Advanced Math

conjugate and RootOf

The conjugate command now has extended support for RootOf expressions. You can now find the conjugate of the following:

- Indexed RootOfs
- RootOf expressions with a numerical selector and real coefficients

The following examples return unevaluated in Maple 2015 and earlier.

> $conjugate\left(RootOf\left(x^5+x^2-1, index=2\right)\right)$

$$RootOf\left(\_Z^5+\_Z^2-1, index=5\right)$$

> $conjugate\left(RootOf\left(x^2+\sqrt{3}\,x+1, -1+I\right)\right)$

$$RootOf\left(\_Z^2+\sqrt{3}\,\_Z+1, -1-\mathrm{I}\right)$$

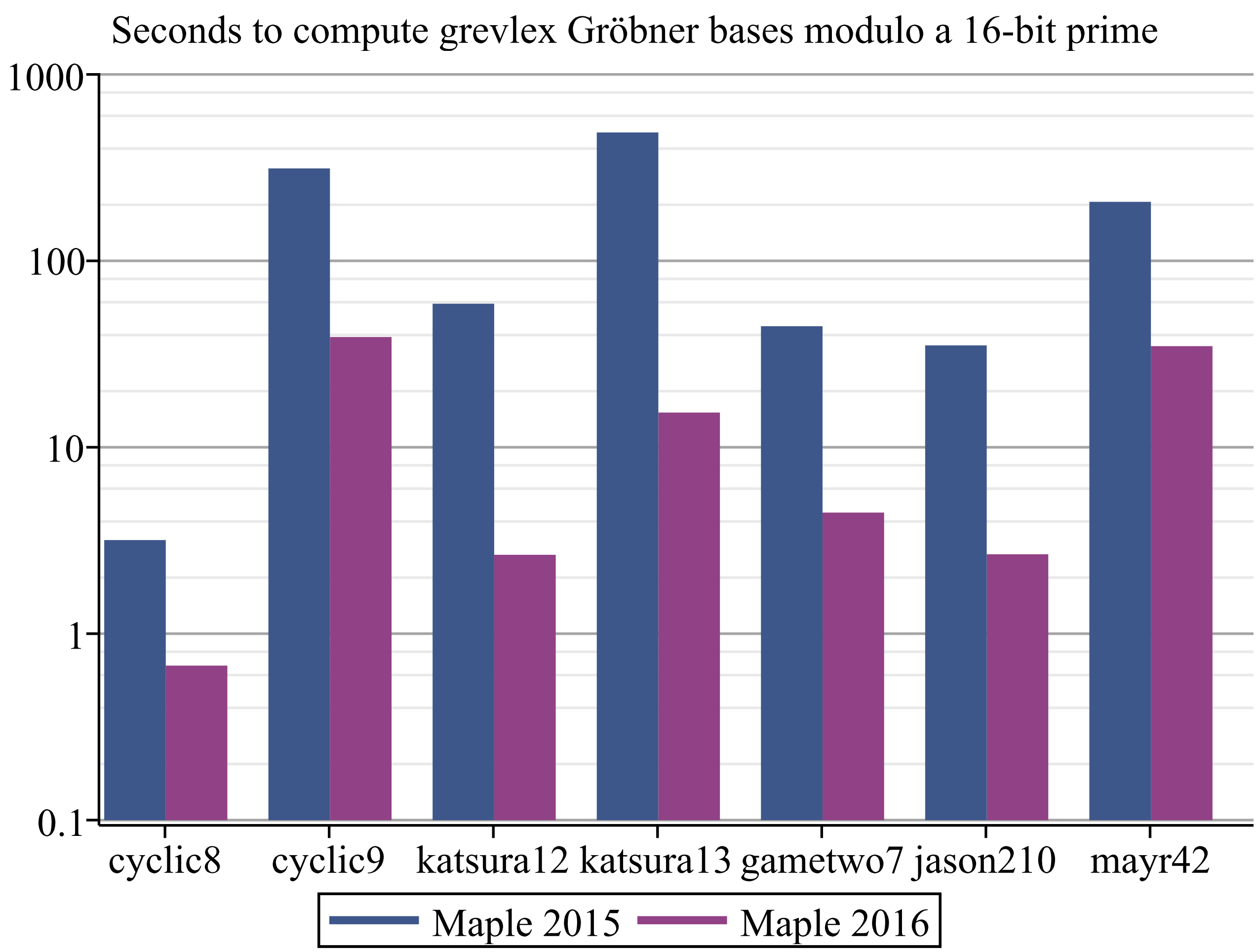
eval

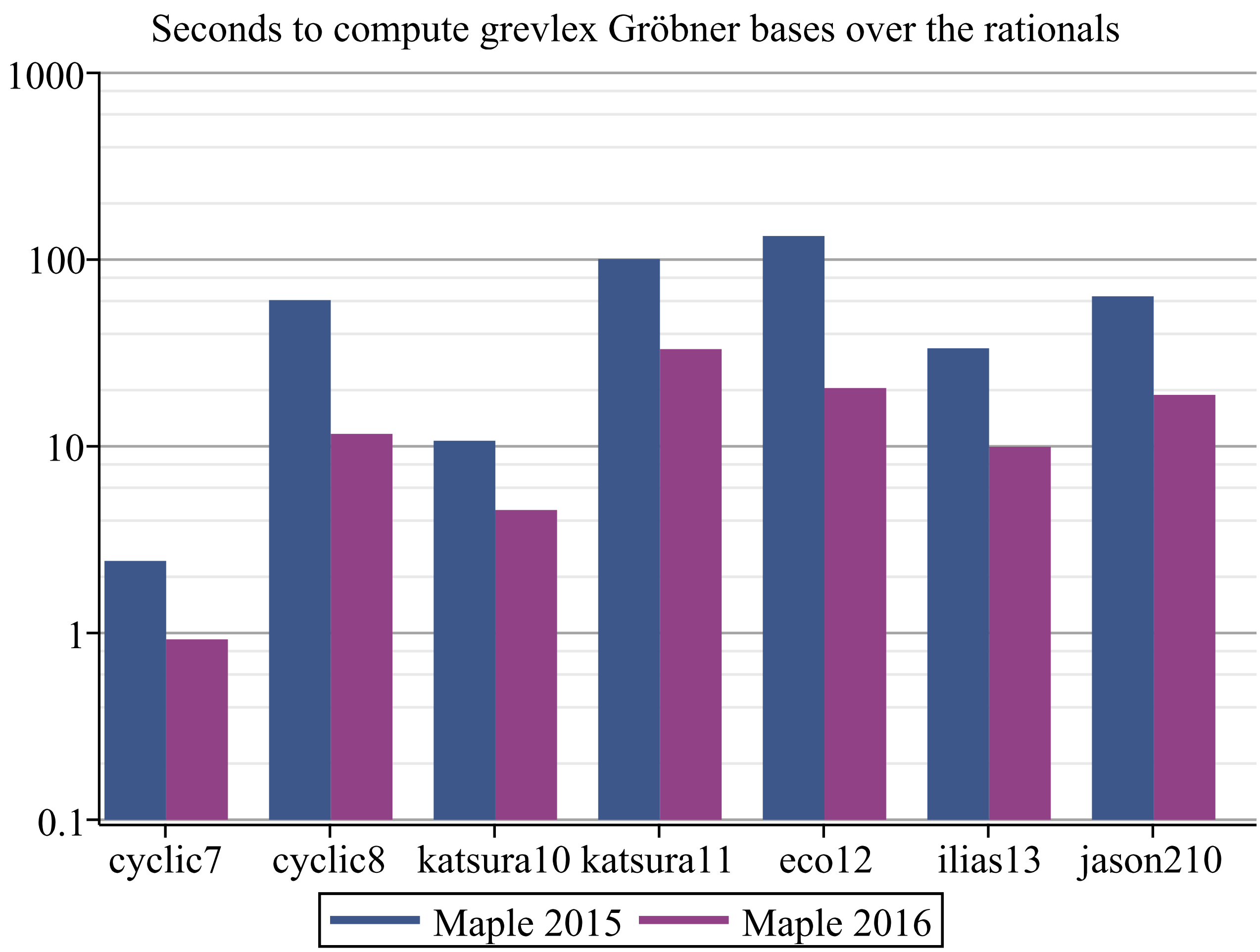
For indefinite integrals, sums, and products, as well as for differentiations, the eval command now supports an additive change of the variable.

> $\left.\int f(x)\,\mathrm{d}x\right\vert_{x=x+c}$ $\int f(x+c)\,\mathrm{d}x$	> $\left.\sum_{x} f(x)\right\vert_{x=x+c}$ $\sum_{x} f(x+c)$
> $\left.f'(x)\right\vert_{x=x+c}$ $\mathrm{D}(f)(x+c)$	> $\left.\prod_{x} f(x)\right\vert_{x=x+c}$ $\prod_{x} f(x+c)$

Gröbner Bases

Maple 2016 includes a new C implementation of the F4 algorithm for computing Gröbner bases, replacing FGb. The new code is generally faster and uses multiple threads. The benchmarks below show real time on a quad core Intel Core i5 4590 3.3 GHz computer using a logarithmic scale. The new code supports primes up to $2^{31}-1$, an increase over FGb's 16-bit primes. To compute over the rationals, Maple uses Chinese remaindering and rational reconstruction.





product

Maple 2016 includes improved handling of "product over RootOf" cases. The following example returns unevaluated in Maple 2015 and earlier:

$$> \prod_{R = RootOf(z^2 - 2)} (x - RootOf(z^2 - R, z))$$

$$x^4 - 2$$

Series and Limit Computations

A number of improvements were made to series and limit computations in Maple.

The following series and asymptotic functions were added:

- Asymptotic expansions of Airy functions at $-\infty$
- Series and asymptotic expansions of hypergeometric functions
- Series expansions of abs and signum in the real case

- Series expansion of GAMMA function at a symbolic pole
- Asymptotic expansion of incomplete GAMMA function w.r.t. the parameter
- Asymptotic expansion of Hurwitz Zeta function
- Series and asymptotic expansions of harmonic numbers
- Series expansions of ln and related functions with a logarithmic branch cut depending on a real parameter were improved.
- Finally, limit computations of oscillating functions were improved.

