What's New in Maple 2016

Student Multivariate Calculus

Six new commands have been added and two previous commands have been augmented. The new commands are:

- CrossProduct
- diff
- DotProduct
- Norm
- Normalize
- TripleScalarProduct

The modified commands are MultiInt and SurfaceArea.

- The Norm and Normalize commands default to the Euclidean norm over the real numbers.
- The DotProduct and TripleScalarProduct commands calculate over the real numbers.
- These, and the CrossProduct command, support the Lines & Planes portion of the typical multivariate calculus course.
- Just as in the VectorCalculus packages, the top-level diff command now automatically maps over the components of a vector when implemented within this package.

The existing MultiInt and SurfaceArea commands have been updated with the capability to integrate over the same regions that are known to the modified int command in the VectorCalculus packages. The net effect of these improvements is to reduce the number of times a student of multivariate calculus needs to work outside the Student MultivariateCalculus package.

Maple can solve many problems in Multivariate Calculus using several methods: using typeset math, using the context sensitive menu, and lastly, by Maple commands. Solutions using each of these three methods are shown for the first three of the following examples.

Example 1: Computing a Norm and Normalizing a Vector

Step	Instructions	Results
Load the Student :-MultivariateCalculus package.	From the **Tools** menu, select **Load Package**, then **Student Multivariate Calculus.**	Loading Student:-MultivariateCalculus
Define the vector **V**.	• Enter: $V = \langle a, b \rangle$. • Right-click on this expression and select **Assign Name** from the context menu.	$\mathbf{V} = \langle a, b \rangle \xrightarrow{\text{assign}}$
Find the norm of **V** using typeset math.	• Type the norm bars. • From the context menu, select **Evaluate and Display Inline.**	$\lVert \mathbf{V} \rVert = \sqrt{a^2 + b^2}$
Alternatively, find the norm using context menu options.	• Type: V. • From the context menu, select **Evaluate and Display Inline.** • Again, from the context menu select **Student Multivariate Calculus**, then select **Norm.**	$\mathbf{V} = \begin{bmatrix} a \\ b \end{bmatrix} \xrightarrow{\text{norm}} \sqrt{a^2 + b^2}$
Normalize V using the context menu.	• Type: V. • From the context menu select **Evaluate and Display Inline.** • Again, from the context menu select **Student Multivariate Calculus** then select **Normalize.**	$\mathbf{V} = \begin{bmatrix} a \\ b \end{bmatrix} \xrightarrow{\text{normalize}} \begin{bmatrix} \frac{a}{\sqrt{a^2 + b^2}} \\ \frac{b}{\sqrt{a^2 + b^2}} \end{bmatrix}$
Use the Norm command to find the norm of V.	Type: "Norm(V)" and press **Enter**.	$Norm(\mathbf{V}) = \sqrt{a^2 + b^2}$

Use the Normalize command to normalize V.	Type: "Normalize(V)" and press **Enter**.	$Normalize(\mathbf{V}) = \begin{bmatrix} \frac{a}{\sqrt{a^2+b^2}} \\ \frac{b}{\sqrt{a^2+b^2}} \end{bmatrix}$

Example 2: Computing Cross Products, Dot Products and Triple Scalar Products

Step	Instructions	Results
Load the Student:-MultivariateCalculus package.	• From the **Tools** menu, select **Load Package,** then **Student Multivariate Calculus.**	Loading Student:-MultivariateCalculus.
Define Vectors A, B and C.	• Enter: $A = \langle a, b, c\rangle$ • Right-click on this expression and select **Assign Name** from the context menu. • Repeat for B and C (see Results column).	$\mathbf{A} = \langle a, b, c\rangle \xrightarrow{\text{assign}}$ $\mathbf{B} = \langle u, v, w\rangle \xrightarrow{\text{assign}}$ $\mathbf{C} = \langle p, q, r\rangle \xrightarrow{\text{assign}}$
Vector Products via Typeset Math (See Common Symbols Palette).		
Compute the vector products using typeset math.	• Context Menu: Evaluate and Display Inline.	
Compute $\mathbf{A} \cdot \mathbf{B}$ using typeset math.	• Type: A. • From the Common Symbols palette, select · • Type: B. • Right-click on the expression.	$\mathbf{A} \cdot \mathbf{B} = a\,u + b\,v + c\,w$

