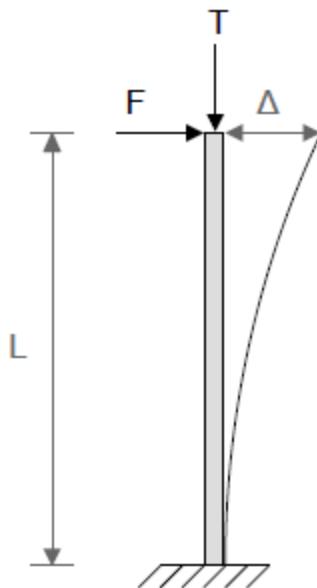


Lateral Drift of a Cantilevered Column

This application calculates the lateral deflection of a fully fixed cantilevered column, taking into account the P-Delta and large displacement effects.

Reference:

Edward L. Wilson: Static & Dynamic Analysis of Structures, 4th Edition, 2004, p. 120-121



Flexural rigidity $EI := 5 \times 10^7$

Length $L := 7.5$

Axial load $T := -350 \times 10^3$

Lateral loads $F := \begin{bmatrix} 20 \times 10^3 \\ 0 \end{bmatrix}$

Response for one step

Stiffness matrix $K_E := \frac{EI}{L^3} \cdot \begin{bmatrix} 12 & 6 \cdot L \\ 6 \cdot L & 4 \cdot L^2 \end{bmatrix}$

Geometric matrix $K_G := \frac{T}{30 \cdot L} \cdot \begin{bmatrix} 36 & 3 \cdot L \\ 3 \cdot L & 4 \cdot L^2 \end{bmatrix}$

Displacement vector for elastic response

$$u_e := K^{-1} \cdot F = \begin{bmatrix} 0.056 \\ -0.011 \end{bmatrix}$$

Displacement vector when P-Delta effects are included

$$u_{eg} := (K_e + K_g)^{-1} \cdot F = \begin{bmatrix} 0.067 \\ -0.013 \end{bmatrix}$$

4x4 stiffness matrix

$$K_e \text{ full} := \frac{EI}{L^3} \cdot \begin{bmatrix} 12 & 6 \cdot L & -12 & -6 \cdot L \\ 6 \cdot L & 4 \cdot L^2 & -6 \cdot L & -2 \cdot L^2 \\ -12 & -6 \cdot L & 12 & -6 \cdot L \\ -6 \cdot L & -2 \cdot L^2 & -6 \cdot L & 4 \cdot L^2 \end{bmatrix}$$

4x4 geometric matrix

$$K_g \text{ full} := \frac{T}{30 \cdot L} \cdot \begin{bmatrix} 36 & 3 \cdot L & -36 & 3 \cdot L \\ 3 \cdot L & 4 \cdot L^2 & -3 \cdot L & -L^2 \\ -36 & -3 \cdot L & 36 & -3 \cdot L \\ 3 \cdot L & -L^2 & -3 \cdot L & 4 \cdot L^2 \end{bmatrix}$$

Response for n steps

Number of steps

$$n := 10$$

Axial force applied in step i

$$T_{inc} := \text{Vector}(n, i \mid \frac{i}{n} \cdot T)$$

Increment of lateral load applied in step i

$$F_{inc} := \text{Vector}(n, i \mid \frac{1}{n} \cdot F)$$

Geometric matrix for step i

$$K_{gi} := \text{Vector}(n, i \mid \frac{T_{inc}[i]}{30 \cdot L} \cdot \begin{bmatrix} 36 & 3 \cdot L \\ 3 \cdot L & 4 \cdot L^2 \end{bmatrix})$$

Displacement increment due to load

$$u_{eg_inc} := \text{Vector}(n, i \mid (K_e + K_{gi}[i])^{-1} \cdot F_{inc}[i])$$

Update total

$$u_{eg_total} := \text{add}(u_{eg_inc}[i], i = 1 .. n)$$

$$u_{eg_total} = \begin{bmatrix} 0.062 \\ -0.012 \end{bmatrix}$$