)$

$R := \text{Statistics:-Sample}(\text{Normal}(1.02, 0.0714), N)$

$V_{cc} := \text{Statistics:-Sample}(\text{Normal}(3, 0.03), N)$

$P := \text{Statistics:-Sample}(\text{Normal}(4.8 \times 10^{-4}, 0.000024), N)$

Simulate the output voltage

$\text{results} := V_{out} \sim (R_1, R_2, R_3, R_4, R_5, R_6, R, V_{cc}, P)$

Mean and standard devision of result

$\mu := \text{Statistics:-Mean}(\text{results}) = 4.677$

$\sigma := \text{Statistics:-StandardDeviation}(\text{results}) = 0.393$

Plot a histogram

$p1 := \text{Statistics:-}Histogram(\text{results}, \text{labels} = [\text{"Voltage (V)"}, \text{"Distribution"}],$
$\text{labeldirections} = [\text{horizontal, vertical}], \text{labelfont} = [\text{Arial}, 11], \text{size} = [800, 500],$
$\text{title} = \text{"Worst Case Circuit Analysis using Monte Carlo Simulation"},$
$titlefont = [\text{Arial}, 14], \text{axesfont} = [\text{Arial}], \text{thickness} = 0.2)$

$p2 := \text{Statistics:-}DensityPlot(\text{Normal}(\mu, \sigma), \text{range} = \min(\text{results}) .. \max(\text{results}),$
$\text{color} = \text{black}, \text{thickness} = 2)$

$p3 := \text{plottools:-}line([\mu - 2\cdot\sigma, 0], [mu - 2\cdot\sigma, 1], \text{color} = \text{"DarkRed"},$
$thickness = 4), \text{plottools:-}line([\mu + 2\cdot\sigma, 0], [\mu + 2\cdot\sigma, 1],$
$\text{color} = \text{"DarkRed"}, \text{thickness} = 4)$

$\text{plots:-display}(p1, p2, p3) =$

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