Step-by-Step Solutions

Simplification Steps

Maple 2022 includes new commands for showing the steps needed to manipulate algebraic expressions in order to reduce them to their simplest form.

In general the generated steps try to find that hard balance between being too verbose and too cryptic. The SimplifySteps command errs on the side of adding more steps, and is aimed to help someone who wants to learn what the steps are, even for fundamentals like adding fractions. Depending on the problem it will adjust somewhat; recognizing higher level problems for which it will decide to skip more easy-level steps.

Here are some examples of different categories of problems that SimplifySteps as well as related command FractionSteps can handle:

$with(Student{:-}Basics):$

Fractions

There is a dedicated FractionSteps command that goes into slightly more detail than SimplifySteps

$FractionSteps(\text{"1/2 + 1/6"})$

Let's Simplify Fractions

$$\frac{1}{2}+\frac{1}{6}$$

- Find fractions to get lowest common denominator of 6

$$\frac{3}{3}\cdot\left(\frac{1}{2}\right)+\frac{1}{1}\cdot\left(\frac{1}{6}\right)$$

- Multiply

$$\frac{3\cdot 1}{6}+\frac{1\cdot 1}{6}$$

- Add numerators

$$\frac{(3\cdot 1+1\cdot 1)}{6}$$

- Multiply $3\cdot 1$

$$\frac{3+1\cdot 1}{6}$$

- Multiply $1\cdot 1$

$$\frac{3+1}{6}$$

- Add $3+1$

$$\frac{4}{6}$$

- Cancel out factor of 2

$$\frac{2}{3}$$

SimplifySteps("1/2 + 1/6")

Let's simplify

$$\frac{1}{2}+\frac{1}{6}$$

- Find fractions to get lowest common denominator of 6

$$\frac{3}{3}\cdot\left(\frac{1}{2}\right)+\frac{1}{1}\cdot\left(\frac{1}{6}\right)$$

- Multiply

$$\frac{3\cdot 1}{6}+\frac{1\cdot 1}{6}$$

- Add fractions

$$\frac{2}{3}$$

Radicals

SimplifySteps("sqrt(6)*sqrt(10)/sqrt(12)")

Let's simplify

$$\frac{\sqrt{6}\cdot\sqrt{10}}{\sqrt{12}}$$

- Pull out a factor of $\sqrt{4}=2$ from $\sqrt{12}$

$$\frac{\sqrt{6}\,\sqrt{10}}{\left(2\sqrt{3}\right)}$$

- Multiply in order to rationalize the denominator

$$\frac{\sqrt{3}}{\sqrt{3}}\cdot\left(\frac{\sqrt{6}\,\sqrt{10}}{2\sqrt{3}}\right)$$

- Multiply the denominator

$$\frac{\sqrt{3}\cdot\left(\sqrt{6}\,\sqrt{10}\right)}{6}$$

- Factor roots

$$\frac{\sqrt{3}\cdot\left(\sqrt{2}\cdot\sqrt{3}\cdot\left(\sqrt{2}\cdot\sqrt{5}\right)\right)}{6}$$

- Combine $\left\{\sqrt{2}\cdot\sqrt{2},\sqrt{3}\cdot\sqrt{3}\right\}$

$$\frac{3\cdot\left(2\cdot\sqrt{5}\right)}{6}$$

- Multiply $3\cdot\left(2\sqrt{5}\right)$

$$\frac{\left(6\sqrt{5}\right)}{6}$$

- Cancel out factor of 6

$$\sqrt{5}$$

SimplifySteps("sqrt(6)*sqrt(10)/(12)^(1/3)")

Let's simplify

$$\frac{\sqrt{6}\cdot\sqrt{10}}{12^{\frac{1}{3}}}$$

- Multiply in order to rationalize the denominator

$$\frac{12^{2/3}}{12^{2/3}}\cdot\left(\frac{\sqrt{6}\sqrt{10}}{12^{1/3}}\right)$$