New Series Expansions

The following series expansions could not be computed in earlier versions of Maple:

${}_2F_1$ functions at 1

> $series\left(\mathrm{hypergeom}\left([1,1],\left[-\frac{1}{3}\right],1-x\right),x\right)$

$$-\frac{8}{9}\frac{\pi\sqrt{3}}{x^{7/3}}+\frac{32}{27}\frac{\pi\sqrt{3}}{x^{4/3}}-\frac{16}{81}\frac{\pi\sqrt{3}}{x^{1/3}}+\frac{4}{7}-\frac{32}{729}\pi\sqrt{3}\,x^{2/3}+\frac{6}{35}x-\frac{40}{2187}\pi\sqrt{3}\,x^{5/3}+\frac{36}{455}x^2-\frac{64}{6561}\pi\sqrt{3}\,x^{8/3}+\frac{81}{1820}x^3+\mathrm{O}\left(x^{11/3}\right)$$

The case where the lower parameter minus the sum of the upper parameters is an integer is supported.

> $series\left(\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{2}\right],[1],1-x\right),x\right)$

$$\frac{\frac{2}{\pi}}{x}+\frac{1}{2}\frac{-\ln(x)-1+4\ln(2)}{\pi}+\frac{1}{2}\frac{-\frac{3}{8}\ln(x)-\frac{13}{16}+\frac{3}{2}\ln(2)}{\pi}x+\frac{1}{2}\frac{-\frac{15}{64}\ln(x)-\frac{9}{16}+\frac{15}{16}\ln(2)}{\pi}x^2+\frac{1}{2}\frac{-\frac{175}{1024}\ln(x)-\frac{5255}{12288}+\frac{175}{256}\ln(2)}{\pi}x^3+\frac{1}{2}\frac{-\frac{2205}{16384}\ln(x)-\frac{11291}{32768}+\frac{2205}{4096}\ln(2)}{\pi}x^4+\mathrm{O}\left(x^5\right)$$

The Γ function at a symbolic pole.

> $series(\Gamma(x),x=n,3)$ assuming $n::nonposint$

$$\frac{\frac{1}{(-1)^n \Gamma(1-n)}}{x-n} + \frac{\Psi(1-n)}{(-1)^n \Gamma(1-n)}$$

$$+ \frac{\frac{1}{6} \frac{\pi^2}{(-1)^n} - \frac{\frac{1}{2} \Psi(1, 1-n) \Gamma(1-n) + \frac{1}{2} \Psi(1-n)^2 \Gamma(1-n)}{(-1)^n \Gamma(1-n)} + \frac{\Psi(1-n)^2}{(-1)^n}}{\Gamma(1-n)} (x$$

$$- n) + \frac{1}{\Gamma(1-n)} \Bigg($$

$$- \frac{1}{(-1)^n \Gamma(1-n)} \left(-\frac{1}{6} \Psi(2, 1-n) \Gamma(1-n) - \frac{1}{2} \Psi(1, 1-n) \Psi(1-n) \Gamma(1-n) \right.$$

$$\left. - \frac{1}{6} \Psi(1-n)^3 \Gamma(1-n) \right)$$

$$- \frac{\Psi(1-n) \left(\frac{1}{2} \Psi(1, 1-n) \Gamma(1-n) + \frac{1}{2} \Psi(1-n)^2 \Gamma(1-n) \right)}{(-1)^n \Gamma(1-n)}$$

$$+ \frac{1}{6} \frac{\left(3 \Psi(1-n)^2 + \pi^2 - 3 \Psi(1, 1-n)\right) \Psi(1-n)}{(-1)^n} \Bigg) (x-n)^2$$

$$+ \frac{1}{\Gamma(1-n)} \Bigg(\frac{7}{360} \frac{\pi^4}{(-1)^n} - \frac{1}{(-1)^n \Gamma(1-n)} \left(\frac{1}{24} \Psi(3, 1-n) \Gamma(1-n) \right.$$

$$+ \frac{1}{6} \Psi(2, 1-n) \Psi(1-n) \Gamma(1-n) + \frac{1}{8} \Psi(1, 1-n)^2 \Gamma(1-n) + \frac{1}{4} \Psi(1, 1$$

$$\left. - n) \Psi(1-n)^2 \Gamma(1-n) + \frac{1}{24} \Psi(1-n)^4 \Gamma(1-n) \right) - \frac{1}{(-1)^n \Gamma(1-n)} \Big(\Psi(1$$

$$- n) \left(-\frac{1}{6} \Psi(2, 1-n) \Gamma(1-n) - \frac{1}{2} \Psi(1, 1-n) \Psi(1-n) \Gamma(1-n) - \frac{1}{6} \Psi(1 \right.$$

$$\left. - n)^3 \Gamma(1-n) \right) \Big)$$

$$- \frac{1}{6} \frac{1}{\Gamma(1-n) (-1)^n} \Big(\left(3 \Psi(1-n)^2 + \pi^2 - 3 \Psi(1, 1-n)\right) \left(\frac{1}{2} \Psi(1, 1-n) \Gamma(1 \right.$$

$$-n)+\frac{1}{2}\Psi(1-n)^2\Gamma(1-n)\Big)\Big)$$

$$+\frac{1}{6}\frac{\left(\Psi(1-n)^3+\Psi(1-n)\pi^2-3\Psi(1,1-n)\Psi(1-n)+\Psi(2,1-n)\right)\Psi(1-n)}{(-1)^n}\Bigg)$$

$$(x-n)^3+O\left((x-n)^4\right)$$

The harmonic function at a negative integer.

