Education

Maple 2023 includes a number of improvements to support teaching and learning of mathematics and science.

Step-by-Step Solutions

Maple 2023 improves the existing suite of commands for showing step-by-step solutions to standard math problems. It also adds some new methods as follows:

Implicit Differentiation Steps

The function f whose rule is given by $f(x) = x^2 + x + 1$, is said to be defined *explicitly*. The function $y(x)$ whose rule must be extracted from an equation of the form $F(x, y) = 0$ is said to be defined *implicitly*.

A simple example is the circle, defined by $x^2 + y^2 = 9$, where $y_{\pm}(x) = \pm\sqrt{9 - x^2}$ are two different *explicit* functions that can be extracted from the equation of the circle. The semicircle above the x-axis is defined by $y_{+}(x) = \sqrt{9 - x^2}$; and below, by $y_{-}(x) = -\sqrt{9 - x^2}$.

Implicit differentiation is a technique by which $y'(x)$ can be obtained without necessarily having to solve for $y(x)$ explicitly. It is merely the Chain rule applied to the *identity* $F(x, y(x)) = 0$.

Maple can show you the steps required to implicitly differentiate with the new command ImplicitDiffSolution.

$$Student:\text{-}Calculus1:\text{-}ImplicitDiffSolution\left(x^2 + y^2 = 9, y, x\right)$$

Implicit Differentiation Steps

$x^2 + y^2 = 9$

- Rewrite y as a function $y(x)$:

  $x^2 + y(x)^2 = 9$

- Differentiate the left side

  $$\frac{d}{dx}\left(x^2 + y(x)^2\right)$$

  - 1. Apply the **sum** rule
    - Recall the definition of the **sum** rule

      $$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

      $f(x) = x^2$

      $g(x) = y(x)^2$

      This gives:

      $$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y(x)^2\right)$$

  - 2. Apply the **power** rule to the term $\frac{d}{dx}\left(x^2\right)$
    - Recall the definition of the **power** rule

      $$\frac{\partial}{\partial x}\left(x^n\right) = n\, x^{n-1}$$

    - This means:

      $$\frac{d}{dx}\left(x^2\right) = 2 \cdot x^1$$

    - So,

      $$\frac{d}{dx}\left(x^2\right) = 2 \cdot x$$

      We can rewrite the derivative as:

      $$(2\,x) + \frac{d}{dx}\left(y(x)^2\right)$$

  - 3. Apply the **chain** rule to the term $y(x)^2$
    - Recall the definition of the **chain** rule

      $$\frac{d}{dx} f(g(x)) = f'(g(x)) \left(\frac{d}{dx} g(x)\right)$$

    - Outside function

      $f(x) = x^2$

Complete the Square Steps

Completing the square is a standard approach that takes a trinomial of degree 2 and rewrites it as a binomial made up of a perfect square plus a remainder. This is a useful method for getting a quadratic into a form that is easier to work with, and is often used as a first step in solving a quadratic equation.

There is a new command CompleteSquareSteps that shows the algebraic steps required to complete the square:

$$Student:\text{-}Basics:\text{-}CompleteSquareSteps(3\,x^2 + 2\,x + 1, x)$$

$$3 \cdot x^2 + 2 \cdot x + 1$$

- Add and subtract $\frac{1}{3} \cdot \left(\frac{2}{2}\right)^2$

$$3\,x^2 + 2\,x + \frac{1}{3} \cdot \left(\frac{2}{2}\right)^2 - \frac{1}{3} \cdot \left(\frac{2}{2}\right)^2 + 1$$

- Simplify terms

$$3\,x^2 + 2\,x + \frac{1}{3} - \frac{1}{3} + 1$$

- The first 3 terms can be regrouped as a perfect square

$$3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} + 1$$

- Simplify the remaining term

$$3\left(x + \frac{1}{3}\right)^2 + \frac{2}{3}$$

Long Division Result

Maple 2023 adds a new option to the LongDivision command that makes it clear how the inputs relate back to the computed result, especially when the remainder is not zero.

In the examples below, the division is carried out, and then, below the long division, it shows an answer derived from the long division that is equal to the dividend/divisor.

$$Student\text{:-}Basics\text{:-}LongDivision(\ 2, 3, digits = 3,\ 'appendresult' = true\ )$$

$$\begin{array}{rrl} & & \underline{.666} \\ 3 &) & 2.000 \\ & & \underline{18} \\ & & 20 \\ & & \underline{18} \\ & & 20 \\ & & \underline{18} \\ & & 2 \end{array}$$

$$= 0.666 + \frac{1}{1500}$$

$$Student\text{:-}Basics\text{:-}LongDivision\left(\ 3 \cdot x^2 + 2 \cdot x + 1, x + 3, 'appendresult' = true\ \right)$$

$$\begin{array}{rrl} & & \underline{3\,x \quad -7} \\ x + 3 &) & 3\,x^2 + 2\,x + 1 \\ & & \underline{3\,x^2 + 9\,x} \\ & & -7\,x + 1 \\ & & \underline{-7\,x - 21} \\ & & 22 \end{array}$$

$$= 3\,x - 7 + \frac{22}{x + 3}$$

New Physics Courseware Support: Mechanics

Maple 2023 now has Physics Courseware support for Mechanics. This new set of content is a helpful complement for a physics mechanics course. It contains typical symbolic problems and shows how they can be solved in a Maple worksheet, demonstrating how computer algebra can support the learning activity.

The material covers several key topics such as equations of motion, curvilinear coordinates, conservation laws, integration of the equations of motion, Kepler's problem, oscillations, rigid-body motion and non-inertial coordinate systems. It utilizes the Physics:-Vectors package to handle abstract vectors as well as projections using unit vectors.