	• From the context menu select **Evaluate and Display Inline**.	
Compute $\mathbf{A} \times \mathbf{B}$ using typeset math.	• Type: A. • From the Common Symbols palette select × • Type: B. • Right-click on the expression. • From the context menu select **Evaluate and Display Inline**.	$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} b\,w - c\,v \\ -a\,w + c\,u \\ a\,v - b\,u \end{bmatrix}$
Compute $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$using typeset math.	• Type: $\mathrm{A} \cdot (\mathrm{B} \times \mathrm{C})$, selecting the operators from the Common Symbols palette. • Right-click on the expression. • From the context menu select **Evaluate and Display Inline**.	$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) =$ $a\,(-q\,w + r\,v) + b\,(p\,w - r\,u)$ $+ c\,(-p\,v + q\,u)$
Vector Products via the Context Menu		
Compute $\mathbf{A} \cdot \mathbf{B}$ from the context menu.	• Enter: A,B. • Right-click on A,B and select **Evaluate and Display Inline** from the context menu. • Right-click on the output from the previous step and select **Student Multivariate Calculus**, then **Dot Product**.	$\mathbf{A}, \mathbf{B} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} u \\ v \\ w \end{bmatrix} \xrightarrow{\text{dot product}}$ $a\,u + b\,v + c\,w$

Compute $\mathbf{A} \times \mathbf{B}$ from the context menu.	• Enter: A,B. • Right-click on A,B and select **Evaluate and Display Inline** from the context menu. • Right-click on the output from the previous step and select **Student Multivariate Calculus**, then **Cross Product**.	$\mathbf{A}, \mathbf{B} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} u \\ v \\ w \end{bmatrix} \xrightarrow{\text{cross product}} \begin{bmatrix} b\,w - c\,v \\ -a\,w + c\,u \\ a\,v - b\,u \end{bmatrix}$
Compute $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ from the context menu.	• Enter: A,B,C. • Right-click on A,B,C and select **Evaluate and Display Inline** from the context menu. • Right-click on the output from the previous step and select **Student Multivariate Calculus**, then **Triple Scalar Product**.	$\mathbf{A}, \mathbf{B}, \mathbf{C} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \begin{bmatrix} p \\ q \\ r \end{bmatrix} \xrightarrow{\text{scalar triple product}} a\,(-q\,w + r\,v) + b\,(p\,w - r\,u) + c\,(-p\,v + q\,u)$
Vector Products via the DotProduct, CrossProduct, and TripleScalarProduct Commands		
Compute $\mathbf{A} \cdot \mathbf{B}$ using the DotProduct command.	• Type: DotProduct(A,B). • Press Enter.	$DotProduct(\mathbf{A}, \mathbf{B}) = a\,u + b\,v + c\,w$
Compute $\mathbf{A} \times \mathbf{B}$ using the CrossProduct command.	• Type: CrossProduct(A,B). • Press Enter.	$CrossProduct(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} b\,w - c\,v \\ -a\,w + c\,u \\ a\,v - b\,u \end{bmatrix}$

Compute $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$ using the TripleScalarProduct command.	• Type: TripleScalarProduct(A, B,C). • Press Enter.	$\mathit{TripleScalarProduct}(\mathbf{A},\mathbf{B},\mathbf{C}) = a\,(-q\,w+r\,v)+b\,(p\,w-r\,u)+c\,(-p\,v+q\,u)$

Example 3: Differentiating a Vector

Step	Instructions	Results
Load the Student:-MultivariateCalculus package.	• From the **Tools** menu, select **Load Package,** then Student Multivariate Calculus.	Loading Student:-MultivariateCalculus
Define vector $\mathbf{V}$.	• Type: $\mathbf{V}=\langle f(x),g(x)\rangle$ • From the context menu, select **Assign Name**.	$\mathbf{V}=\langle f(x),g(x)\rangle \xrightarrow{\text{assign}}$
Define vector $\mathbf{v}$.	• Type: $\mathbf{v}=\langle x(t),y(t)\rangle$ • From the context menu select **Assign Name**	$\mathbf{v}=\langle x(t),y(t)\rangle \xrightarrow{\text{assign}}$
Differentiate via Typeset Math		
Differentiate $\mathbf{V}$.	• Type $\mathbf{V}$. • Select $'$ from the Common Symbols palette. • Press Enter.	$\mathbf{V}'=\begin{bmatrix}\frac{d}{dx}f(x)\\ \frac{d}{dx}g(x)\end{bmatrix}$
Differentiate $\mathbf{v}$.	• From the Calculus palette, select $\dot{A}$. • Replace A with v. • Press Enter.	$\dot{\mathbf{v}}=\begin{bmatrix}\frac{d}{dt}x(t)\\ \frac{d}{dt}y(t)\end{bmatrix}$

