Advanced Math

Maple 2017 includes numerous cutting-edge updates in a variety of branches of mathematics.

GroupTheory

The GroupTheory package has been extended and improved in several respects for Maple 2017. Most notable are an implementation of the Burnside-Dixon-Schneider algorithm to compute the (ordinary) character table of a finite group, and the redesign of the database architecture for small groups that allows for new and more flexible search options for the SearchSmallGroups command. In addition, the NumGroups command has been improved for this release. Finally, most objects produced by the GroupTheory package now have custom context-sensitive menus available.

Improvements to the SearchSmallGroups command

You can now search for groups whose specified subgroups and quotients have a designated order, as well as a specific group ID. Previously, if you wanted to find groups whose center had order equal to 4, you would have to have specified individual searches for each of the (two) groups of order 4 separately and then manually combine the results:

$with(GroupTheory):$

$A := SearchSmallGroups(order = 1..20, center = [4, 1])$

$$A := [4, 1], [16, 6], [16, 13]$$

$B := SearchSmallGroups(order = 1..20, center = [4, 2])$

$$B := [4, 2], [16, 3], [16, 4], [16, 11], [16, 12]$$

A, B

$$[4, 1], [16, 6], [16, 13], [4, 2], [16, 3], [16, 4], [16, 11], [16, 12]$$

Now it is possible to simply indicate the order in a single invocation of the command:

$SearchSmallGroups(order = 1..20, center = 4)$

$$[4, 1], [4, 2], [16, 3], [16, 4], [16, 6], [16, 11], [16, 12], [16, 13]$$

This works for all the subgroup and quotient properties that are supported by the SearchSmallGroups command.

In addition, several new properties have been added to the SearchSmallGroups command. These are: **classnumber** (the number of conjugacy classes), **centralquotient** (the order, or group ID of the quotient of the group by its center), **fittingquotient** (the quotient of the group by its Fitting subgroup) and **frattiniquotient** (the quotient of the group by its Frattini subgroup).

Computing Character Tables of Finite Groups

Maple 2017 includes a new command, CharacterTable, in the GroupTheory package to compute the ordinary character table of a finite group.

$with(GroupTheory):$

$G := Alt(4)$

$$G := \mathbf{A}_4$$

$ct := CharacterTable(G)$

$$ct := \begin{bmatrix} C & 1a & 2a & 3a & 3b \\ |C| & 1 & 3 & 4 & 4 \\ X1 & 1 & 1 & 1 & 1 \\ X2 & 1 & 1 & -\frac{1}{2} - \frac{I\sqrt{3}}{2} & -\frac{1}{2} + \frac{I\sqrt{3}}{2} \\ X3 & 1 & 1 & -\frac{1}{2} + \frac{I\sqrt{3}}{2} & -\frac{1}{2} - \frac{I\sqrt{3}}{2} \\ X4 & 3 & -1 & 0 & 0 \end{bmatrix}$$

The first line of the character table displays the class labels, while the second row indicates the size of the corresponding conjugacy class. The characters themselves occupy succeeding rows, with the value of each character on a conjugacy class in the corresponding column of the table.

Character tables are represented as Maple objects with a number of methods. For example, the CharacterDegrees command returns a list of the degrees of the irreducible characters of the group, along with their multiplicities.

$CharacterDegrees(ct)$

$$[[1, 3], [3, 1]]$$

The Display method supports the inclusion of additional information about the character table, such as the associated (prime) power maps and the Frobenius-Schur indicator values.

Display(*ct*, *showpowermaps*, *showindicator*)

	C	1a	2a	3a	3b
	|C|	1	3	4	4
	c^2	1a	1a	3b	3a
	c^3	1a	2a	1a	1a
ν_2					
1	χ_1	1	1	1	1
0	χ_2	1	1	$-\frac{1}{2} - \frac{I\sqrt{3}}{2}$	$-\frac{1}{2} + \frac{I\sqrt{3}}{2}$
0	χ_3	1	1	$-\frac{1}{2} + \frac{I\sqrt{3}}{2}$	$-\frac{1}{2} - \frac{I\sqrt{3}}{2}$
1	χ_4	3	−1	0	0

