Deriving a Least Cost Pipe Diameter Equation

This application implements a simple economic analysis to determine the optimum diameter of a horizontal pipeline with liquid being conveyed by a pump.

The analysis demonstrates that the most economic pipe diameter is proportional to

- the friction factor to the power of 1/6 (and hence is not signficantly influenced by this factor)
- the square root of the mass flowrate

Reference:
Nevers, de Noel, *Fluid Mechanics for Chemical Engineers,* 2nd Edition, 2nd edition, McGraw Hill, 1991

Theoretical derivation

- PP is an empirical constant that determines the cost of supplies and labor, and has dimensions of \$/(diameter in inches × length in feet)
- CC is an empirical constant that determines the capital charge, and has dimensions of 1/year
- PC is an empirical constant that determines the annual pumping cost, and has dimensions of \$/(hp × year)
- m is the mass flowrate
- pDia and pLen are the pipe diameter and length

Costs of supplies and labor

$$\mathrm{PurchasePrice} := \mathrm{PP} \cdot \mathrm{pDia} \cdot \mathrm{pLen}$$

Yearly capital charge

$$\mathrm{AnnualCapitalCharge} := \mathrm{CC} \cdot \mathrm{PurchasePrice}$$

Yearly pumping cost

$$\mathrm{AnnualPumpingCost} := \mathrm{PC} \cdot \mathrm{PumpPower}$$

Pump power for a horizontal pipe

$$\mathrm{PumpPower} := \frac{m^3 \cdot 2 \cdot f \cdot \mathrm{pLen} \cdot (4/\pi)^2}{\rho^2 \cdot \mathrm{pDia}^5}$$

Total annual cost

$$\mathrm{TotalAnnualCost} := \mathrm{PC} \cdot \mathrm{PumpPower} + \mathrm{CC} \cdot \mathrm{PP} \cdot \mathrm{pDia} \cdot \mathrm{pLen}$$

$$\mathrm{TotalAnnualCost} = \frac{32 \cdot \mathrm{PC} \cdot m^3 \cdot f \cdot \mathrm{pLen}}{\pi^2 \cdot \rho^2 \cdot \mathrm{pDia}^5} + \mathrm{CC} \cdot \mathrm{PP} \cdot \mathrm{pDia} \cdot \mathrm{pLen}$$

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The most economic pipe diameter is given by setting the derivative to zero, and solving for the pipe diameter

$$\text{sol} := solve\left(\frac{d}{dpDia}\text{TotalAnnualCost} = 0, \text{pDia}\right)$$

$$\text{sol}[1] = \frac{160^{1/6}\cdot\left(PC\cdot m^3\cdot f\cdot CC^5\cdot PP^5\cdot\pi^4\cdot\rho^4\right)^{1/6}}{CC\cdot PP\cdot\pi\cdot\rho}$$

Parameters and Solution

Liquid density

$$\rho := 62.3\,\text{lb}\cdot\text{ft}^{-3}$$

Mass flowrate

$$m := 200\ \text{gal}\cdot\text{min}^{-1}\cdot\rho = 12.592\ \frac{\text{kg}}{\text{s}}$$

Friction factor

$$f := 0.0042$$

Economic parameters

$$PC := 270\ \text{HP}^{-1}\cdot\text{year}^{-1}$$

$$PP := 2\ \text{inch}^{-1}\cdot\text{ft}^{-1}$$

$$CC := 0.4\ \text{year}^{-1}$$

Hence the most economic pipe diameter is

$$\text{sol}[1] = \frac{1.509\cdot\left(\frac{1}{\text{HP}\cdot\text{yr}}\cdot\left(\frac{\text{kg}}{\text{s}}\right)^3\cdot\left(\frac{1}{\text{yr}}\right)^5\cdot\left(\frac{1}{\text{in}\cdot\text{ft}}\right)^5\cdot\left(\frac{\text{lb}}{\text{ft}^3}\right)^4\right)^{1/6}}{1\ \frac{1}{\text{yr}}\cdot\frac{1}{\text{in}\cdot\text{ft}}\ \frac{\text{lb}}{\text{ft}^3}}$$

$$\text{simplify}(\text{sol}[1]) = 0.088\ \text{m}$$