- Multiply the denominator

$$\frac{12^{2/3}\cdot\left(\sqrt{6}\sqrt{10}\right)}{12}$$

- Factor roots

$$\frac{3^{2/3}\cdot\left(2\,2^{1/3}\right)\cdot\left(\sqrt{2}\cdot\sqrt{3}\cdot\left(\sqrt{2}\cdot\sqrt{5}\right)\right)}{12}$$

- Combine $\left\{3^{2/3}\cdot\sqrt{3},2^{1/3}\cdot\sqrt{2}\cdot\sqrt{2}\right\}$

$$\frac{2\cdot\left(\left(3\,3^{1/6}\right)\cdot\left(\left(2\,2^{1/3}\right)\cdot\sqrt{5}\right)\right)}{12}$$

- Multiply $3\,3^{1/6}\cdot\left(2\,2^{1/3}\sqrt{5}\right)$

$$\frac{2\cdot\left(6\sqrt{5}\,2^{1/3}\,3^{1/6}\right)}{12}$$

- Multiply $2\cdot\left(6\sqrt{5}\,2^{1/3}\,3^{1/6}\right)$

$$\frac{\left(12\sqrt{5}\,2^{1/3}\,3^{1/6}\right)}{12}$$

- Cancel out factor of 12

$$\sqrt{5}\,2^{1/3}\,3^{1/6}$$

Exponents

SimplifySteps("(2*x^3*y^3)*(3*x^1*y^2)^2")

Let's simplify

$2 \cdot x^3 \cdot y^3 \cdot \left(3 \cdot x \cdot y^2\right)^2$

- Evaluate exponent $\left(3\,x\,y^2\right)^2$

  $2 \cdot x^3 \cdot y^3 \cdot \left(9\,x^2\,y^4\right)$

- Multiply $2 \cdot x^3 \cdot y^3 \cdot \left(9\,x^2\,y^4\right)$

  $\left(18\,x^5\,y^7\right)$

SimplifySteps("x^3*x^5")

Let's simplify

$x^3 \cdot x^5$

- Apply the product rule $a^n\,a^m = a^{n+m}$ to add exponents with common base

  x^{3+5}

- Add exponents

  x^8

- Solution

  x^8

SimplifySteps("(b^n)^m")

Let's simplify

$\left(b^n\right)^m$

- Apply the integer power of a power rule, $\left(a^n\right)^m = a^{n\,m}$

  $b^{n \cdot m}$

SimplifySteps("y^5/y^4")

Let's simplify

$\frac{y^5}{y^4}$

- Cancel out factor of y^4 provided $y^4 \neq 0$

  y

SimplifySteps("y^(-5)/y^4")

Let's simplify

$$\frac{y^{-5}}{y^4}$$

- Divide assuming $y^4 \neq 0$

$$\frac{1}{y^9}$$

SimplifySteps("123^5/123^4")

Let's simplify

$$\frac{123^5}{123^4}$$

- Apply the quotient rule: $\frac{a^n}{a^m} = a^{n-m}$

$$123^1$$

- Evaluate exponent 123^1

$$123$$

Logs

SimplifySteps("log[10](2)/log[10](5)+log[10](3)")

Let's simplify

$$\frac{\log_{10}(2)}{\log_{10}(5)} + \log_{10}(3)$$

- Use the log rule, $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$ to express as a single logarithm

$$\left(\log_5(2)\right) + \log_{10}(3)$$

- Solution

$$\log_5(2) + \log_{10}(3)$$

Note: This is different than how Maple's simplify command treats expressions like this, always converting to ln:

