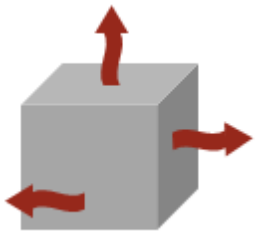
Forecasting Natural Gas Heating Cost for a Building

This application estimates the cost of heating a building with natural gas. The analysis accounts for
- the energy content and cost of natural gas
- heat loss from four walls a roof, and windows
- thickness and R value of insulation



The application uses real heating degree days data for a cold winter month in Kitchener, Ontario.

Humid Air Properties

Density

$$\rho := \frac{1}{\text{ThermophysicalData:-Property}(V, Tdb = 290\ K, P = 1\ atm, R = 0.5, HumidAir)}$$

$$\rho = 1.206\ \frac{kg}{m^3}$$

Specific heat capacity

$$Cp := \text{ThermophysicalData:-Property}(C, Tdb = 290\ K, P = 1\ atm, R = 0.5, HumidAir)$$

$$Cp = 1.011\ \frac{kJ}{kg\ K}$$

Heat capacity per unit volume

$$airCapacityPerVolume := Cp \cdot \rho = 1.220\ \frac{kJ}{K\ m^3}$$

Natural Gas and Furnace Properties

Cost of Natural gas per unit volume at standard conditions

$$costPerVolume := 0.0972\ m^{-3}$$

Energy content of natural gas

$$energyPerVolume := 0.0373\ GJ \cdot m^{-3}$$

Walls and Ceiling

Area of side1, side2, side3, side 4 and ceiling

$$A_{wallceiling} := \begin{bmatrix} 60 \\ 60 \\ 60 \\ 60 \\ 60 \end{bmatrix} m^2$$

Depth of insulation

$$insulationDepth_{wallceiling} := [9cm,\ 9\ cm,\ 9cm,\ 9cm,\ 15\ cm]$$

$$insulationDepth_{wallceiling} := \begin{bmatrix} 9 \\ 9 \\ 9 \\ 9 \\ 15 \end{bmatrix} cm$$

R value of insulation (typical value for fiberglass batts)

$$Rp := 22\ K \cdot m \cdot W^{-1}$$

Windows

Area of windows on all four sides and ceiling

$$A_{windows} := [5.4\ m^2,\ 12\ m^2,\ 2.5\ m^2, 0, 0]$$

$$A_{windows} := \begin{bmatrix} 5.4 \\ 12 \\ 2.4 \\ 0 \\ 0 \end{bmatrix} m^2$$

R value of windows on all four sides and ceiliing

$$R_{windows} := \begin{bmatrix} 0.35 \\ 0.35 \\ 0.35 \\ 0.35 \\ 10^8 \end{bmatrix} \frac{m^2 \cdot K}{W}$$

Miscellanous

Volume of building interior

$volumeBuilding := 400\ m^3$

Furnace efficiency

$furnaceEfficiency := 0.95$

Number of air exchanges per hour

$airExchanges := 0.8\ hour^{-1}$

Mean HDD for Kitchener, Ontario in January

$HDD := 713\ degC \cdot day$

Cost of Natural Gas

Cost per unit energy content of natural gas

$$\text{costPerEnergy} := \frac{\text{costPerVolume}}{\text{energyPerVolume}} = 2.606 \ \frac{1}{\text{GJ}}$$

Effective cost per unit energy, given the furnace efficiency

$$\text{costPerEnergyEffective} := \frac{\text{costPerEnergy}}{\text{furnaceEfficiency}} = 2.743 \ \frac{1}{\text{GJ}}$$

Energy Losses in One Month Through Various Paths

R values of all four sides and ceiling

$$R_{\text{wallceiling}} := \text{Rp} \cdot \text{insulationDepth}_{\text{wallceiling}} = \begin{bmatrix} 1.980 \ \frac{\text{K} \cdot \text{s}^3}{\text{kg}} \\ 1.980 \ \frac{\text{K} \cdot \text{s}^3}{\text{kg}} \\ 1.980 \ \frac{\text{K} \cdot \text{s}^3}{\text{kg}} \\ 1.980 \ \frac{\text{K} \cdot \text{s}^3}{\text{kg}} \\ 3.300 \ \frac{\text{K} \cdot \text{s}^3}{\text{kg}} \end{bmatrix}$$

Heat loss through all four walls and ceiling

$$Q_{\text{wallceiling}} := \text{HDD} \cdot \text{add}\left(\frac{A_{\text{wallceiling}}[i] - A_{\text{windows}}[i]}{R_{\text{wallceiling}}[i]}, i = 1..5\right) = 7.97 \times 10^3 \ \text{MJ}$$

Heat loss through windows

$$Q_{\text{windows}} := \text{HDD} \cdot \text{add}\left(\frac{A_{\text{windows}}[i]}{R_{\text{windows}}[i]}, i = 1..5\right) = 3.48 \times 10^3 \ \text{MJ}$$

Heat loss through air exchanges

$Q_{exchange} := HDD \cdot volumeBuilding \cdot airExchanges \cdot airCapacityPerVolume = 6.68 \times 10^{3}\ MJ$

Total Heating Cost

Total energy loss in one month

$Q_{total} := Q_{wallceiling} + Q_{windows} + Q_{exchange} = 1.81 \times 10^{4}\ MJ$

Total cost of natural gas used in heating home for one month

$cost := Q_{total} \cdot costPerEnergyEffective = 49.745$