Differentiate via Context Menu		
Differentiate $\mathbf{V}$.	• Enter: V • Right-click on $\mathbf{V}$. • From the context menu, select **Evaluate and Display Inline**. • Select the output, then from the context menu, select **Student Multivariate Calculus**, then **Differentiate**, then select **With Respect To** x.	$\mathbf{V} = \begin{bmatrix} f(x) \\ g(x) \end{bmatrix} \xrightarrow{\text{differentiate}} \begin{bmatrix} \frac{d}{dx} f(x) \\ \frac{d}{dx} g(x) \end{bmatrix}$
Differentiate $\mathbf{v}$.	• Enter: v • Right-click on $\mathbf{v}$. • From the context menu, select **Evaluate and Display Inline**. • Select the output, then from the context menu, select **Student Multivariate Calculus**, then **Differentiate**, then select **With Respect To** x.	$\mathbf{v} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \xrightarrow{\text{differentiate}} \begin{bmatrix} \frac{d}{dt} x(t) \\ \frac{d}{dt} y(t) \end{bmatrix}$
Differentiate via the Differentiation Operator in the Calculus Palette		
Differentiate $\mathbf{V}$.	• From the Calculus palette, select the differentiation operator, $\frac{d}{dx} f$. • Replace f with V. • Right-click the expression and select **Evaluate and Display Inline**.	$\frac{d}{dx} \mathbf{V} = \begin{bmatrix} \frac{d}{dx} f(x) \\ \frac{d}{dx} g(x) \end{bmatrix}$
Differentiate $\mathbf{v}$.	• From the Calculus palette, select the differentiation operator, $\frac{d}{dx} f$. • Replace f with v and replace x with t. • Right-click the expression and select **Evaluate and Display**	$\frac{d}{dt} \mathbf{v} = \begin{bmatrix} \frac{d}{dt} x(t) \\ \frac{d}{dt} y(t) \end{bmatrix}$

	Inline.	
Differentiation via the Modified diff Command		
Differentiate $\mathbf{V}$.	• Type "diff(V,x)". • Press Enter.	$diff(\mathbf{V}, x) = \begin{bmatrix} \frac{d}{dx} f(x) \\ \frac{d}{dx} g(x) \end{bmatrix}$
Differentiate $\mathbf{v}$.	• Type "diff(v,x)". • Press Enter.	$diff(\mathbf{v}, t) = \begin{bmatrix} \frac{d}{dt} x(t) \\ \frac{d}{dt} y(t) \end{bmatrix}$

Example 4: Integrating a Function Over a Region

Integrate $f(x, y) = x\,y$ over the upper half of the circle whose center is $\langle 1, 2 \rangle$ and whose radius is R.

Instructions	Results
• From the **Tools** menu, select **Load Package,** then **Student Multivariate Calculus**.	Loading Student:-MultivariateCalculus
Use the MultiInt Command.	
$MultiInt\left(x\,y, [x, y] = Sector\left(Circle\left(\langle 1, 2 \rangle, R, [r, \theta]\right), 0, \pi\right), output = integral\right)$ $\int_0^R \int_0^\pi (r\cos(\theta) + 1)\,(r\sin(\theta) + 2)\,r\,d\theta\,dr$	

$MultiInt(x\,y,\ [x,y]=Sector(Circle(\langle 1,2\rangle, R, [r,\theta]), 0, \pi), output=value) = \pi R^2 + \frac{2}{3} R^3$
$MultiInt(x\,y,\ [x,y]=Sector(Circle(\langle 1,2\rangle, R, [r,\theta]), 0, \pi), output=steps)$ $$\int_0^R \int_0^\pi (r\cos(\theta)+1)\,(r\sin(\theta)+2)\,r\,d\theta\,dr$$ $$=\int_0^R \left(r\left(-\frac{r^2\cos(\theta)^2}{2}+2\,r\sin(\theta)-r\cos(\theta)+2\,\theta\right)\Bigg\vert_{\theta=0..\pi}\right) dr$$ $$=\int_0^R (2\,\pi\,r+2\,r^2)\,dr$$ $$=\left(\pi\,r^2+\frac{2}{3}\,r^3\right)\Bigg\vert_{r=0..R}$$ $$\pi R^2+\frac{2}{3}R^3$$

In addition to Circle and Sector, the MultiInt command "knows" the additional predefined regions Ellipse (to which Sector also applies), Parallelepiped (a rectangular box with sides parallel to the coordinate planes), Rectangle, Sphere, Tetrahedron, Triangle, and the general Region.