$simplify\left(\frac{\log[10](2)}{\log[10](5)} + \log[10](3)\right)$

$$\frac{\ln(2)}{\ln(5)} + \frac{\ln(3)}{\ln(2) + \ln(5)}$$

$SimplifySteps(\ \%log[10](100) + \%log[10](1000)\ )$

Let's simplify

$\log_{10}(100) + \log_{10}(1000)$

- Evaluate $\log_{10}(100)$

$2 + \log_{10}(1000)$

- Evaluate $\log_{10}(1000)$

$2 + 3$

- Add $2 + 3$

5

$SimplifySteps(\text{"5*log[z](z^4)"})$

Let's simplify

$5 \cdot \log_z\left(z^4\right)$

- Apply the log rule $\log_a\left(m^n\right) = n \log_a(m)$

$5 \cdot \left(4 \log_z(z)\right)$

- Apply the log rule $\log_a(a) = 1$

$5 \cdot 4$

- Multiply $5 \cdot 4$

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You can also use the new PowerSteps command to get step by step results for problems with radicals, exponents, and logarithms.

Trig

$SimplifySteps(\ \text{"1+cot(x)^2"}\ )$

Let's simplify

$1 + \cot(x)^2$

- Apply Pythagoras trig identity, $\cot(x)^2 = \csc(x)^2 - 1$

$1 + \left(\csc(x)^2 - 1\right)$

- Apply Reciprocal Function trig identity, $\csc(x) = \frac{1}{\sin(x)}$

$$\left(\frac{1}{\sin(x)}\right)^2$$

- Evaluate

$$\frac{1}{\sin(x)^2}$$

$SimplifySteps("\sin(\mathrm{Pi} + \mathrm{Pi} + x)")$

Let's simplify

$$\sin(\pi + \pi + x)$$

- Add $\pi + \pi + x$

$$\sin((2\pi + x))$$

- Evaluate $\sin(2\pi + x)$

$$\sin(x)$$

$$SimplifySteps\left(\frac{\sec(x)^2 - 1}{\sec(x)^2}\right)$$

Let's simplify

$$\frac{\sec(x)^2 - 1}{\sec(x)^2}$$

- Apply Reciprocal Function trig identity, $\sec(x) = \frac{1}{\cos(x)}$

$$\frac{\sec(x)^2 - 1}{\left(\frac{1}{\cos(x)}\right)^2}$$

- Apply Pythagoras trig identity, $\sec(x)^2 = 1 + \tan(x)^2$

$$\left(\left(1 + \tan(x)^2\right) - 1\right)\cos(x)^2$$

- Apply Quotient trig identity, $\tan(x) = \frac{\sin(x)}{\cos(x)}$

$$\left(\frac{\sin(x)}{\cos(x)}\right)^2\cos(x)^2$$

- Evaluate

$$\sin(x)^2$$

$SimplifySteps\left(\left(-\cos(x)^2 + 1\right)\left(1 + \cot(x)^2\right)\right)$

Let's simplify

$$\left((-1)\cdot\cos(x)^2 + 1\right)\cdot\left(1 + \cot(x)^2\right)$$

- Apply Pythagoras trig identity, $\cot(x)^2 = \csc(x)^2 - 1$

$$\left(-\cos(x)^2 + 1\right)\left(1 + \left(\csc(x)^2 - 1\right)\right)$$

- Apply Reciprocal Function trig identity, $\csc(x) = \frac{1}{\sin(x)}$

  $$\left(-\cos(x)^2 + 1\right)\left(\frac{1}{\sin(x)}\right)^2$$

- Apply Pythagoras trig identity, $\cos(x)^2 = 1 - \sin(x)^2$

  $$\frac{-\left(1 - \sin(x)^2\right) + 1}{\sin(x)^2}$$

- Evaluate

  $$1$$

You can also use the new TrigSteps command to get these step by step results.