To access the characters themselves, use the Character command:

c2 := *Character*(*ct*, 2)

$$c2 := \left\langle \text{character: 1a} \rightarrow 1, \text{2a} \rightarrow 1, \text{3a} \rightarrow -\frac{1}{2} - \frac{I\sqrt{3}}{2}, \text{3b} \rightarrow -\frac{1}{2} + \frac{I\sqrt{3}}{2} \text{ for } \mathbf{A}_4 \right\rangle$$

GroupOrder(*Kernel*(*c2*))

4

Indicator(*c2*)

0

Improvements to the NumGroups Command

The NumGroups command has been updated for Maple 2017 so that all cases for arithmetically "small" group orders of the form p^3q, p^2qr and p^2q^2 are now handled. Also, an improved algorithm for computing the number of groups of a square-free order has been implemented that allows you to compute the number of groups for much larger square-free orders than in previous releases.

```
> NumGroups( 139565314102386439 ); # 16253^3 * 32507
```

16263

```
> NumGroups( mul( ithprime( i ), i = 100 .. 1000 ) );
```

27713402698845946768149019108756751520967372957812367222372050177142623558828\
8099930603520000000

Series and Limits

The asympt and limit commands can now handle asymptotic cases of the incomplete Γ function where both arguments tend to infinity and their quotient remains finite.

$asympt(\Gamma(x, 2x), x)$

$$\frac{\left(\frac{1}{x} - \frac{2}{x^2} + \frac{10}{x^3} - \frac{82}{x^4} + \frac{938}{x^5} + O\left(\frac{1}{x^6}\right)\right) 2^x}{(e^x)^2 \left(\frac{1}{x}\right)^x}$$

$asympt(\Gamma(x, x+1), x, 3)$

$$\frac{\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{\frac{1}{x}}}{2} - \frac{4}{3\,x} + \frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{1}{x}\right)^{3/2}}{24} + \frac{86}{135\,x^2} + \frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{1}{x}\right)^{5/2}}{576} + O\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x}\right)^x e^x}$$

$\lim_{x \to \infty} (\Gamma(x, 1) - \Gamma(x))$

$$0$$

The limit command has also been enhanced for the case of limits of quotients of two bivariate analytic functions.

Slode

The Slode package computes formal series expansions for linear ODEs. The commands FPseries, FTseries and dAlembertian_series_sol now support inhomogeneous ODEs where the right-hand side is d'Alembertian.

$ode := y'(x) - y(x) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n-1} \Gamma(k+1)\right) \frac{x^n}{n!}$

$$ode := y'(x) - y(x) = \sum_{n=0}^{\infty} \frac{\left(\sum_{k=0}^{n-1} \Gamma(k+1)\right) x^n}{n!}$$

$Slode\text{:-}dAlembertian\_series\_sol(ode, y(x))$

$$\left(\sum_{\_n=1}^{\infty} \frac{1}{\_n!} \left(\left(\prod_{\_k=1}^{\_n-1} \frac{\_k+1}{\_k}\right) \left(\left(\sum_{\_n1=1}^{\_n-1} \frac{\_n1 \left(\prod_{\_k=1}^{\_n1-1} \frac{\_k^2}{(\_k+1)^2}\right)}{\_n1+1}\right)\right.\right.\right.$$

$$+\left(\sum_{\_n1=1}^{\_n-1}\frac{\_n1\left(\prod_{\_k=1}^{\_n1-1}\frac{\_k^2}{(\_k+1)^2}\right)\left(\sum_{\_n2=1}^{\_n1-1}\frac{(\_n2+1)\left(\prod_{\_k=1}^{\_n2-1}\frac{(\_k+1)^2}{\_k}\right)}{\_n2}\right)}{\_n1+1}\right)\Bigg)x^{\_n}\Bigg)\Bigg)$$

$$+\_C_0\left(\sum_{\_n=0}^{\infty}\frac{x^{\_n}}{\_n!}\right)$$

Symbolic Summation

The output format for definite sums involving binomials has been improved, so that the answer is expressed again in terms of binomials, if possible, and also more simplified.

