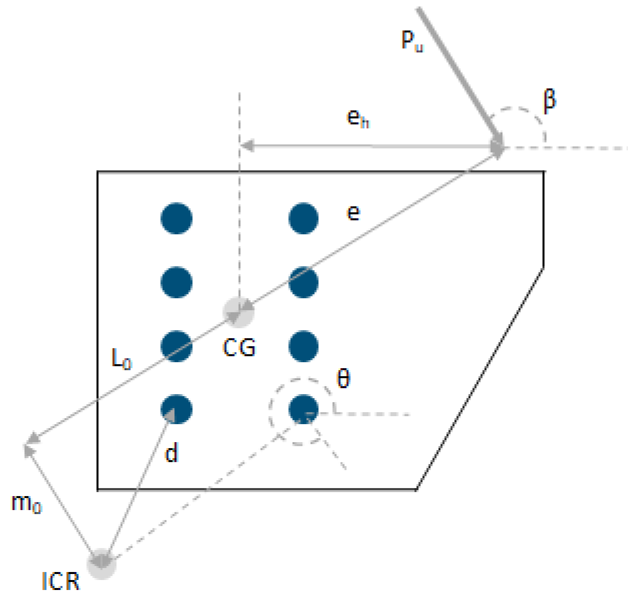
Bolt Group Coefficient for Eccentric Loads

This application calculates the bolt coefficient for eccentrically loaded bolt groups using the Instantaneous Center of Rotation method



The bolt coefficient C is the ratio of the factored force (or available strength) of the bolt group P_u and the shear capacity of a single bolt ϕr_n,

$$C = \frac{P_u}{\phi r_n}$$

Once the coefficient is known, a bolt group can be designed for any load.

Traditionally, bolt group coefficients are extracted by using tabulated values in the AISC Steel Construction Manual. However, these tables are limited to common bolt patterns, and specific load eccentricities and angles. Non-tabulated values must be extracted by using linear interpolation.

This Maple worksheet, however, calculates the bolt group coefficient for any bolt and load configuration by implementing the theory used to generate the tables.

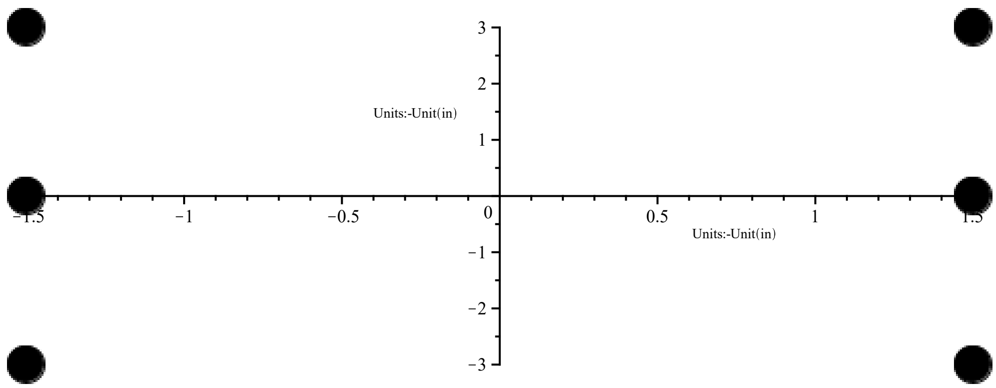
The results agree with those presented in *AISC Manual of Steel Construction: Load and Resistance Factor Design*, 2nd Edition.