Calculus

Showing the steps to solving an integral, limit, or derivative has been available in past versions of Maple via the Student:-Calculus1:-ShowSolution command. You can now also access those step by step solutions through SimplifySteps, further unifying the ability to do step by step solutions using a single command.

$$SimplifySteps\left(x^2 + \int x \, dx\right)$$

Let's simplify

$$x^2 + \int x \, dx$$

- Integral to evaluate

  $$\int x \, dx$$

  1. Apply the **power** rule to the term $\int x \, dx$

  Recall the definition of the **power** rule, for n ≠ -1

  $$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

  This means:

  $$\int x \, dx = \frac{x^{1+1}}{1+1}$$

  So,

  $$\int x \, dx = \frac{x^2}{2}$$

We can rewrite the integral as:

$$\frac{x^2}{2}$$

- Sub evaluated integral back in expression

$$\frac{3x^2}{2}$$

Steps for Sketching a Curve

Maple 2022 includes a new command for showing the steps needed to sketch the graph of an expression by identifying the basic function and the transformations done to the function. Various kinds of expressions are handled, including trig, logs, and polynomials to pick just a few. Here are some examples:

$with(Student{:}\text{-}Basics):$

$$CurveSketchSteps\left(2\sin\left(3x+\frac{\pi}{3}\right)+1\right)$$

Let's plot $2\sin\left(3x+\frac{\pi}{3}\right)+1$

- Compared to the plot of $\sin(x)$, we have a vertical stretch by a factor of 2

$$2\cdot\sin\left(3\cdot\left(x+\frac{\pi}{9}\right)\right)+1$$

- Then, we have a horizontal compression by a factor of $\frac{1}{3}$

$$2\cdot\sin\left(3\cdot\left(x+\frac{\pi}{9}\right)\right)+1$$

- Then, we have a vertical shift of 1

$$2\cdot\sin\left(3\cdot\left(x+\frac{\pi}{9}\right)\right)+1$$

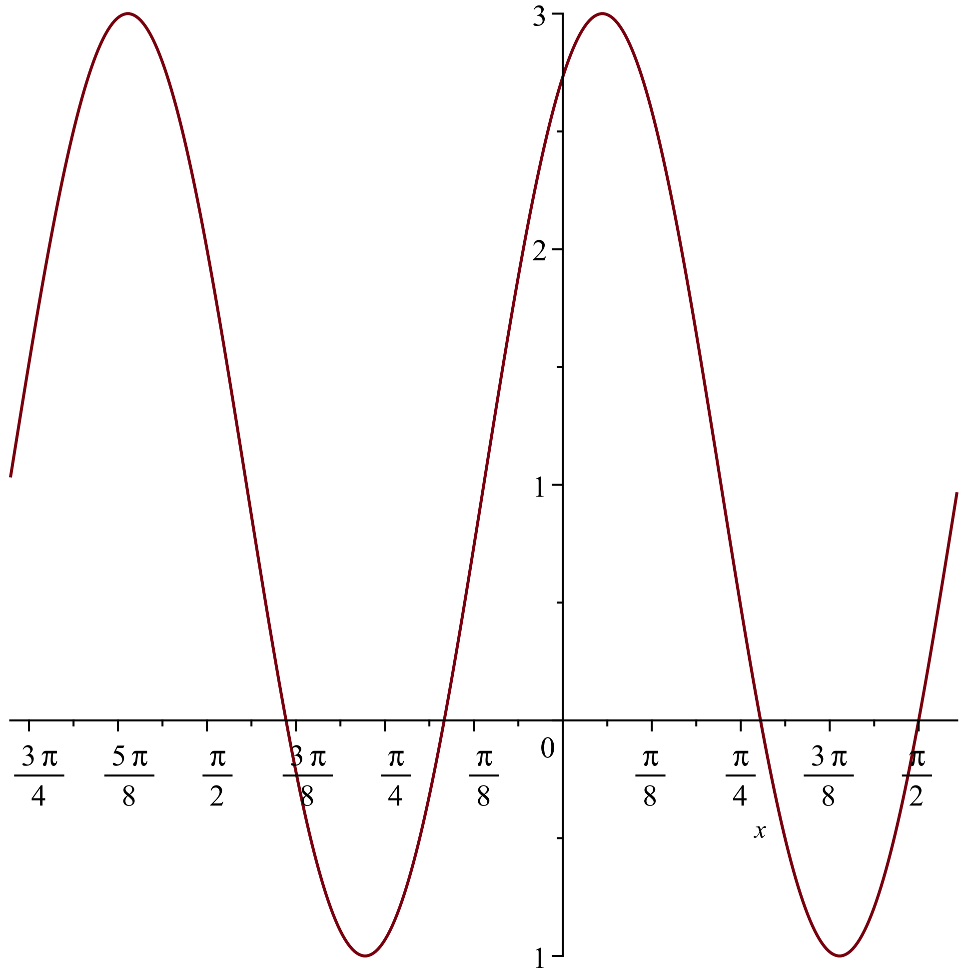
- Then, we have a horizontal shift of $-\frac{\pi}{9}$