Maple 2017	Maple 2016
$\sum_{k=0}^{n}\binom{2n}{2k}^2$ $\frac{(-1)^n\binom{2n}{n}}{2}+\frac{\binom{4n}{2n}}{2}$	$\sum_{k=0}^{n}\binom{2n}{2k}^2$ $\left(\frac{2^{2n+\frac{5}{2}}\sqrt{2}\,\Gamma\left(2n+\frac{1}{2}\right)n}{4}-\frac{\Gamma\left(2n+\frac{1}{2}\right)4^n}{2}+\frac{(-1)^n 2^{2n+2}\Gamma\left(n+\frac{1}{2}\right)^2 n}{2\sqrt{\pi}}-\frac{(-1)^n 4^n\Gamma\left(n+\frac{1}{2}\right)^2}{2\sqrt{\pi}}\right)\Big/\left(\Gamma\left(n+\frac{1}{2}\right)\Gamma(n+1)\,(4n-1)\right)$

$\sum_{k=0}^{n} (-1)^k \binom{2n}{k} \binom{2n-k}{n}^2 \frac{2n+1}{2n+1+k}$ $\frac{\binom{2n}{n}\binom{4n+1}{n}}{\binom{3n+1}{n}}$	$\sum_{k=0}^{n} (-1)^k \binom{2n}{k} \binom{2n-k}{n}^2 \frac{2n+1}{2n+1+k}$ $\left(64\,\Gamma\left(2n+\frac{3}{2}\right)\Gamma\left(n+\frac{3}{2}\right)^3 1024^n\,729^{-n}\right) \Big/ \left(27\,(2n+1)^2\,\Gamma\left(n+\frac{4}{3}\right)^2\Gamma\left(n+\frac{2}{3}\right)^2\Gamma(n+1)\right)$
$\sum_{k=0}^{n} \binom{4n+1}{2n-2k} \binom{n+k}{k}$ $4^n \binom{3n}{n}$	$\sum_{k=0}^{n} \binom{4n+1}{2n-2k} \binom{n+k}{k}$ $\frac{\sqrt{3}\,\Gamma\left(n+\frac{1}{3}\right)\Gamma\left(n+\frac{2}{3}\right)108^n}{2\pi\,\Gamma(2n+1)}$
$\sum_{k=0}^{n} \binom{n}{k} \binom{x}{r+k}$ $\binom{n+x}{n+r}$	$\sum_{k=0}^{n} \binom{n}{k} \binom{x}{r+k}$ $\frac{\binom{x}{r}\Gamma(r+1)\,\Gamma(n+1+x)}{\Gamma(x+1)\,\Gamma(n+1+r)}$
$\sum_{k=0}^{n} (-1)^k \binom{x}{n-k} \binom{n+k}{k} \frac{n-k}{(n+k)\cdot 2^{n+k}}$ $\binom{\frac{x}{2}}{n}$	$\sum_{k=0}^{n} (-1)^k \binom{x}{n-k} \binom{n+k}{k} \frac{n-k}{(n+k)\cdot 2^{n+k}}$ $-\frac{x\,(-1)^n\,\Gamma\left(n-\frac{x}{2}\right)}{2\,\Gamma\left(1-\frac{x}{2}\right)\Gamma(n+1)}$

$\sum_{k=0}^{2n} \frac{(-1)^k \binom{2n}{k} \binom{2k}{k}}{\binom{n+k}{k} 2^{2k}}$ $\frac{\binom{6n}{3n}}{16^n \binom{2n}{n}}$	$\sum_{k=0}^{2n} \frac{(-1)^k \binom{2n}{k} \binom{2k}{k}}{\binom{n+k}{k} 2^{2k}}$ $\frac{\sqrt{3}\,\Gamma\left(n+\frac{1}{6}\right)\Gamma\left(n+\frac{5}{6}\right)}{3\,\Gamma\left(n+\frac{1}{3}\right)\Gamma\left(n+\frac{2}{3}\right)}$