Example 5: Calculating the Surface Area of a Region

Calculate the surface area of that portion of the surface $z=3-x^2/4-y^2/3$ that is defined over the triangle whose vertices are $(0,0)$, $(3,0)$, $(1,2)$.

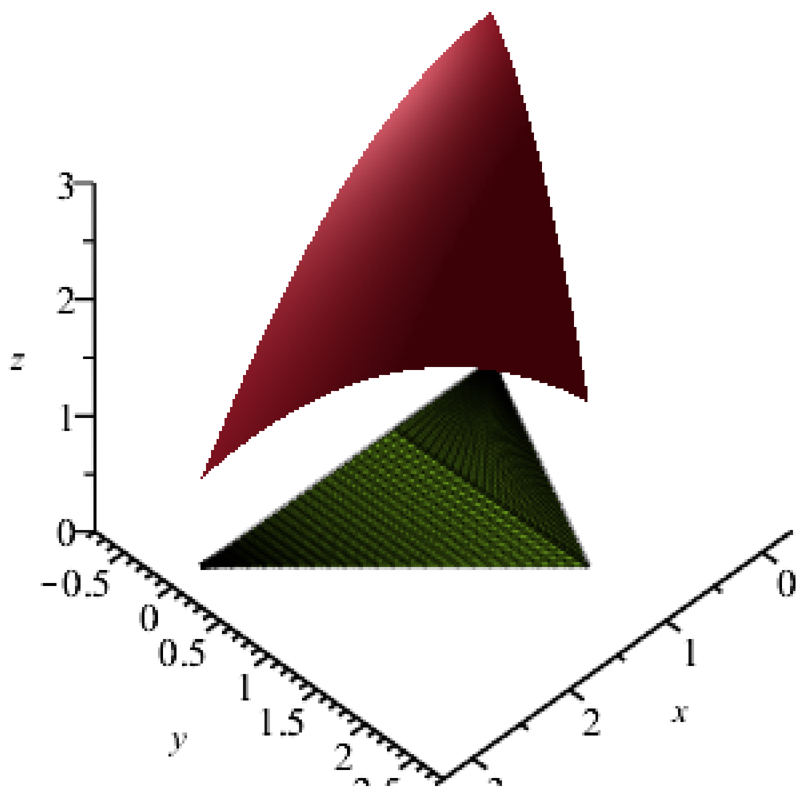
Instructions	Result
• From the **Tools** menu, select **Load Package,** then **Student Multivariate Calculus**.	Loading Student:-MultivariateCalculus.

• Type: $Z=3-x^2/4-y^2/3$ • Right-click on Z and select **Assign Name**.	$Z=3-x^2/4-y^2/3 \xrightarrow{\text{assign}}$

Use SurfaceArea **Command.**

$SurfaceArea(Surface(\langle x, y, Z\rangle, [x, y] = Triangle(\langle 0, 0\rangle, \langle 3, 0\rangle, \langle 1, 2\rangle)), output = plot, axes = frame, labels = [x, y, z])$

Surface Area



Visualizing surface area for $(x, y, 3 - \frac{1}{4}x^2 - \frac{1}{3}y^2)$ over the triangle with vertices at (0, 0), (3, 0), (1, 2).

$SurfaceArea(Surface(\langle x, y, Z\rangle, [x, y] = Triangle(\langle 0, 0\rangle, \langle 3, 0\rangle, \langle 1, 2\rangle)), output = integral)$

$$-\left(\int_0^1 \int_{2x}^0 \frac{1}{6}\sqrt{9x^2+16y^2+36}\, dy\, dx\right) - \left(\int_1^3 \int_{3-x}^0 \frac{1}{6}\sqrt{9x^2+16y^2+36}\, dy\, dx\right)$$

$evalf(SurfaceArea(Surface(\langle x, y, Z\rangle, [x, y] = Triangle(\langle 0, 0\rangle, \langle 3, 0\rangle, \langle 1, 2\rangle)))) = 4.028651111$