> $series(\mathrm{harmonic}(-1+n), n)$

$$-n^{-1}+\frac{1}{6}\pi^2 n-\zeta(3)n^2+\frac{1}{90}\pi^4 n^3-\zeta(5)n^4+\frac{1}{945}\pi^6 n^5+O(n^6)$$

> $series(\mathrm{harmonic}(-2+n, 2), n)$

$$-n^{-2}-1+(-2+2\zeta(3))n+\left(-3-\frac{1}{30}\pi^4\right)n^2+(-4+4\zeta(5))n^3+\left(-5-\frac{1}{189}\pi^6\right)n^4$$

$$+(-6+6\zeta(7))n^5+O(n^6)$$

New Asymptotic Expansions

The following asymptotic expansions could not be computed in earlier versions of Maple:

Airy functions at $-\infty$:

> $series(\mathrm{AiryAi}(x), x=-\infty)$

$$\frac{\sin\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{1/4}}{\sqrt{\pi}}-\frac{5}{48}\frac{\cos\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{7/4}}{\sqrt{\pi}}$$

$$-\frac{385}{4608}\frac{\sin\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{13/4}}{\sqrt{\pi}}$$

$$+\frac{85085}{663552}\frac{\cos\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{19/4}}{\sqrt{\pi}}+O\left(\left(-\frac{1}{x}\right)^{25/4}\right)$$

> $series(\mathrm{AiryBi}(1, x), x=-\infty)$

$$\frac{\sin\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)}{\sqrt{\pi}\left(-\frac{1}{x}\right)^{1/4}}+\frac{7}{48}\frac{\cos\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{5/4}}{\sqrt{\pi}}$$

$$+\frac{455}{4608}\frac{\sin\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{11/4}}{\sqrt{\pi}}$$

$$-\frac{95095}{663552}\frac{\cos\left(\frac{2}{3}(-x)^{3/2}+\frac{1}{4}\pi\right)\left(-\frac{1}{x}\right)^{17/4}}{\sqrt{\pi}}+\mathrm{O}\left(\left(-\frac{1}{x}\right)^{23/4}\right)$$

Hypergeometric functions w.r.t. the argument, when $p \le q+1$, where p and q are the number of upper and lower parameters, respectively:

> $asympt\left(\mathrm{hypergeom}\left(\left[-\frac{1}{3}\right],[2],x\right),x\right)$

$$\left(-\frac{1}{3}\frac{\left(\frac{1}{x}\right)^{7/3}}{\Gamma\left(\frac{2}{3}\right)}-\frac{28}{27}\frac{\left(\frac{1}{x}\right)^{10/3}}{\Gamma\left(\frac{2}{3}\right)}-\frac{980}{243}\frac{\left(\frac{1}{x}\right)^{13/3}}{\Gamma\left(\frac{2}{3}\right)}-\frac{127400}{6561}\frac{\left(\frac{1}{x}\right)^{16/3}}{\Gamma\left(\frac{2}{3}\right)}\right.$$

$$\left.+\mathrm{O}\left(\left(\frac{1}{x}\right)^{19/3}\right)\right)\mathrm{e}^{x}$$

> $asympt\left(\mathrm{hypergeom}\left(\left[\frac{1}{3}\right],[1,2],x\right),x\right)$

$$\left(\frac{1}{4}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{13/12}}{\pi^{3/2}}+\frac{23}{192}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{19/12}}{\pi^{3/2}}\right.$$

$$+\frac{8363}{55296}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{25/12}}{\pi^{3/2}}+\frac{237325}{884736}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{31/12}}{\pi^{3/2}}$$

$$+\frac{930728275}{1528823808}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{37/12}}{\pi^{3/2}}+\frac{123511351105}{73383542784}\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{1}{x}\right)^{43/12}}{\pi^{3/2}}$$

$$\left.+\mathrm{O}\left(\left(\frac{1}{x}\right)^{49/12}\right)\right)\mathrm{e}^{\frac{2}{\sqrt{\frac{1}{x}}}}$$

Cases where the two upper parameters of a ${}_2F_1$ function differ by an integer are handled as well:

> $asympt\left(\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{2}\right],\left[\frac{3}{2}\right],x\right),x\right)$

$$-\frac{\frac{1}{4}\,\mathrm{I}}{\left(\frac{1}{x}\right)^{3/2}}+\frac{\frac{3}{4}\,\mathrm{I}}{\sqrt{\frac{1}{x}}}-\mathrm{I}\left(\frac{3}{16}\ln(x)+\frac{3}{16}\,\mathrm{I}\,\pi+\frac{9}{32}+\frac{3}{8}\ln(2)\right)\sqrt{\frac{1}{x}}+\frac{1}{32}\,\mathrm{I}\left(\frac{1}{x}\right)^{3/2}$$

$$+\frac{3}{512}\,\mathrm{I}\left(\frac{1}{x}\right)^{5/2}+\frac{1}{512}\,\mathrm{I}\left(\frac{1}{x}\right)^{7/2}+\mathrm{O}\left(\left(\frac{1}{x}\right)^{9/2}\right)$$