Symbolic Integration

The int command now takes into account equality assumptions:

$$\int_0^{\mathrm{Pi}} \cos(n\,x)\,\mathrm{d}x \text{ assuming } n=0$$

$$\pi$$

$$\int x^n\,\mathrm{d}x \text{ assuming } n=-1;$$

$$\ln(x)$$

Some improvements were made in the Risch algorithm:

$$\int \sqrt{\frac{1}{u\,(a-u)}}\,\left(8\,u^2+6\,u+3\right)\mathrm{d}u \text{ assuming } 0<u;$$

$$-\frac{1}{\sqrt{-u\,(a-u)}}\left(\sqrt{\frac{1}{u\,(a-u)}}\,(a-u)\,u\,\left(3\,a^2\ln\left(-\frac{a}{2}+u+\sqrt{-a\,u+u^2}\right)+6\sqrt{-a\,u+u^2}\,a+4\sqrt{-a\,u+u^2}\,u+3\,a\ln\left(-\frac{a}{2}+u+\sqrt{-a\,u+u^2}\right)+6\sqrt{-a\,u+u^2}+3\ln\left(-\frac{a}{2}+u+\sqrt{-a\,u+u^2}\right)\right)\right)$$

$$\int_0^{\frac{1}{2}} \frac{2\sin(u)+2}{\sqrt{\cos(u)-u}}\,\mathrm{d}u$$

$$4-2\sqrt{-2+4\cos\left(\frac{1}{2}\right)}$$

$$\int_0^1 \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{1-\cos(z)}}\,dz$$

$$-\frac{1}{3}\,\frac{1}{(e^I+1)\sqrt{-(e^I-1)^2 e^{-I}}}\Big(6\,I\,e^I\ln(-e^I+1)\sqrt{(e^I+1)^2 e^{-I}}+12\,I\,e^I\,\mathrm{polylog}(3,e^I)\sqrt{(e^I+1)^2 e^{-I}}-6\,I\ln(-e^I+1)\sqrt{(e^I+1)^2 e^{-I}}-12\,I\,\mathrm{polylog}(3,e^I)\sqrt{(e^I+1)^2 e^{-I}}+12\,\zeta(3)\sqrt{-(e^I-1)^2 e^{-I}}\,e^I+12\,e^I\,\mathrm{polylog}(2,e^I)\sqrt{(e^I+1)^2 e^{-I}}+e^I\sqrt{(e^I+1)^2 e^{-I}}+12\,\zeta(3)\sqrt{-(e^I-1)^2 e^{-I}}-12\,\mathrm{polylog}(2,e^I)\sqrt{(e^I+1)^2 e^{-I}}-\sqrt{(e^I+1)^2 e^{-I}}\Big)$$

$$\int -\frac{x^5 (x^2)^{1/4}\sqrt{x^4}}{\sqrt{-x^2+1}\,(7x^2-5)}\,dx$$

$$-\frac{2}{11319}\,\frac{(147x^4+354x^2+865)(x^2-1)(x^2)^{1/4}\sqrt{x^4}}{x^2\sqrt{-x^2+1}}-\frac{1}{x^3\sqrt{-x^2+1}}\left(\left(\frac{10180}{79233}\,\frac{\sqrt{x+1}\sqrt{-2x+2}\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{x+1},\frac{1}{2}\sqrt{2}\right)}{\sqrt{x^3-x}}+\frac{125}{33614}\,\frac{\sqrt{35}\sqrt{x+1}\sqrt{-2x+2}\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{x+1},-\frac{1}{-1-\frac{1}{7}\sqrt{35}},\frac{1}{2}\sqrt{2}\right)}{\sqrt{x^3-x}\left(-1-\frac{1}{7}\sqrt{35}\right)}-\frac{125}{33614}\,\frac{\sqrt{35}\sqrt{x+1}\sqrt{-2x+2}\sqrt{-x}\,\mathrm{EllipticPi}\left(\sqrt{x+1},-\frac{1}{-1+\frac{1}{7}\sqrt{35}},\frac{1}{2}\sqrt{2}\right)}{\sqrt{x^3-x}\left(-1+\frac{1}{7}\sqrt{35}\right)}\right)(x^2)^{1/4}\sqrt{x^4}\sqrt{(x^2-1)\,x}\right)$$

$$\int \sqrt{x+|x+2|^2}\,dx$$

$$\frac{1}{4}(2x+5)\sqrt{x^2+5x+4}-\frac{9}{8}\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+4}\right)$$