$$2\cdot\sin\left(3\cdot\left(x+\frac{\pi}{9}\right)\right)+1$$

- Apply the horizontal shift and stretch to the range, $x=-2\pi..2\pi$

$$-2\pi\cdot\left(\frac{1}{3}\right)+\frac{\pi}{9}..2\pi\cdot\left(\frac{1}{3}\right)+\frac{\pi}{9}=-2.443460953..1.745329252$$

- We can now plot using the information extracted



CurveSketchSteps$\left(2x^2+4x+10\right)$

Let's plot $2x^2+4x+10$

$2\cdot x^2+4\cdot x+10$

- Complete the square

$2\,(x+1)^2+8$

- With the expression in vertex form we can extract valuable information

$2\cdot(x+1)^2+8$

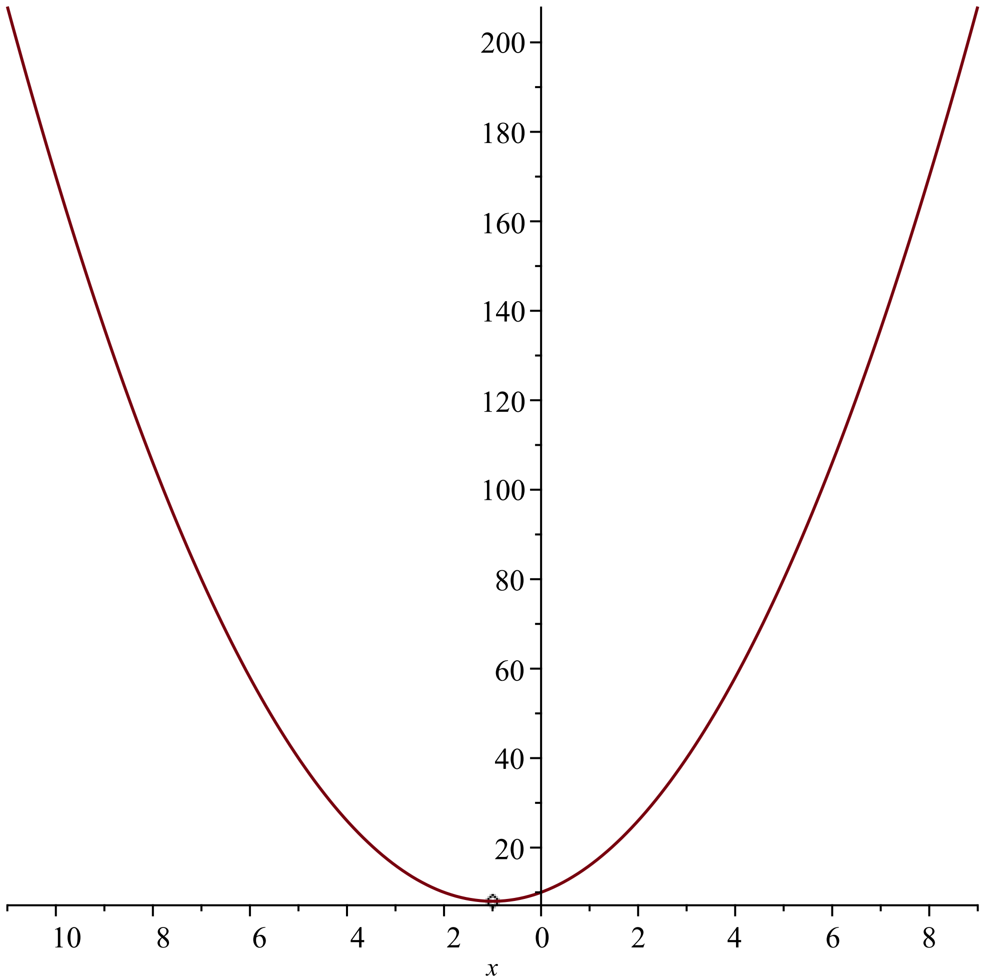
- The coefficient 2 of the $(x+1)^2$ term indicates a parabola that opens up and has a vertical stretch of 2

