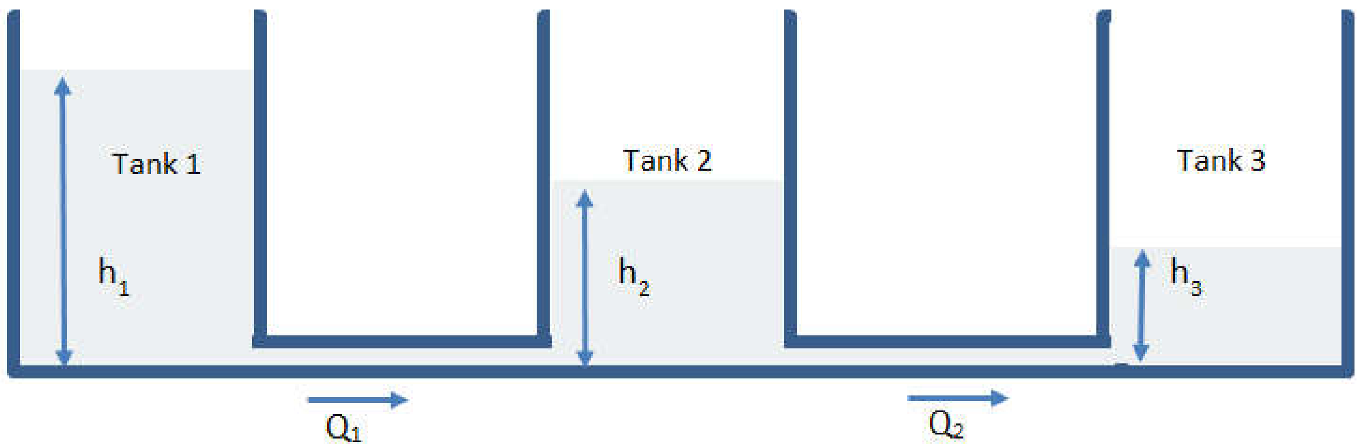
Interacting Tank Reservoirs

Introduction

This worksheet models liquid flow between three tanks connected by two pipes (the first pipe connecting Tank 1 and 2, and the second pipe connecting Tank 2 and 3).



The flow is opposed by pipe friction, and the level of liquid in each tank oscillates to an equilibrium. Differential equations that describe the dynamic change in liquid height in each tank and a momentum balance are solved numerically.

> *restart*;

Physical Parameters

Cross-sectional area of tanks

> $A_1 := 1 : A_2 := 5 : A_3 := 2 :$

Diameter, length, and roughness of pipe

> $Dia := 0.3 : L := 100 : e := 0.001 :$

Density and viscosity of liquid

> $\rho := 1000 : \mu := 0.001 :$

Gravitational constant

> $g := 9.81 :$

Momentum Balance

Friction Factor

```
> friction :=proc(Q)
  local Rey, fL, fT :
   if type(Q, numeric) then
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```
Rey := 4 Q ρ / (evalf(π) Dia μ) :
fL := 64 / Rey :
fT := 1 / (1.8 log10(6.9/Rey + (e/(3.7 Dia))^1.11))^2 :

  if Rey > 0 and Rey < 2000 then
    return fL :
  elif Rey ≥ 2000 and Rey < 4000 then
    return fL + ((fT − fL) · (Rey − 2000)) / (4000 − 2000)

  elif Rey ≥ 4000 then
    return fT
  else
    return 0
  end if;
else
 return 'friction'(Q)
end if
end proc:
```

$$Rey := \frac{4\,Q\,\rho}{evalf(\pi)\;Dia\,\mu} :$$

$$fL := \frac{64}{Rey} :$$

$$fT := \frac{1}{\left(1.8\log10\left(\frac{6.9}{Rey} + \left(\frac{e}{3.7\,Dia}\right)^{1.11}\right)\right)^{2}} :$$

Differential Equations

The rate of change of liquid height in Tank 1

$$> \; height_1 := \frac{d}{dt} H_1(t) = -\frac{Q_1(t)}{A_1} :$$

The rate of change of liquid height in Tank 2

$$> \; height_2 := \frac{d}{dt} H_2(t) = \frac{Q_1(t) - Q_2(t)}{A_2} :$$

The rate of change of liquid height in Tank 3

$$> \; height_3 := \frac{d}{dt} H_3(t) = \frac{Q_2(t)}{A_3} :$$

A momentum balance

$$> \; momentumBalance_1 := \frac{d}{dt} Q_1(t) = \frac{\pi\,Dia^2\,g\,H_1(t)}{4\,L} - \frac{\pi\,Dia^2\,g\,H_2(t)}{4\,L} - \frac{2\cdot friction\left(\mathrm{abs}\left(Q_1(t)\right)\right)\,\mathrm{abs}\left(Q_1(t)\right)\cdot Q_1(t)}{\pi\,Dia^3} :$$