Some improvements in definite integration:

$$\int_{\frac{\pi}{2}}^{\ln(3) I} \csc(x) \, dx$$

$$\frac{1}{2} I \pi - \ln(2)$$

Integration of symbolic powers of hyperbolic functions can now be handled:

$$\int \cosh(x)^n \, dx \text{ assuming } posint;$$

$$\left(\sum_{i=0}^{\left\lceil \frac{n}{2} \right\rceil - 1} \frac{\left(\prod_{j=1}^{i} \left(1 + \frac{1}{n - 2j}\right) \right) \cosh(x)^{n-2i-1}}{n} \right) \sinh(x) + \left(\prod_{j=0}^{\left\lceil \frac{n}{2} \right\rceil - 1} \left(1 - \frac{1}{n-2j}\right) \right) x$$

Improved simplification of integrals:

$$simplify\left(\int_a^b \mathrm{Im}(x) \, dx \right) \text{ assuming } a = 0, b = 2;$$

$$0$$

$$simplify\left(\int_1^2 \mathrm{Im}(x) \, dx \right)$$

$$0$$

Better error handling:

$$IntegrationTools{:}\text{-}Change\left(\int_a^b f(x) \, dx, x = -x \right)$$

Error, (in IntegrationTools:-Change) no new integration variable found in transformation equation

Improvements in **inttrans**:

$$inttrans[fouriersin]\left(\ln\left(\left| \frac{a^2 + x^2}{b^2 - x^2} \right| \right), x, y \right) \text{ assuming } a > 0, b > 0;$$

$$\frac{\sqrt{2} \left(2 \ln\left(\frac{a}{b}\right) - e^{-a y} \mathrm{Ei}(a y) - e^{a y} \mathrm{Ei}(-a y) + 2 \cos(b y) \mathrm{Ci}(b y) + 2 \sin(b y) \mathrm{Si}(b y) \right)}{\sqrt{\pi} \, y}$$

NumberTheory

With the latest additions in Maple 2017, the NumberTheory package now completely replaces the deprecated **numtheory** package. It is recommended that you use the **NumberTheory** package instead of **numtheory**.

The new commands are ChineseRemainder, IthFermat, Radical and SimplestRational.

$with(NumberTheory):$

$FermatDF := (n) \rightarrow DataFrame(\langle \langle seq(2\hat{}(\text{`}2\text{`}\hat{}i) + 1, i \textbf{ in } n) \rangle | \langle seq(length(IthFermat(i)), i \textbf{ in } n) \rangle | \langle seq(IthFermat(i, output = prime), i \textbf{ in } n) \rangle | \langle seq(IthFermat(i, output = completelyfactored), i \textbf{ in } n) \rangle \rangle, rows = [op(n)], columns = [F_n, \text{`\# of digits`}, \text{`Prime`}, \text{`Completely Factored`}]) :$

The following displays a data frame containing information on the first 8 Fermat numbers.

$FermatDF([seq(0..7)]);$

	F_n	# of digits	Prime	Completely Factored
0	3	1	true	true
1	2^2+1	1	true	true
2	$2^{2^2}+1$	2	true	true
3	$2^{2^3}+1$	3	true	true
4	$2^{2^4}+1$	5	true	true
5	$2^{2^5}+1$	10	false	true
6	$2^{2^6}+1$	20	false	true
7	$2^{2^7}+1$	39	false	true

assume, is, coulditbe

Better handling of multiple or complicated assumptions:

$is(b \neq 0)$ assuming $a \neq 0, a = b^2$

$$true$$

$$c1 := a0 :: \left(RealRange\left(Open(0), Open\left(\frac{1}{4}\right)\right)\right), x0 :: real, x1 :: (RealRange(Open(0), Open(1))), \left(\frac{1 - a0 - x1}{1 - x1}\right) :: real, (-a0 - x1) :: (RealRange(Open(-1), \infty)), (a0 + x1) :: (RealRange(-\infty, Open(1))), \left(\frac{1 - a0 - x1}{1 - x1} - x0\right) :: (RealRange(Open(0), \infty)), (1 - a0$$

