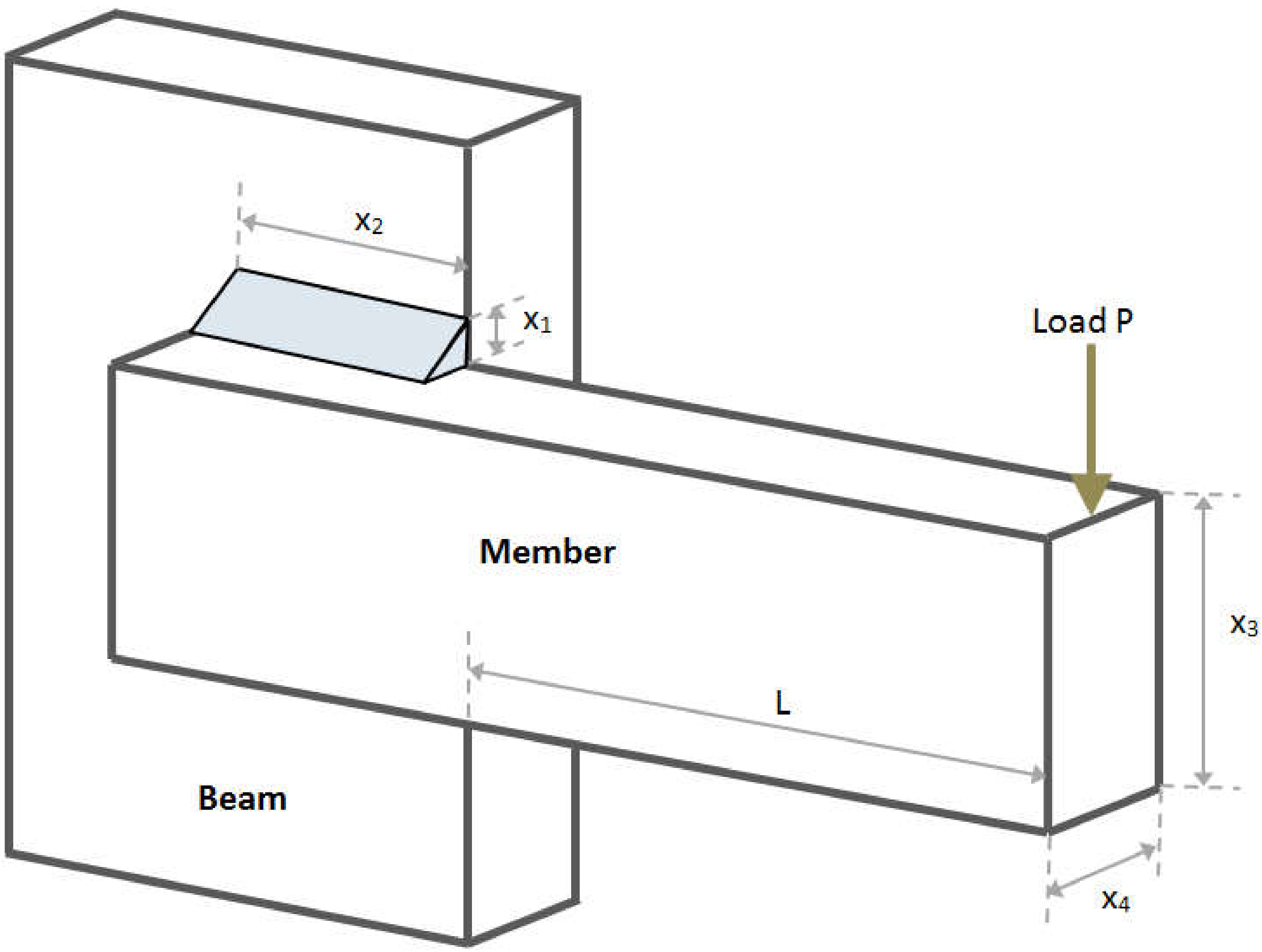
Welded Beam Design Optimization

Introduction

The diagram illustrates a rigid member welded onto a beam. A load is applied to the end of the member.



The total cost of production is equal to the labor costs (a function of the weld dimensions) plus the cost of the weld and beam material.

The beam is to be optimized for minimum cost by varying the weld and member dimensions x_1, x_2, x_3 and x_4. The constraints include limits on the shear stress, bending stress, buckling load and end deflection. The variables x_1 and x_2 are usually integer multiples of 0.0625 inch, but for this application are assumed continuous.

Reference: K. Ragsdell and D. Phillips. Optimal Design of a Class of Welded Structures using Geometric Programming. J. Eng. Ind., 98(3):1021–1025, 1976.