> $momentumBalance_2 := \frac{d}{dt} Q_2(t) = \frac{\pi Dia^2 g H_2(t)}{4 L} - \frac{\pi Dia^2 g H_3(t)}{4 L} - \frac{2 \cdot friction(\mathrm{abs}(Q_2(t)))\, \mathrm{abs}(Q_2(t)) \cdot Q_2(t)}{\pi Dia^3}$:

Initial Conditions

> $initialConditions := Q_1(0) = 0, Q_2(0) = 0, H_1(0) = 1.5, H_2(0) = 1.2, H_3(0) = 2$:

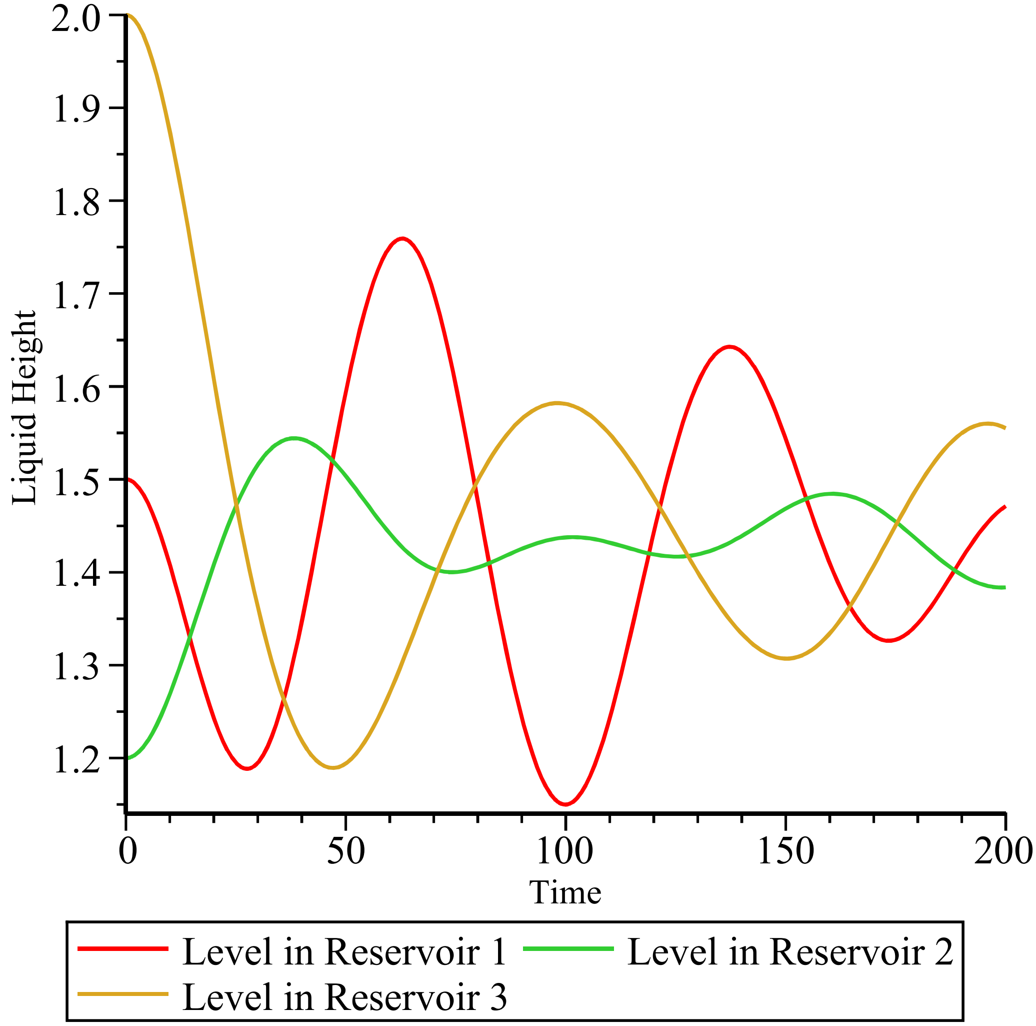
Numerical Solution of Governing Equations

> $res := dsolve(\{height_1, height_2, height_3, momentumBalance_1, momentumBalance_2, initialConditions\},$
$\{H_1(t), H_2(t), H_3(t), Q_1(t), Q_2(t)\},$
$numeric, output = listprocedure, known = friction)$:

> $H_1 := subs(res, H_1(t))$: $H_2 := subs(res, H_2(t))$: $H_3 := subs(res, H_3(t))$:
$Q_1 := subs(res, Q_1(t))$: $Q_2 := subs(res, Q_2(t))$:

Results

> $plot([H_1(t), H_2(t), H_3(t)], t = 0..200,$
legend = (["Level in Reservoir 1","Level in Reservoir 2", "Level in Reservoir 3"]),
labels = (["Time","Liquid Height"]),
labeldirections = ([*horizontal*, *vertical*]));


2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
Liquid Height
0
50
100
150
200
Time
Level in Reservoir 1
Level in Reservoir 2
Level in Reservoir 3