Hypergeometric functions w.r.t. a parameter, when the function is actually elementary:

$> \ asympt\left(\mathrm{hypergeom}\left([k],[\ ],\frac{z}{k}\right),k\right)$

$$\mathrm{e}^{z}+\frac{1}{2}\frac{\mathrm{e}^{z}z^{2}}{k}+\frac{\mathrm{e}^{z}\left(\frac{1}{3}z^{3}+\frac{1}{8}z^{4}\right)}{k^{2}}+\frac{\mathrm{e}^{z}\left(\frac{1}{4}z^{4}+\frac{1}{6}z^{5}+\frac{1}{48}z^{6}\right)}{k^{3}}$$

$$+\frac{\mathrm{e}^{z}\left(\frac{1}{5}z^{5}+\frac{13}{72}z^{6}+\frac{1}{24}z^{7}+\frac{1}{384}z^{8}\right)}{k^{4}}+\mathrm{O}\left(\frac{1}{k^{5}}\right)$$

$> \ convert\left(\mathrm{hypergeom}\left([k],[\ ],\frac{z}{k}\right),elementary\right)$

$$\left(\frac{k-z}{k}\right)^{-k}$$

Hurwitz ζ and incomplete Γ functions w.r.t. the parameter:

$> \ asympt\left(\mathrm{Zeta}\left(0,\frac{1}{3},v\right),v\right)$

$$\frac{1}{\left(\frac{1}{v}\right)^{2/3}}+\frac{1}{2}\left(\frac{1}{v}\right)^{1/3}-\frac{1}{54}\left(\frac{1}{v}\right)^{4/3}+\frac{7}{7290}\left(\frac{1}{v}\right)^{10/3}+\mathrm{O}\left(\left(\frac{1}{v}\right)^{16/3}\right)$$

$> \ asympt(\Gamma(v,x),v)$

$$\frac{1}{\left(\frac{1}{v}\right)^{v}\mathrm{e}^{v}}\left(\sqrt{2}\sqrt{\pi}\sqrt{\frac{1}{v}}+\frac{1}{12}\sqrt{2}\sqrt{\pi}\left(\frac{1}{v}\right)^{3/2}+\frac{1}{288}\sqrt{2}\sqrt{\pi}\left(\frac{1}{v}\right)^{5/2}\right.$$

$$-\frac{139}{51840}\sqrt{2}\sqrt{\pi}\left(\frac{1}{v}\right)^{7/2}-\frac{571}{2488320}\sqrt{2}\sqrt{\pi}\left(\frac{1}{v}\right)^{9/2}$$

$$\left.+\frac{163879}{209018880}\sqrt{2}\sqrt{\pi}\left(\frac{1}{v}\right)^{11/2}+\mathrm{O}\left(\left(\frac{1}{v}\right)^{13/2}\right)\right)$$

The harmonic function:

$> \ asympt(\mathrm{harmonic}(n),n)$

$$\ln(n)+\gamma+\frac{1}{2n}-\frac{1}{12n^{2}}+\frac{1}{120n^{4}}+\mathrm{O}\left(\frac{1}{n^{6}}\right)$$

> $asympt\left(\text{harmonic}\left(n, \frac{2}{3}\right), n\right)$

$$-\frac{1}{\left(\frac{1}{n}\right)^{1/3}} + \zeta\left(\frac{2}{3}\right) - \frac{5}{6}\left(\frac{1}{n}\right)^{2/3} + \frac{25}{54}\left(\frac{1}{n}\right)^{5/3} - \frac{10}{27}\left(\frac{1}{n}\right)^{8/3} + \frac{239}{729}\left(\frac{1}{n}\right)^{11/3} - \frac{220}{729}\left(\frac{1}{n}\right)^{14/3} + O\left(\left(\frac{1}{n}\right)^{17/3}\right)$$

Two-sided and One-sided Expansions of abs and signum

Maple can now compute two-sided expansions of signum at finite nonzero points.

> $series(\text{signum}(a+x), x, 4)$ assuming $x :: real$

$$\text{signum}(a) + \left(-\frac{a\,\Re(a)}{|a|^3} + \frac{1}{|a|}\right)x + \left(\frac{a\left(-\frac{1}{2\,|a|^2} + \frac{3}{2}\frac{\Re(a)^2}{|a|^4}\right)}{|a|} - \frac{\Re(a)}{|a|^3}\right)x^2 + \left(\frac{a\left(\frac{3}{2}\frac{\Re(a)}{|a|^4} - \frac{5}{2}\frac{\Re(a)^3}{|a|^6}\right)}{|a|} + \frac{-\frac{1}{2\,|a|^2} + \frac{3}{2}\frac{\Re(a)^2}{|a|^4}}{|a|}\right)x^3 + O(x^4)$$

One-sided expansions of abs and signum at 0 and asymptotic expansions can now also be computed.

