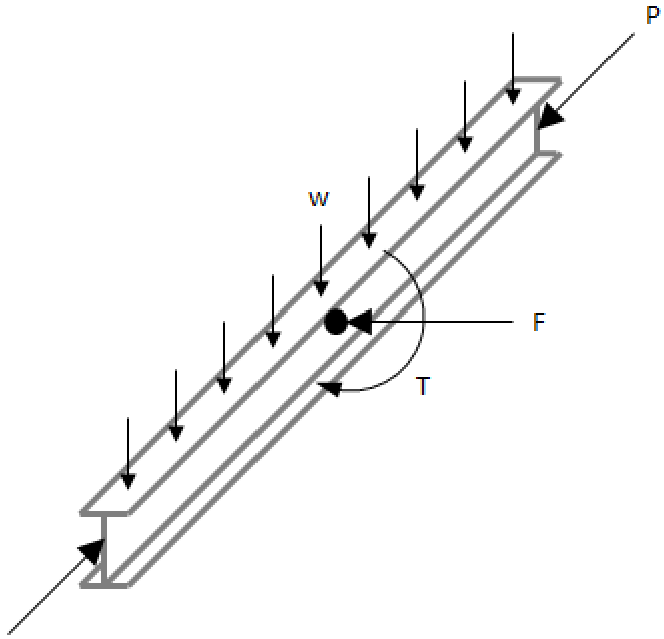
Simply-Supported Beam with Torsional and Lateral Loading

This application analyzes a simply-supported beam with torsional and lateral loading for a W10X54 steel beam (as defined by the AISC Steel Shapes Database).



References:
- Simplified Design for Torsional Loading of Rolled Steel Members, Lin, P.H., Engineering Journal, AISC, 1977

- 2010 Specification for Structural Steel Buildings (ANSI/AISC 360/10), Fourth Printing (https://www.aisc.org/content.aspx?id=2884)

Parameters

Warping Constant $Cw := 1.2 \times 10^{3}\ inch^{6}$

Torsional moment of inertia $JT := 1.51\ inch^{4}$

Elastic section modulus about the X-axis $Sx := 60\ inch^{3}$

Elastic section modulus about the Y-axis $Sy := 20.6\ inch^{3}$

Cross sectional area of member $A := 15.8\ inch^{2}$

Plastic section modulus about the x-axis $Zx := 66.6\ inch^3$

Moment of inertia about the x-axis $Ix := 303\ inch^4$

Moment of inertia about the y-axis $Iy := 103\ inch^4$

Overall depth of member $d := 10.1\ inch$

Radius of gyration about the x-axis $rx := 4.37\ inch$

Gravity distributed load
$w := 1.15\ kipf \cdot ft^{-1}$

Lateral load in middle
$F := 5\ kipf$

Torsion at mid-span
$T := 5.1\ kipf \cdot ft$

Axial load
$P := 96\ kipf$

Beam length
$L := 15\ ft$

Beam yield stress
$Fy := 50\ ksi$

Vertical bending unbraced length
$Lb := 15\ ft$

Axial vertical unbraced length
$Lx := 15\ ft$

Axial horizontal unbraced length
$Ly := 7.5\ ft$

Young's modulus
$E := 29000\ ksi$

Shear modulus
$G := 11200\ ksi$

Tortional property (Philip, 1977)
$\lambda := \sqrt{\frac{G \cdot JT}{E \cdot Cw}} = 0.868\ \frac{1}{m}$

Governing Moments at Middle of Span

Flexural moments

$Mx := w \cdot L^2 / 8 = 32.344\ kipf \cdot foot$

$My := F \cdot L / 4 = 25.422\ kN\ m$

$M0 := T \cdot L / (4 \cdot d) = 30.808\ kN\ m$

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$\beta := \frac{4 \cdot \sinh(\lambda \cdot L/2)^2}{\lambda \cdot L \cdot \sinh(\lambda \cdot L)} = 0.485$

Torsional moment
$MT := \beta \cdot M0 = 1.495 \times 10^4\ J$

Check Torsional Capacity (AISC 360-10 H3.3 & Philip p100)

Maximum combined normal stress at the load point $fbx := \frac{Mx}{Sx} + \frac{2 \cdot MT}{Sy} = 1.332 \times 10^8\ Pa$

$Fnx := Fy / \Omega = \frac{50}{\Omega}\ ksi$

Safety factor for compression $\Omega := 1.67$

If this is less than 1, then design is satisfactory $fbx/Fnx = 0.645$

Check Combined Compression and Bending Capacity (AISC 360-10, H1)

$Mrx := (Mx/Sx + 2 \cdot MT/Sy) \cdot Sx = 130.949\ \mathbf{kN\ m}$

Effective length factor $K := 0.85$

Elastic buckling stress

$$Fe := \frac{\pi^2 \cdot E}{(K \cdot L/rx)^2} = 233.495\ \mathbf{ksi}$$

Critical stress

$$Fcr := 0.658^{\frac{Fy}{Fe}} \cdot Fy = 45.714\ \mathbf{ksi}$$

Allowable axial strength

$Pn := Fcr \cdot A = 3212.839\ \mathbf{kN}$

$Pc := Pn/\Omega = 1923.855\ \mathbf{kN}$

Available flexural strength (Chapter F AISC 360-10)

$Mn := \min(Fy \cdot Zx, Fy \cdot Sx) = 338.954\ \mathbf{kN\ m}$

$Mcx := Mn/\Omega = 202.967\ \mathbf{kN\ m}$

This is greater than Mrx so it is satisfactory

$Mcy := Mn/\Omega = 2.030 \times 10^5\ \mathbf{J}$

This should be below 1 for a satisfactory design

$$\frac{P}{Pc} + \frac{8}{9} \cdot \left(\frac{Mrx}{Mcx} + \frac{My}{Mcy} \right) = 0.907$$

Determine Deflections

Max twist angle (Lin, p100 eq4) in degrees

$$\phi := \frac{T}{2 \cdot G \cdot JT \cdot \lambda} \cdot \left(\frac{\lambda \cdot L}{2} - \frac{2 \cdot \sinh(\lambda \cdot L/2)}{\sinh(\lambda \cdot L)} \right) \cdot \sinh\left(\frac{\lambda \cdot L}{2} \right) = 0.502$$

$$I3 := Ix \cdot \cos\left(\frac{(90 - \phi) \cdot \pi}{180} \right)^2 + Iy \cdot \sin\left(\frac{(90 - \phi) \cdot \pi}{180} \right)^2 = 103.015\ \mathbf{in^4}$$

$$I4 := Ix \cdot \cos\left(\frac{(90 - \phi) \cdot \pi}{180} \right)^2 + Iy \cdot \sin\left(\frac{(90 - \phi) \cdot \pi}{180} \right)^2 = 103.015\ \mathbf{in^4}$$

Vertical deflection at middle

$$\Delta vert := \frac{5 \cdot w \cdot L^4}{384 \cdot E \cdot I3} = 0.011\ \mathbf{m}$$

Horizontal deflection at middle

$$\Delta horiz := \frac{F \cdot L^3}{48 \cdot E \cdot I4} = 0.005\ \mathbf{m}$$