$2\cdot(x+1)^2+8$

- We have a horizontal shift of -1 and a vertical shift of 8 which gives a vertex of $(-1,8)$

$2\cdot(x+1)^2+8$

- We can now plot using the information extracted



CurveSketchSteps$(4\,x + 10,$ *output* = *typeset*)

Let's plot $4\,x + 10$

$4 \cdot x + 10$

- This is a line; find two points and draw a line through them

  $y = 4\,x + 10$

- Set $x = 0$ to solve for y intercept

  $y = 10$

- This gives a y intercept of (0,10)

  $y = 10$

- Set expresson to 0 to solve for x intercept

  $0 = 4 \cdot x + 10$

- Subtract $4 \cdot x$ from both sides

  $0 \quad 4 \cdot x = 4 \cdot x + 10 \quad 4 \cdot x$

- Simplify

  $(-4) \cdot x = 10$

- Divide both sides by -4

$$\frac{(-4) \cdot x}{-4} = \frac{10}{-4}$$

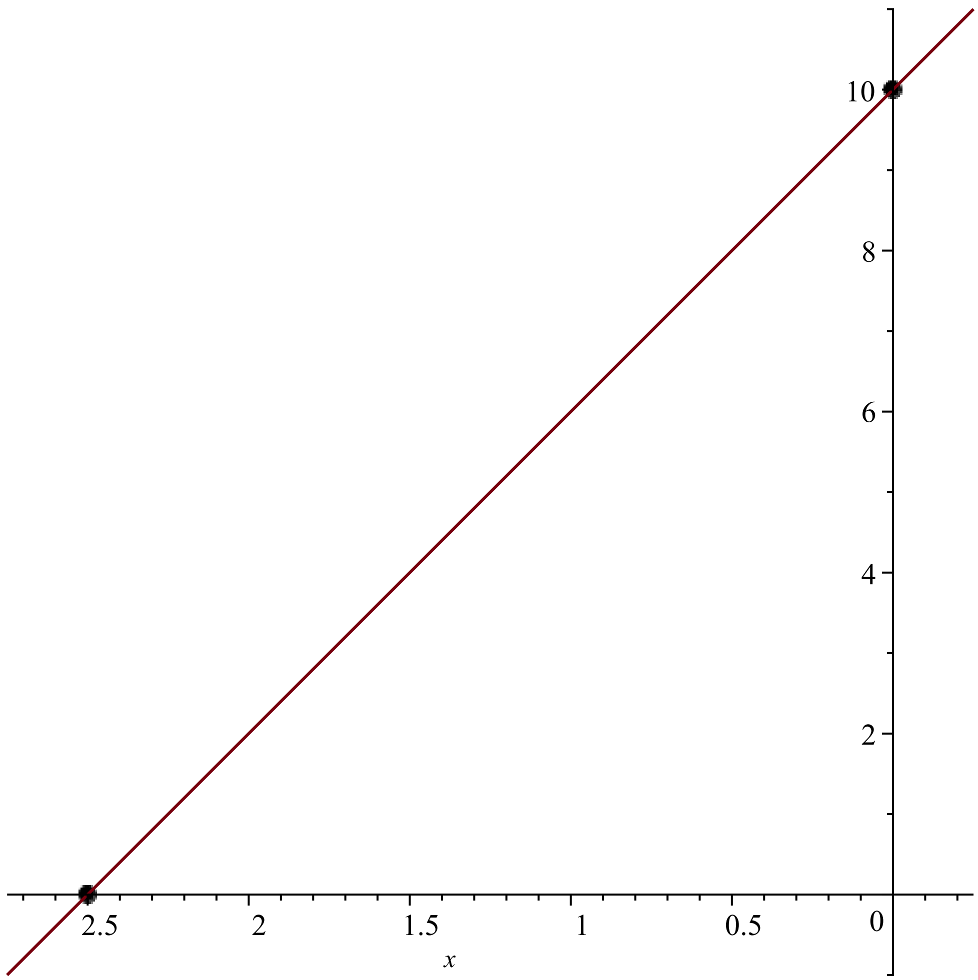
- Simplify

$$x = \frac{5}{2}$$

- This gives an x intercept of ($\frac{5}{2}$,0)