> $series(\text{abs}(\sin(x)), x)$ assuming $x > 0$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + O(x^7)$$

> $series(\text{signum}(\sin(x)), x)$ assuming $x > 0$

$$1$$

> $asympt\left(\text{abs}\left(\text{arccot}\left(x^2 - \frac{x}{11}\right)\right), x\right)$

$$\frac{1}{x^2} + \frac{1}{11\,x^3} + \frac{1}{121\,x^4} + \frac{1}{1331\,x^5} + O\left(\frac{1}{x^6}\right)$$

Expansions of Functions with a Logarithmic Branch Cut Depending on a Real Parameter

For functions ln, arctan, arccot, arctanh, arccoth, Ei, Ci and Chi depending on a parameter, Maple now computes series expansions that are correct for all real values of the parameter (and for all complex values of the series variable sufficiently close to the expansion point).

> $series(\ln(a+x), x)$ assuming $a :: real$

$$\ln(|a|) + \frac{1}{2} \mathrm{I} \left(\mathrm{signum}(a) \, \mathrm{csgn}(\mathrm{I}\,(a+x)) - \mathrm{csgn}(\mathrm{I}\,(a+x))\right) \pi + \frac{1}{a} x - \frac{1}{2\,a^2} x^2 + \frac{1}{3\,a^3} x^3 - \frac{1}{4\,a^4} x^4 + \frac{1}{5\,a^5} x^5 + \mathrm{O}(x^6)$$

> *series*(ln(*a x*), *x*) assuming *a* :: *real*

$$\ln(|a|) + \ln(x) + \frac{1}{2} \mathrm{I} \left(\mathrm{csgn}(\mathrm{I}\,x) - \mathrm{signum}(a) \, \mathrm{csgn}(\mathrm{I}\,x)\right) \pi$$

> *series*(arctanh(*a* + *x*), *x*, 4) assuming *a* :: *real*

$$-\mathrm{I}\left(\frac{1}{4} \mathrm{csgn}(-\mathrm{I}\,\mathrm{signum}(a) + \mathrm{I}\,a + \mathrm{I}\,x)\, \pi \left(1 + \mathrm{signum}(a^2 - 1)\right) + \mathrm{I}\,\Re(\mathrm{arctanh}(a))\right) + \frac{1}{-a^2+1} x - \frac{a}{(a^2-1)\,(-a^2+1)} x^2 - \frac{\frac{1}{3} \mathrm{I} \left(-\frac{\mathrm{I}}{a^2-1} + \frac{4\,\mathrm{I}\,a^2}{(a^2-1)^2}\right)}{-a^2+1} x^3 + \mathrm{O}(x^4)$$

> *series*(Chi(*a x*), *x*) assuming *a* :: *real*

$$\gamma + \ln(|a|) + \ln(x) + \frac{1}{2} \mathrm{I} \left(\mathrm{csgn}(\mathrm{I}\,x) - \mathrm{signum}(a) \, \mathrm{csgn}(\mathrm{I}\,x)\right) \pi + \frac{1}{4} a^2 x^2 + \frac{1}{96} a^4 x^4 + \mathrm{O}(x^6)$$

Limits of Oscillating Functions

Limit computations for functions containing oscillating terms were improved. The following limits could not be computed in Maple 2015 or earlier.

> $\lim_{x \to \infty} \left(e^{\mathrm{i}\,x} \tan(x)\right)$

undefined

> $\lim_{x \to \infty} \left(-\frac{\sinh(x)\, \mathrm{BesselJ}(1, x) \sqrt{x}}{\cosh(x) - \cos(x)^2}\right)$

$$-\frac{\sqrt{2}}{\sqrt{\pi}} .. \frac{\sqrt{2}}{\sqrt{\pi}}$$

> $\lim_{x \to \infty} \left((-1)^x - \left(\frac{1}{x^2} - 1\right)^x\right)$

0

Symbolic Integration

The results for definite integration of rational functions have been improved. In certain cases when the denominator is of degree 4 or higher, the result is now simpler.

Maple 2016 gives:

> $$\int_0^{\infty} \frac{1}{x^4 - x + 1}\, dx$$

$$-\left(\sum_{\_R = RootOf(\_Z^4 - \_Z + 1)} \frac{\ln(-\_R)}{4\,\_R^3 - 1}\right)$$

where Maple 2015 produced:

> $$\int_0^{\infty} \frac{1}{x^4 - x + 1}\, dx$$

$$-\left(\sum_{\_R = RootOf(229\,\_Z^4 + 18\,\_Z^2 - 8\,\_Z + 1)} \_R \ln\left(\frac{2061}{64}\,\_R^3 + \frac{687}{64}\,\_R^2 + \frac{391}{64}\,\_R - \frac{27}{64}\right)\right)$$

And Maple 2016 gives the much simpler:

> $$\int_0^{\infty} \frac{1}{(x^4 + 6\,x + 1)^3}\, dx$$

$$\frac{1530171}{18852964} - \frac{3}{18852964} \sum_{\_R = RootOf(\_Z^4 + 6\,\_Z + 1)} \frac{(1944\,\_R^2 - 33429\,\_R + 24169)\ln(-\_R)}{2\,\_R^3 + 3}$$

compared to the Maple 2015 result for the same problem:

> $$\int_0^{\infty} \frac{1}{(x^4 + 6\,x + 1)^3}\, dx$$

$$\frac{1530171}{18852964} - \frac{3}{9426482} \sum_{\_R = RootOf(8684\,\_Z^4 - 194281841538\,\_Z^2 + 201567506232261\,\_Z - 1531566190081040464)} \_R \ln\left(\frac{1620561084493871952}{9953026735779065741014153282 9}\,\_R^3 + \frac{2530026158794014504}{4584535576130384956708499 9}\,\_R^2 - \frac{1250116823365209175734115238 98}{2160802104337635172374172677717 59}\,\_R - \frac{1530662817658400954319300 3}{4584535576130384956708499 9}\right)$$

In addition, Maple can now compute more definite integrals that could not be computed in Maple 2015 or earlier.

