Maximum Flow Rate in Open-Channel Flow for a Circular Pipe

This application determines the greatest attainable flowrate in a circular pipe partially filled with water.



The Manning formula is employed to calculate the open-channel flow of water:

$$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{2}$$

where

- Q is the flowrate
- n is an empirical coefficient
- A is the cross-sectional area of flow
- R is the hydraulic raduis
- S is the incline of the channel

Manning formula

$$\mathbf{Q} \coloneqq \frac{1.49}{n} \cdot \mathbf{A} \cdot \mathbf{R}^{\frac{2}{3}} \cdot \mathbf{S}_{0}^{\frac{1}{2}}$$

Flow area for a partially filled circular pipe

$$\mathsf{A} := \pi \cdot \mathsf{r}^2 - \mathsf{r}^2 \cdot \frac{\theta - \sin(\theta)}{2}$$

Wetted perimiter and hydraulic radius

$$P := 2 \!\cdot\! \pi \!\cdot\! r \!-\! r \!\cdot\! \theta$$

$$R := \frac{A}{P} = \frac{3.14 \cdot r^2 - r^2 \cdot (0.50 \cdot \theta - 0.50 \cdot \sin(\theta))}{-r \cdot \theta + 6.28 \cdot r}$$

The Manning formula then becomes

$$Q = \frac{\left(4.68 \cdot r^{2} - 1.49 \cdot r^{2} \cdot \left(0.50 \cdot \theta - 0.50 \cdot \sin(\theta)\right)\right) \cdot \left(\frac{3.14 \cdot r^{2} - r^{2} \cdot \left(0.50 \cdot \theta - 0.50 \cdot \sin(\theta)\right)}{-r \cdot \theta + 6.28 \cdot r}\right)^{2/3} \cdot \sqrt{S_{0}}}{n}$$

Parameters

Maximum flow rate

Find the value of theta that maximizes Q

$$\begin{split} n &:= 0.013 \qquad S_0 := 0.0001 \qquad r := 4 \\ res &:= Optimization:-Maximize(Q) \\ Q_{maxflow} &:= res[1] = 98.38 \end{split}$$

$$\theta_{maxflow} \coloneqq rhs(res[2, 1]) = 1.005$$