Parameters

Bolt locations

$\text{boltLoc} := \text{Matrix}([[-1.5,-3],[1.5,-3],[-1.5,0],[1.5,0],[-1.5,3],[1.5,3]])\,\text{inch}$

plots:-pointplot(boltLoc, style = point, symbol = solidcircle, symbolsize = 40) =



Shear strength of a single bolt $\quad \phi r_n := 96081\ \mathbf{N}$

Horizontal component of force eccentricity with respect to the centroid of the bolt group $\quad e_h := 2\ \mathbf{inch}$

Force angle to horizontal axis $\quad \beta := \frac{75\cdot\pi}{180} = 1.309$

Calculation

Translate the bolt locations so that the centroid is at the origin

$$X := \text{boltLoc}[\,..,1] -\sim \frac{\text{add}(i, i \ \mathbf{in}\ \text{boltLoc}[\,..,1])}{6}$$

$$Y := \text{boltLoc}[\,..,2] -\sim \frac{\text{add}(i, i \ \mathbf{in}\ \text{boltLoc}[\,..,2])}{6}$$

Eccentricity $\quad e := e_h\cdot\sin(\beta) = 1.932\ \mathbf{in}$

Adjusted beta $\quad \beta := \left\| \begin{cases} \beta & e > 0 \\ \beta + 0.5\cdot\pi & \text{otherwise} \end{cases} \right\| = 1.309$

Instantaneous center of rotation (ICR)

$$X_0 := (L_0, m_0) \quad -L_0 \cdot \sin(\beta) - m_0 \cdot \cos(\beta)$$

$$Y_0 := (L_0, m_0) \quad L_0 \cdot \cos(\beta) - m_0 \cdot \sin(\beta)$$

Bolt angle to ICR

$$\theta := (L_0, m_0) \quad \arctan\sim\left(Y - \sim Y_0(L_0, m_0), X - \sim X_0(L_0, m_0)\right) - \sim 0.5 \cdot \pi$$

Bolt distance to ICR

$$d := (L_0, m_0) \quad \left(\left(X - \sim X_0(L_0, m_0)\right)^{\sim 2} + \left(Y - \sim Y_0(L_0, m_0)\right)^{\sim 2}\right)^{\sim \frac{1}{2}}$$

Bolt displacement from Crawford and Kulak (1971)

$$dmax := (L_0, m_0) \quad \max(d(L_0, m_0))$$

$$\Delta := (L_0, m_0) \quad \frac{d(L_0, m_0)}{\sim dmax(L_0, m_0)} \cdot 0.34$$

Load-deformation relationship from Crawford and Kulak (1971)

$$Rn := (L_0, m_0) \quad \phi r_n \cdot \left(1 - \sim \exp\sim\left(-10 \cdot \Delta(L_0, m_0)\right)\right)^{\sim 0.55}$$

Optimization

The sum of the bolt forces in the vertical and horizontal directions are equal to the applied shear and axial loads

$$forceX := (P_0, L_0, m_0) \quad P_0 \cdot \sin(\beta) + add\left(i, i \textbf{ in } Rn(L_0, m_0) \cdot \sim \sin\sim\left(\theta(L_0, m_0)\right)\right) = 0$$

$$forceY := (P_0, L_0, m_0) \quad P_0 \cdot \cos(\beta) + add\left(i, i \textbf{ in } Rn(L_0, m_0) \cdot \sim \cos\sim\left(\theta(L_0, m_0)\right)\right) = 0$$

The moment of the bolt forces about the ICT is equal to the moment of the applied load

$$moment := (P_0, L_0, m_0) \quad P_0 \cdot (L_0 + e) - add\left(i, i \textbf{ in } Rn(L_0, m_0) \cdot \sim d(L_0, m_0)\right) = 0$$

Solve the force and moment balance

$$res := fsolve\left(\left\{forceX(P_0, L_0, m_0), forceY(P_0, L_0, m_0), moment(P_0, L_0, m_0)\right\}, \left\{P_0 = 1\mathrm{N}, m_0 = 1\mathrm{m}, L_0 = 1\mathrm{m}\right\}\right)$$

$$res = \left\{L_0 = 0.091\ \mathbf{m}, P_0 = 4.292 \times 10^5\ \mathbf{N}, m_0 = -0.005\ \mathbf{m}\right\}$$

Ultimate tensile stress of bolt group and bolt coefficient

$$P_u := eval(P_0, res) = 4.292 \times 10^5\ \mathbf{N} \qquad C := \frac{P_u}{\phi r_n} = 4.467$$