> $$\int_0^{\infty} \frac{\pi}{2} - \arctan(x^9)\, dx$$

$$-\frac{1}{2}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{(\_R^3+1)\ln(-\_R)}{\_R(2\_R^3-1)}-\frac{1}{3}\pi\sqrt{3}$$

> $\int_0^\infty \cos(x)\cdot \mathrm{BesselJ}(3,x)\,\mathrm{d}x$

$$\infty$$

> $\int_{-1}^{1}\left|1+\frac{1}{x^5+x^2+2}+\frac{1}{(x^5+x^2+2)^3}\right|\mathrm{d}x$

$$\frac{2564314191}{1260823328}-\left(\sum_{\_R=RootOf(\_Z^5+\_Z^2+2)}\frac{\ln(-1-\_R)}{\_R(5\_R^3+2)}\right)-\frac{3}{1260823328}\sum_{\_R=RootOf(\_Z^5+\_Z^2+2)}\frac{(675729\_R^3-19964900\_R^2-5401458\_R+75189729)\ln(-1-\_R)}{\_R(5\_R^3+2)}+\sum_{\_R=RootOf(\_Z^5+\_Z^2+2)}\frac{\ln(1-\_R)}{\_R(5\_R^3+2)}+\frac{3}{1260823328}\sum_{\_R=RootOf(\_Z^5+\_Z^2+2)}\frac{(675729\_R^3-19964900\_R^2-5401458\_R+75189729)\ln(1-\_R)}{\_R(5\_R^3+2)}$$

Symbolic Summation

Maple 2016 includes a number of improvements to Maple's symbolic summation engine:

- Improved handling of definite parametric sums
- New option *formal* for *sum*
- Support for Jacobi Theta sums
- Support for piecewise expressions with more than two branches
- Improved divergence testing for infinite sums
- Better support for doubly infinite sums

Parametric Sums and Option Formal

Maple 2016 includes several improvements for parametric sums:

- The scope of the option *parametric* was extended so it now works for more types of definite sums.
- For infinite sums, Maple is now more careful regarding potentially divergent parametric sums. The behavior can be controlled using assumptions, the

_EnvFormal environment variable, or, equivalently, a new option *formal* to the sum command.

By default, Maple returns a generic answer for certain types of parametric definite hypergeometric sums.	With option *parametric*, a complete case distinction is now returned for hypergeometric sums with a single parameter that is valid for all integer values of the parameter:
> $sum\left((-1)^k \operatorname{binomial}(m, k)\ k, k=0..m\right)$ 0 > $sum(\operatorname{binomial}(n, 4\,k), k=0..n)$ $\frac{1}{4}\,2^n + \frac{1}{4}\,(1+I)^n + \frac{1}{4}\,(1-I)^n$	> $sum\left((-1)^k \operatorname{binomial}(m, k)\ k, k=0..m, parametric\right)$ $\begin{cases} 0 & m \le 0 \\ -1 & m = 1 \\ 0 & 2 \le m \end{cases}$ > $sum(\operatorname{binomial}(n, 4\,k), k=0..n, parametric)$ $\begin{cases} \sum_{k=0}^{n} \operatorname{binomial}(n, 4\,k) & n \le -1 \\ 1 & n = 0 \\ \frac{1}{4}\,2^n + \frac{1}{4}\,(1+I)^n + \frac{1}{4}\,(1-I)^n & 1 \le n \end{cases}$

The behavior for infinite parametric sums of geometric, hypergeometric, polylog, or Zeta type has changed.

Without any assumptions on the parameter, such sums now return unevaluated.	The same sums with appropriate assumptions:
> $\sum_{k=0}^{\infty} k^2 x^k$ $\sum_{k=0}^{\infty} k^2 x^k$	> $\sum_{k=0}^{\infty} k^2 x^k$ assuming $-1 < x < 1$ $-\frac{x\,(x+1)}{(x-1)^3}$