$$x = \frac{5}{2}$$

- By connecting through the two points we can plot the line



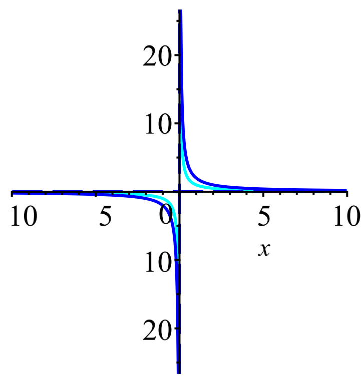
$CurveSketchStepsL\left(\frac{2}{3x+2}\right)$

Let's plot $\frac{2}{3x+2}$

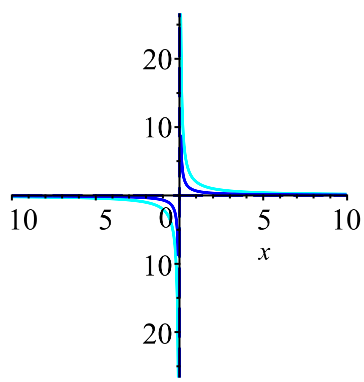
- Rewrite the equation in the following form

$$\frac{2}{3\cdot\left(x+\frac{2}{3}\right)}+0$$

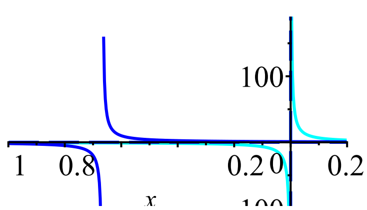
- Compared to the plot of $\frac{1}{x}$, we have a vertical stretch by a factor of 2

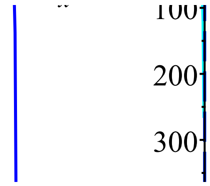


- Then, we have a horizontal compression by a factor of $\frac{1}{3}$

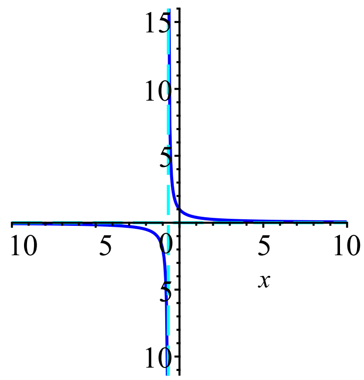


- Then, we have a horizontal shift of $\frac{2}{3}$





- The final plot with asymptotes in cyan at $y = 0$ and $x = \frac{2}{3}$ is



For more information, see the help page [Student:-Basics:-CurveSketchSteps].