$-x1) :: real, (x0 + a0 + x1) :: (RealRange(Open(1), \infty)):$

$\mathrm{Re}(x0)$ assuming $c1$

$$x0$$

$is(x0, imaginary)$ assuming $c1$

$$false$$

$c2 := a0 :: \left(RealRange\left(Open(0), Open\left(\frac{1}{4}\right)\right)\right), x0 :: real, x1 :: (RealRange(Open(0), \infty)),$
$\left(\frac{1 - a0 - x1}{1 - x1}\right) :: real, (1 - a0) :: real, (a0 + x1) :: (RealRange(-\infty, Open(1))),$
$\left(\frac{1 - a0 - x1}{1 - x1} - x0\right) :: (RealRange(Open(0), \infty)), (1 - a0 - x1) :: real, (x0 + a0 + x1)$
$:: (RealRange(Open(1), \infty)):$

$is(x1 \neq 1)$ assuming $c2$

$$true$$

$c3 := \mathrm{And}(a1 :: \mathrm{Not}(nonposint),$
$a2 :: \mathrm{Not}(nonposint),$
$b1 :: \mathrm{Not}(nonposint),$
$b2 :: \mathrm{Not}(nonposint),$
$c :: \mathrm{Not}(nonposint),$
$(b1 - a1) :: \mathrm{Not}(nonposint),$
$(a1 - b1) :: \mathrm{Not}(nonposint),$
$(b2 - a2) :: \mathrm{Not}(nonposint),$
$(a2 - b2) :: \mathrm{Not}(nonposint),$
$(c - (a1 + a2)) :: \mathrm{Not}(nonposint),$
$(c - (b1 + a2)) :: \mathrm{Not}(nonposint),$
$(c - (a1 + b2)) :: \mathrm{Not}(nonposint),$
$(c - (b1 + b2)) :: \mathrm{Not}(nonposint)):$

$is(c3)$ assuming $c3$

$$true$$

Improved handling of SetOf:

$is(\{1, 2\}$ **subset** $SetOf(\mathbb{Z}))$

$$true$$

Functional properties added:

$is(\ln(|p|) < 0)$ assuming $|p| < 1$

$$true$$

Logic added to **is** to take into account functional properties during simplification:

$is\left(|x|^2 < 1\right)$ assuming $|x| < 1$

$$true$$

$is(|x| < 1)$ assuming $|x|^2 < 1$

$$true$$

abs factors out positive values:

$is\left(\left|\frac{x+5}{3}\right| < 1\right)$ assuming $|x+5| < 3$

$$true$$

Simplification of properties:

$`property/+`(LinearProp(4,\mathbb{Z},0), LinearProp(-6,\mathbb{Z},0))$

$$LinearProp(2, integer, 0)$$

$`property/+`\left(LinearProp\left(\frac{2}{3}, \mathbb{Z}, 0\right), RealRange\left(0, Open\left(\frac{2}{3}\right)\right)\right)$

$$real$$

Improved handling of specified RootOfs:

$c5 := t2 > 0, t2 < 2\,\pi:$

$r1, r2, r3, r4 := RootOf(2\,z^2 + 2\,z - 1, index=1), RootOf(2\,z^2 + 2\,z - 1, index=2), RootOf(2\,z^2 - 2\,z - 1, index=1), RootOf(2\,z^2 - 2\,z - 1, index=2):$

$is\left(0 < (r2-r1)\cdot\left(\tan\left(\frac{\theta}{2}\right) - r3\right) - (r3-r4)\cdot r3\right)$

$$false$$

More care is taken with unspecified RootOfs. **is** now only returns true if the condition is true for all possible values of the RootOf. **coulditbe** returns true if any possible value of the RootOf returns true:

$is(RootOf(\_Z^3 + \_Z), real)$

$$false$$

$is(I \cdot RootOf(\_Z^2 - \_Z), real)$

$$false$$

$is(RootOf(\_Z^3 + \_Z), imaginary)$

$$true$$

$is(I \cdot RootOf(\_Z^2 - \_Z), imaginary)$

$$true$$

$coulditbe(RootOf(\_Z^3 + \_Z), Non(real))$

$$true$$

Improvements when processing **Non** properties:

$(AndProp, OrProp)(2, Non(0))$

$$2, Non(0)$$

$is(\nu < 3)$ assuming $\nu \le 3, \nu :: \mathrm{Not}(\mathbb{Z})$

$$true$$

Improvements made to solving linear inequalities:

$is\left(j + \frac{n}{Y_0(4)} < 0\right)$ assuming $1 \le j, j \le n$

$$true$$

$is\left(\frac{2L}{1 - x - y} < 1\right)$ assuming $L = \frac{1}{2}, x > 0, y > 0, x + y < L$

$$false$$

Extra knowledge added regarding odd, even:

$is(x, posint)$ assuming $x :: odd, x > 0$

$$true$$

$is(x, posint)$ assuming $x :: even, x > 0$

$$true$$

Improvements regarding properties **GaussianInteger** and **prime**:

$is(n < 3)$ assuming $n :: prime, n :: even$

$$true$$

$is(5\,p, prime)$ assuming $p :: prime$

$$false$$

$c4 := a$ **in** $SetOf(prime)$ **minus** $\{2\}$:

$is(a + 1, prime)$ assuming $c4$

$$false$$

$is(a + 1, even)$ assuming $c4$

$$true$$

$coulditbe(\ x = 7\ )$ assuming $x :: prime, x > 5$

$$true$$

Better treatment of conditions with infinity and floats:

$is(\,(.75 - t)^3 = \infty\,)$ assuming $t :: RealRange(Open(0), Open(.25))$

$$false$$

More simplification of radicals inside **is**:

$is\left(-4^{\frac{1}{2}}, \mathbb{Z}\right)$

$$true$$

$is\left(3 \cdot 4^{\frac{1}{2}}, \mathbb{Z}\right)$

$$true$$

Improved handling of assumptions involving Re and Im:

$is(a :: real)$ assuming $\Im(a) = 0$

$$true$$

$coulditbe(a :: real)$ assuming $\Im(a) > 0$

$$false$$

$\Im(b)$ assuming $b :: imaginary, \Im(b) > -1$

$$-\mathrm{I}\, b$$

$\Im(x)$ assuming $\Im(x) = 0, x :: real$

$$0$$

Improved normalization for LinearProp:

$LinearProp\left(-1, Non(\mathbb{Z}), \frac{5}{2}\right)$

$$LinearProp\left(1, Non(integer), \frac{1}{2}\right)$$

Improved handling of **OrProp** and **Or** conditions:

$is(x \leq -1 \textbf{ or } x \geq 1)$ assuming $x :: OrProp(RealRange(-\infty, -1), RealRange(1, \infty))$

$$true$$

$is(y \leq 0)$ assuming $\mathrm{Or}(\mathrm{Or}(y < 0, y = 0), y > 0)$

$$false$$

$is(\mathrm{Or}(x > 0, y > 0))$

$$false$$

$is(\mathrm{Or}(b :: \mathrm{Not}(nonposint), b < -1))$

$$false$$

$is(\mathrm{Or}(b :: \mathrm{Not}(nonposint), b < 1))$

$$true$$

$is(\mathrm{Or}(b \leq c, c \leq a, a \leq b))$ assuming $real$

$$true$$

More care is taken about assuming variables are real if they are involved in an inequality assumption. The assumption must be an affirmative one:

$is(a, real)$ assuming $\mathrm{Not}(a > b)$

$$false$$

Improved handling of nonstrict inequalities involving infinity:

$is(x + \infty \leq x + \infty)$ assuming $x :: real$

$$true$$

It is now recognized that a and b may not be real in this example:

$is(a > b)$ assuming $(a-b) :: RealRange(Open(0), \infty)$

$$false$$

Improved simplification of piecewise using assumptions:

$piecewise(w < 0, undefined, 0)$ assuming $\mathrm{Not}(w < 0);$

$$0$$

Improved handling of multiple periodic assumptions:

$c6 := n :: \mathbb{Z}, \left(\frac{3\,n}{2}\right) :: posint:$

$is(n, even)$ assuming $c6$

$$true$$

$is(n > 1)$ assuming $c6$

$$true$$

Improvements in **signum** under assumptions:

$\mathrm{signum}\left(\frac{1-a-x2}{1-x2} - 1 + a + x2\right)$ assuming $a > 0, a < 1/4,\ 0 < x2, a + x2 < 1$

$$1$$

Other miscellaneous improvements:

$coulditbe(2\,\pi\,I\,z, posint)$ assuming $z :: \mathbb{Z}$

$$false$$

$is\left(\frac{c}{x} + I, real\right)$ assuming $real$

$$false$$

$is(z = 1)$ assuming $z :: posint$

$$false$$

$coulditbe(x = 0)$ assuming $x :: nonnegint$

$$true$$

Logic

The new Parity command returns the Boolean expression corresponding to the parity function on a set of variables: that is, the function which is true if and only if an odd number of inputs are true.

$with(Logic):$

$Parity(\ true, false, true\ )$

$$false$$

$Parity(\ x \textbf{ xor } y, y \textbf{ xor } z, true\ )$

$$\neg Parity(x, z)$$

$Parity(\ true \$ 2\,n - 1\ )$ assuming $n :: posint$

$$true$$

Other Improvements

- The content command now computes the content of all multivariate polynomials with numeric coefficients.

$content(2.0\,u\,x + 3.5\,u\,v, x)$

$$u$$

- Some extra simplifications of expressions involving LambertW functions have been added:

$LW1 := \mathrm{LambertW}\left(-\frac{72}{13}(\ln(2) + \ln(3))\right):$

$simplify\left(LW1 - \ln\left(-\frac{1}{LW1}\right), \mathrm{LambertW}\right);$

$$\ln\left(\frac{72\ln(2)}{13} + \frac{72\ln(3)}{13}\right)$$

$LW2 := \mathrm{LambertW}\left(-\frac{72}{13}(\ln(2) - \ln(3))\right):$

$simplify\left(LW2 - \ln\left(-\frac{1}{LW2}\right), \mathrm{LambertW}\right);$

$$-2\,\mathrm{I}\,\pi + \ln\left(\frac{72\ln(2)}{13} - \frac{72\ln(3)}{13}\right)$$

- rationalize is more careful about not multiplying and dividing by 0:

$a := \sqrt{687} : b := \left(\frac{3}{256} + \frac{a}{768}\cdot I\right)^{\frac{1}{6}} : c := (9 - a\cdot I)^{\frac{1}{6}} : d := \sqrt{3} : e := 768^{\frac{1}{6}} :$

$z := 1 \Big/ \Big(-6\,\mathrm{I}\,a\,d\,c\,e^5\,b^5 + 54\,d\,c\,e^5\,b^5 + 6\,\mathrm{I}\,a\,d\,c^5\,e\,b + 54\,d\,c^5\,e\,b$
$\quad + 96\,d\sqrt{3\,\mathrm{I}\,a\,e^2\,b^2\,c^4 + 27\,c^4\,e^2\,b^2 + 4608 - 3\,\mathrm{I}\,a\,e^4\,b^4\,c^2 + 27\,c^2\,e^4\,b^4}\Big) :$

$rationalize(z);$

$$\frac{1}{23887872}\,12^{2/3}\left(\mathrm{I}\,12^{2/3}\left(9 - \mathrm{I}\sqrt{229}\sqrt{3}\right)^{1/3}\sqrt{3}\sqrt{229} + 9\,12^{2/3}\left(9 - \mathrm{I}\sqrt{229}\sqrt{3}\right)^{1/3} + 48\left(9 - \mathrm{I}\sqrt{229}\sqrt{3}\right)^{2/3} - 192\,12^{1/3}\right)$$

- Efficiency has been improved for radnormal for certain examples:

$$a := 108 + 12\cdot\sqrt{687}\cdot I : b := a^{\frac{1}{3}} : c := \frac{b^2+48}{b} : d := c^{\frac{1}{4}} : e := 6^{\frac{1}{2}} : f := \sqrt{-\frac{6\cdot\left(b^2 d^2 + 12\, e\cdot b + 48\, d^2\right)}{b}} : g := \frac{e\cdot d^2 + \frac{f}{d}}{12} :$$

$$x1 := g^4 + g + 1 :$$

$$t := time(\ ) : radnormal(x1);\ time(\ ) - t;$$

$$0$$

$$0.047$$