	> $\sum_{k=0}^{\infty} k^2 x^k$ assuming $x > 1$ ∞
> $\sum_{k=0}^{\infty} \mathrm{binomial}(n+k, k)\, x^k$ $\sum_{k=0}^{\infty} \mathrm{binomial}(n+k, k)\, x^k$	> $\sum_{k=0}^{\infty} \mathrm{binomial}(n+k, k)\, x^k$ assuming $-1 < x < 1$ $-\frac{\left(-\frac{1}{x-1}\right)^n}{x-1}$
> $\sum_{k=1}^{\infty} \frac{x^k}{k^2}$ $\sum_{k=1}^{\infty} \frac{x^k}{k^2}$	> $\sum_{k=1}^{\infty} \frac{x^k}{k^2}$ assuming $-1 < x < 1$ $\mathrm{polylog}(2, x)$
> $\sum_{k=1}^{\infty} k^n$ $\sum_{k=1}^{\infty} k^n$	> $\sum_{k=1}^{\infty} k^n$ assuming $\mathrm{Re}(n) < -1$ $\zeta(-n)$ > $\sum_{k=1}^{\infty} k^n$ assuming $n \geq -1$ ∞

Alternatively, formal answers can be obtained by either setting the environment variable $\_EnvFormal := true$, or by specifying the new option *formal*. (This even works for non-parametric divergent sums.)	For geometric, polylog, and Zeta type sums, option *parametric* can also be used:
> $\_EnvFormal := true:$ > $\sum_{k=0}^{\infty} k^2 x^k$	> $sum(k^2 x^k, k = 0 .. \infty, parametric)$

> $-\frac{(1+x)\,x}{(x-1)^3}$ > $\sum_{k=0}^{\infty}\text{binomial}(n+k,k)\,x^k$ $\frac{1}{(-x+1)^{n+1}}$ > $\_EnvFormal := '\_EnvFormal'$: > $sum\left(\frac{x^k}{k^2}, k=1..\infty\right)$ $\sum_{k=1}^{\infty}\frac{x^k}{k^2}$ > $sum\left(\frac{x^k}{k^2}, k=1..\infty, formal\right)$ $\text{polylog}(2,x)$ > $sum(k^3, k=1..\infty, formal)=\zeta(-3)$ $\frac{1}{120}=\frac{1}{120}$	$\begin{cases} -\frac{(1+x)\,x}{(x-1)^3} & \lvert x\rvert < 1 \\ \infty & 1 \leq x \\ undefined & otherwise \end{cases}$ > $sum\left(\frac{x^k}{k^2}, k=1..\text{infinity}, parametric\right)$ $\begin{cases} \text{polylog}(2,x) & x=-1 \textbf{ Or } x=1 \textbf{ Or } \lvert x\rvert < 1 \\ \infty & 1 \leq x \\ undefined & otherwise \end{cases}$ > $sum(k^n, k=1..\text{infinity}, parametric)$ $\begin{cases} \zeta(-n) & 1 < -\Re(n) \\ \infty & -n \leq 1 \\ undefined & otherwise \end{cases}$

Jacobi Theta

Maple now recognizes infinite sums that can be expressed in terms of Jacobi Theta functions.

> $\sum_{n=0}^{\infty} r^{n^2}$ assuming $0 < r < 1$

$$\frac{1}{2}\,\text{JacobiTheta3}(0,r)+\frac{1}{2}$$

> $sum(r^{n^2}, n=0..\infty, parametric)$

$$\begin{cases} \frac{1}{2}\,\text{JacobiTheta3}(0,r)+\frac{1}{2} & \lvert r\rvert < 1 \\ \infty & 1 \leq r \\ undefined & otherwise \end{cases}$$

> $\sum_{n=1}^{\infty} 2^{-n^2-n} \cos((2n+1) \cdot t)$

$$\frac{1}{2} 2^{1/4} \text{JacobiTheta2}\left(t, \frac{1}{2}\right) - \cos(t)$$

Piecewise Sums

Maple now supports piecewise summands with integer branch points and more than two branches.

> $\sum_{n=0}^{\infty} \begin{cases} 0 & n \leq 0 \\ a & n \leq 1 \\ 2^{-n} & \text{otherwise} \end{cases}$

$$a + \frac{1}{2}$$

> $\sum_{n=0}^{\infty} n \begin{cases} 2 & n = 0 \\ 10^n & n < 10 \\ x & n = 10 \\ x^n & \text{otherwise} \end{cases}$ assuming $-1 < x < 1$

$$9876543210 - \frac{x^{11} (10x - 11)}{(x-1)^2} + 10x$$

Sums Diverging to $\pm \infty$

For some non-hypergeometric infinite sums without parameters, Maple now detects when they diverge to $\pm \infty$.

> $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

$$\infty$$

> $\sum_{n=0}^{\infty} \frac{\cosh((2n+1) \cdot \pi)}{\cos((2n+1) \cdot \pi)}$

$$-\infty$$

Doubly Infinite Sums

Maple now has improved support for doubly infinite sums, by splitting them into two one-sided infinite sums.

> $f := i \rightarrow \frac{(-1)^i}{4\,i^2 - 1}$:

> $\sum_{i=-\infty}^{\infty} f(i)$

$$-\frac{1}{2}\,\pi$$

> $\sum_{i=-\infty}^{-1} f(i)$

$$\frac{1}{2} - \frac{1}{4}\,\pi$$

> $\sum_{i=1}^{\infty} f(i)$

$$\frac{1}{2} - \frac{1}{4}\,\pi$$

> $\frac{1}{2} - \frac{1}{4}\,\pi + f(0) + \frac{1}{2} - \frac{1}{4}\,\pi$

$$-\frac{1}{2}\,\pi$$