Parameters

> restart :

Young's modulus (psi)

> $E := 30 \cdot 10^6$:

Shearing modulus for the beam material (psi)

> $G := 12 \cdot 10^6$:

Overhang length of the member (inch)

> $L := 14$:

Design stress of the weld (psi)

> $\tau_{max} := 13600$:

Design normal stress for the beam material (psi)

> $\sigma_{max} := 30000$:

Maximum deflection (inch)

> $\delta_{max} := 0.25$:

Load (lb)

> $P := 6000$:

Cost per unit volume of the weld material (\$ $inch^{-3}$)

> $C_1 := 0.10471$:

Cost per unit volume of the bar (\$ $inch^{-3}$)

> $C_2 := 0.04811$:

Labor cost per unit weld volume (\$ $inch^{-3}$)

> $C_3 := 1$:

Cost Function

Volume of weld material ($inch^3$)

> $V_{weld} := x_1^2 \cdot x_2$:

Volume of bar ($inch^3$)

> $V_{bar} := x_3 \cdot x_4 \cdot (L + x_2)$:

Total material cost to be minimized.

> $f := (x_1, x_2, x_3, x_4) \rightarrow (C_1 + C_3) \cdot V_{weld} + C_2 \cdot V_{bar}$:

Constraints

The shear stress at the beam support location cannot exceed the maximum allowable for the material

> $con1 := \tau_{max} - \tau(x_1, x_2, x_3, x_4) \geq 0$:

The normal bending stress at the beam support location cannot exceed the maximum yield strength for the material

> $con2 := \sigma_{max} - \sigma(x_1, x_2, x_3, x_4) \geq 0$:

The member thickness is greater than the weld thickness

> $con3 := x_4 - x_1 \geq 0$:

> $con4 := C_1 \cdot x_1^2 + C_2 \cdot x_3 \cdot x_4 \cdot (L + x_2) - 5 \leq 0$:

The weld thickness must be larger than a defined minimum

> con5 := $x_1 - 0.125 \geq 0$:

The deflection cannot exceed the maximum deflection

> con6 := $\delta_{max} - \delta(x_1, x_2, x_3, x_4) \geq 0$:

The buckling load is greater than the applied load

> con7 := $Pc(x_1, x_2, x_3, x_4) - P \geq 0$:

Size constraints

> con8 := $x_1 \geq 0.1, x_4 \leq 2.0, x_2 \geq 0.1, x_3 \leq 10$:

Collect all the constraints

> cons := {con1, con2, con3, con4, con5, con6, con7, con8} :

Engineering Relationships

Weld stress

$$> \tau := (x_1, x_2, x_3, x_4) \rightarrow \sqrt{\tau_d^2 + 2 \cdot \tau_d \cdot \tau_{dd} \cdot \frac{x_2}{2R} + \tau_{dd}^2} :$$

Primary stress acting over the weld throat

$$> \tau_d := \frac{P}{\sqrt{2} \cdot x_1 \cdot x_2} :$$

Secondary torsional stress.

$$> \tau_{dd} := \frac{M \cdot R}{J} :$$

Moment of P about center of gravity of weld setup

$$> M := P \cdot \left(L + \frac{x_2}{2} \right) :$$

$$> R := \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2} \right)^2} :$$

Polar moment of inertia of weld

$$> J := 2 \left(x_1 \cdot x_2 \cdot \sqrt{2} \cdot \left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2} \right)^2 \right) \right) :$$

Bar bending stress

$$> \sigma := (x_1, x_2, x_3, x_4) \rightarrow \frac{6 \cdot P \cdot L}{x_4 \cdot x_3^2} :$$

Bar Deflection. To calculate the deflection, assume the bar to be a cantilever of length L

$$> \delta := (x_1, x_2, x_3, x_4) \rightarrow \frac{4 \cdot P \cdot L^3}{E \cdot x_4 \cdot x_3^2} :$$

For narrow rectangular bars, the bar buckling load is approximated by (Timoshenko, S., and J. Gere,Theory of Elastic Stability,McGraw-Hill, New York, 1961, p. 257)

$$> Pc := (x_1, x_2, x_3, x_4) \rightarrow \frac{4.013 \cdot E \cdot \sqrt{\frac{x_3^2 \cdot x_4^6}{36}}}{L^2} \cdot \left(1 - \frac{x_3}{2 \cdot L} \cdot \sqrt{\frac{E}{4 \cdot G}} \right) :$$

Optimzation

> bounds := $x_1 = 0..10, x_2 = 0..10, x_3 = 0..10, x_4 = 0..10$:

Hence the minimum cost and optimized dimensions are

> sol := Optimization:-Minimize$\left(\mathrm{f}\left(x_1, x_2, x_3, x_4\right), \mathrm{cons}, \mathrm{bounds}\right)$

$$sol := [1.72485230854216631, [x_1 = 0.205729639770726, x_2 = 3.47048866582864, x_3 = 9.03662391069483, x_4 = 0.205729639770726]] \quad \textbf{(6.1}$$